MULTIPATH TIME-DELAY ESTIMATION FOR LONG DATA RECORDS

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ABSTRACT

We address the problem of multipath time-delay estimation. When the received data is very long compared to the transmitted signal, the data is expected to consist of a large number of paths. Modeling the entire data becomes computationally expensive. We propose a technique to break the data into short segments and model each segment individually without misfitting or truncating any paths at the ends of any segment. By effectively using overlapping segments, the estimates of time-delays are combined to model the entire data record. The method is extended to the case where only basebanded data are available. The proposed technique is demonstrated on an experimental sea-test data.

1. INTRODUCTION

In multipath time-delay estimation, the waveform $r(t)$ received at a single sensor is modeled as delayed and attenuated replicas of the transmitted signal. This is the result of multiple reflections and attenuation of the signal in the channel. The signal $r(t)$ could also be a beamformed combination of signals received at an array of sensors. It is described mathematically as

$$r(t) = \sum_{k=1}^{M} a_k s(t-\tau_k) + w(t) \quad , \quad 0 \le t \le T \qquad (1)$$

where $s(t)$ is the transmitted signal (pulse), a_k the amplitude (attenuation value) for path k, τ_k the time-delay for path k, M the number of different paths and $w(t)$ the white Gaussian noise corrupting the received signal. We assume that $s(t)$ and M are known and all signals are real-valued. The problem is to estimate the unknown parameters a_k and τ_k from the samples of the received signal.

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In practice, if signals are band pass, their basebanded versions are stored instead of the original signals. Hence only the samples of the basebanded signals are available to estimate the time-delays. Let $\hat{x}(t)$ denote the Hilbert transform of $x(t)$, and $x^a(t)$ $(= x(t) + j\hat{x}(t))$ denote the analytic signal of the real-valued signal $x(t)$. Let $x_{BB}(t)$ denote the baseband version of $x(t)$. It is defined as $x_{BB}(t) = \frac{1}{2}x^a(t)\exp(-j\omega_c t)$, where $\omega_c = 2\pi f_c$ is the center frequency in radians/sec. The baseband version of $y(t) = x(t-\tau)$ is $y_{BB}(t) = \frac{1}{2}y^a(t)\exp(-j\omega_c t) = x_{BB}(t-\tau)\exp(-j\omega_c\tau)$. So the baseband version of the multipath signal (1) is

$$r_{BB}(t) = \sum_{k=1}^{M} A_k s_{BB}(t-\tau_k) + w_{BB}(t) \qquad (2)$$

where $A_k = a_k \exp(-j\omega_c\tau_k)$. The amplitudes in the propagation model are real-valued since the signals $r(t)$ and $s(t)$ are real-valued. Accurate estimates are obtained by imposing real-amplitude constraints. In the original model (1) it amounts to constraining the a_k's to be real-valued whereas in the baseband model (2) it amounts to constraining the A_k's to be complex-valued with a phase angle of $-\omega_c\tau_k$ radians.

Since the noise is white Gaussian, the maximum-likelihood estimates (MLE) and the Least-Squares (LS) estimates of a_k's and τ_k's are the same. The LS estimates of the time-delays can be obtained by minimizing the M-dimensional LS error function. But when the observation interval $[0, T]$ is very large and contains a large number of paths, minimizing the M-dimensional LS error function is not feasible as it would require an enormous amount of computation.

We propose a technique of modeling the data in short segments without misfitting or truncating any paths at the ends of any segment. The estimates of time-delays from all the segments are then combined to model the entire data record. The technique is demonstrated on an experimental sea-test basebanded data.

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2. MODELING OF SHORT DATA SEGMENTS

Long data records may consist of too many paths. This would make modeling the entire data computationally expensive. It is more economical to break the data into short segments and then model each segment individually. Further, while selecting a segment of data, some paths could get truncated at the ends of the segment. So, to model a segment of data and also to account for the truncated paths, time domain weighting is introduced in the original Least-Squares (LS) error function as follows:

$$E(\mathrm{a}, \tau) = \int_{-\infty}^{\infty} g^2(t) \left| r(t) - \sum_{k=1}^{M} a_k s(t-\tau_k) \right|^2 dt \quad (3)$$

$$= \int_{-\infty}^{\infty} \left| g(t)r(t) - \sum_{k=1}^{M} a_k g(t) s(t-\tau_k) \right|^2 dt \quad (4)$$

where $g(t)$ is a non-negative window function which is non-zero only in the selected time interval $t_1 \le t \le t_2$. One could think of this as replacing the data $r(t)$ and each of the hypothesized path $s(t-\tau_k)$ by $g(t)r(t)$ and $g(t)s(t-\tau_k)$, respectively. Expressing the best amplitudes in terms of the delays, we get an error function which is a function of the delays alone ($E(\tau)$). The CRALS algorithm developed in [1] is used to determine the global minimum of the LS error function. Some ill-conditioning problem encountered in LS modeling is addressed in [2].

Suppose T_w is the duration of the transmitted pulse $s(t)$. A path at $t_1 - T_w$ would touch the beginning of the interval $[t_1, t_2]$ and a path at t_2 would touch the end of the interval $[t_1, t_2]$. By minimizing the error function in (4), all time-delays in the interval $[t_1 - T_w, t_2]$ are considered. In this manner, all paths which partly or completely lie in the selected interval of $[t_1, t_2]$ are accounted. The CRALS algorithm is applied by using the selected segment of data $g(t)r(t)$, and by windowing each hypothesized path $s(t-\tau_k)$ by $g(t)$ whenever the LS function and its Jacobian needs to be computed.

In applications such as radar, sonar, and geophysics, only band pass signals are used as they are capable of traveling long distance. Techniques like the Alternating Projection (AP) [3] and Estimate Maximize (EM) algorithms [4], when applied on the LS error function, would involve a sequence of one-dimensional searches of an oscillatory function. Hence they are likely to converge to local minima for closely-spaced paths. In contrast, the CRALS algorithm starts from the envelope of the error function and makes a transition to the actual error function. The CRALS imposes the real-amplitude constraint since the amplitudes in the propagation model are real-valued.

3. MODELING OF LONG DATA RECORD

The schematic diagram in Figure 1 shows the window function $g(t)$ and the time interval of one particular segment. It can be seen that paths with time-delays in the interval $[t_1 - T_w, t_1]$ would overlap in the selected interval of $[t_1, t_2]$. Similarly, paths with time-delays in the interval $[t_2 - T_w, t_2]$ would overlap in the selected interval. Estimates of paths which only partly lie in the selected interval are not as accurate as those of the paths which completely lie in the selected interval. Hence we do not keep the estimates from the "unreliable regions", $[t_1 - T_w, t_1]$ and $[t_2 - T_w, t_2]$, indicated by shaded regions in Figure 1. The next segment is chosen such that it overlaps with the previous one and it covers $[t_2 - T_w, t_2]$ reliably. That is, the new $t_1 := t_2 - T_w$.

While modeling the data in the interval $[t_1, t_2]$ it is convenient to consider $t_1 - T_w$ as the new time origin. However caution must be exercised in imposing the real-amplitude constraint on the baseband model. Suppose t_0 is the new time origin. The new variables for time and delay are $t' = t - t_0$ and $\tau'_k = \tau_k - t_0$, respectively. Substituting these variables in (2), we get

$$\exp(j\omega_c t_0) r_{BB}(t' + t_0) = \sum_{k=1}^{M} A'_k s_{BB}(t' - \tau'_k) + \exp(j\omega_c t_0) w_{BB}(t) \quad (5)$$

where $A'_k = a_k \exp(-j\omega_c \tau'_k)$. By multiplying the data with the the complex scale factor of $\exp(j\omega_c t_0)$, the real-amplitude constraint can be imposed in a similar manner as in (2). For the selected interval $[t_1, t_2]$, the new time origin is chosen as $t_0 = t_1 - T_w$.

4. EXAMPLE WITH BASEBANDED EXPERIMENTAL DATA

The experimental sea-test data used here was provided by the Naval Undersea Warfare Center (NUWC) in Newport, RI. The original received (real-valued) signal $r(t)$ is a narrow band signal with a center frequency of about $f_c = 21.7$ kHz. It is sampled at 80 kHz. Instead of storing the samples of the original $r(t)$, its basebanded version $r_{BB}(t)$ is resampled at a much lower rate and stored. The original modulated signal is multiplied by $\exp(-j2\pi f_c)$, to shift the signal in frequency to the baseband level. It is then decimated by a factor of 16 by low pass filtering and resampling at a lower rate of $80/16 = 5$ kHz. The basebanded signal is complex-valued in general. Figure 3(a) shows the magnitude of

$s_{BB}(t)$. Figure 3(b) shows the magnitude of $r_{BB}(t)$ in solid line. The transmitted signal is also basebanded by the same procedure. The sampling interval of the basebanded signals is $16T_s$, where T_s is the original sampling interval.

The receiver is a 2-dimensional (2-D) array consisting of 52 sensors. A 2-D beamformed signal looking at a particular direction is used in this example as the received signal. The basebanded received signal $r_{BB}(t)$ is 2048 samples long while $s_{BB}(t)$ is only 13 samples long. For sake of illustration, the data is modeled over three overlapping segments. The first segment is $[200 \times 16T_s, 254 \times 16T_s]$. The windowed $r_{BB}(t)$ is zero-padded in the beginning by $T_w/16T_s = 13$ and its DFT is used in the CRALS algorithm [5]. This is equivalent to using original band pass signals. The segment is fitted with 10 paths. The delays in the interval [241, 254] samples are replaced by those from second segment, which is chosen as [141, 295]. Again the delays in the unreliable region are replaced by those from the third segment, which is chosen as [282, 336]. The signal is reconstructed from the estimated amplitudes and time-delays. Figure 3(b) shows the magnitude of the reconstructed signal in dotted line and Figure 3(c) shows the magnitude of the residue after fitting with 19 paths. A plot of the amplitudes and time-delays is shown in Figure 2.

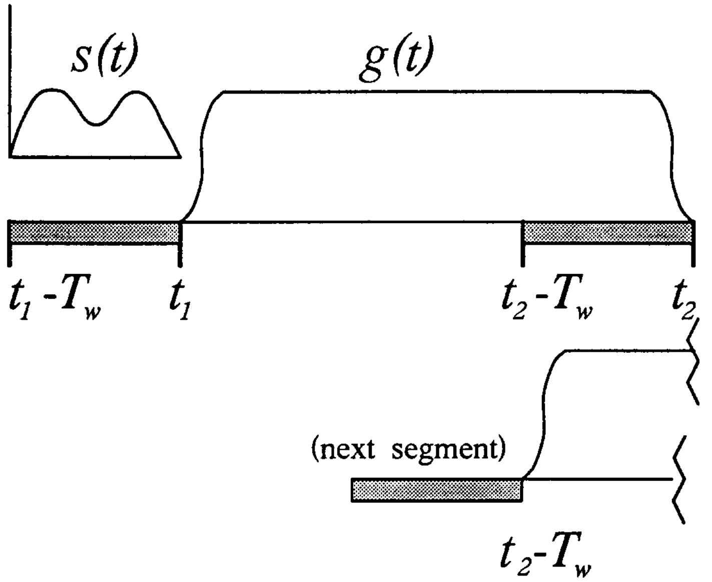
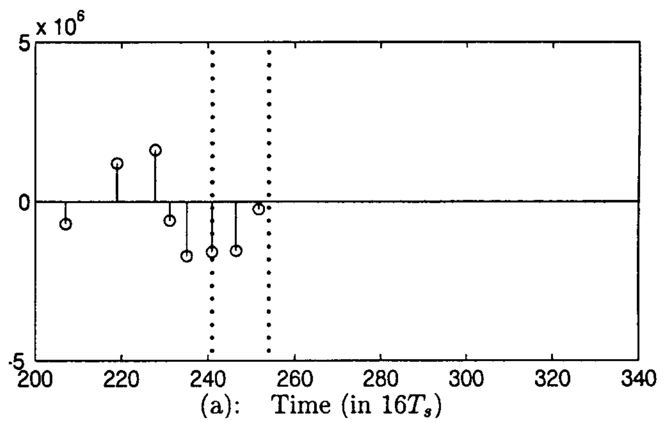


Figure 1: Schematic diagram to show the window function $g(t)$ and the transmitted pulse $s(t)$ on one particular segment. The next segment is partially shown.



5. REFERENCES

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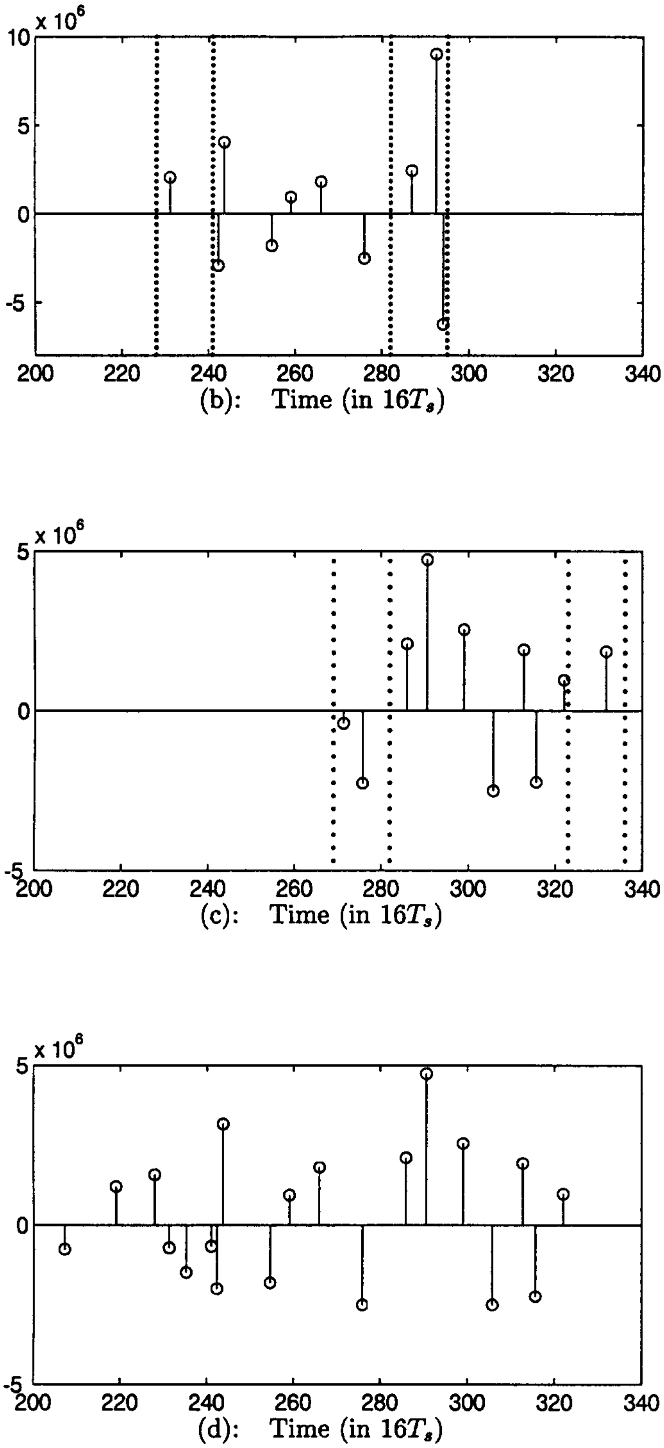

Figure 2: Stem plot of the estimated time-delays and amplitudes in the (a) first segment; (b) second segment; (c) third segment; (d) all the three segments combined. The regions shown within vertical dotted lines are the "unreliable regions."

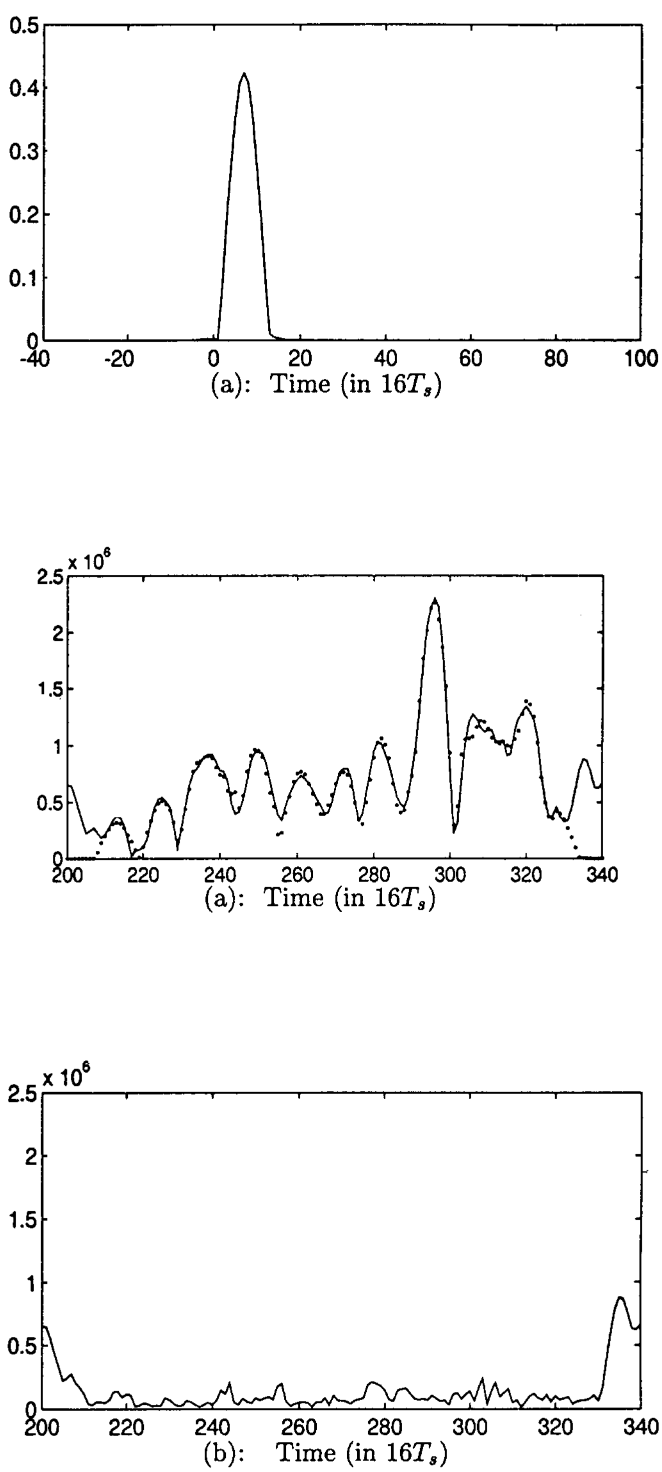

Figure 3: Magnitude of basebanded (a) transmitted signal $s_B B(t)$, (b) received (solid line) and reconstructed signal (dotted line), and (c) residue.