PASSIVE TARGET MOTION ANALYSIS USING MULTIPATH DIFFERENTIAL TIME-DELAY AND DIFFERENTIAL DOPPLER SHIFTS

P. BLANC–BENON AND G. BIENVENU

Thomson-Sintra ASM, BP.57, 06903 Sophia-Antipolis Cedex, France

ABSTRACT

Target Motion Analysis (TMA) is a basic function in passive SONAR, generally using bearings only or bearings and frequency measurements. But due to the arrays whose aperture are practically negligeable considering the target range, and even if the platform moves itself to yield a "synthetic array", the classical TMA methods take a few ten minutes to give an acceptable solution. Hence, this paper presents an enhanced TMA estimator using jointly the bearings and multipath parameters: the differential time-delays and their doppler shifts. The Cramer-Rao lower bounds are studied for two cases of sound propagation: a constant celerity profile and a bilinear one. They both exhibit advantages in terms of a shorter time to get a given precision on the target parameters: its range, depth, and speed vector.

1. INTRODUCTION

Passively locating an acoustic source (the target) which radiates a broadband noise with eventually stable lines is known to be a difficult task specially for actual at-sea situations. The basics of bearings-only TMA has been addressed in a major publication from Nardone et al. [1] in 1984, stating both the problem of target observability and the Cramer-Rao bounds, and establishing a class of efficient batch non-linear estimators (in comparison with extended Kalman filters). The attempt to increase the global TMA performance leads to either the optimization of the platform maneuvers [2] or the addition of new measurements as frequency lines [3], or even to the addition of other platforms [4], [5]. However, specific applications require a quicker identification of the target with a single antenna: the use of the multipath is an answer [6], [7], [8].

In Sec. 2, we introduce the measurement equations for the bearings and for the differential time-delays and their doppler shifts. Sec. 3 states the problem of estimating the 3-dimensional target position and speed via the measurements collected at sample times. Cramer-Rao bounds are established for both a constant sound speed profile (SSP) and a bilinear SSP. Sec. 4 gives results and compare the contribution of the differential doppler shifts during time.

2. PROBLEM STATEMENT

Given a 3-axis coordinate system, we simply recall the primary model and measurement equations used in TMA from bearings at least. The passive array is built to deliver bearings relatively to the platform (via eg. classical beamforming). Knowing its exact position at each time t, the true bearing measurement equation states as

$$\beta(t,X) = \arctan\left(\frac{x(t)-x_P(t)}{y(t)-y_P(t)}\right) , \quad (1)$$

$$\beta_m(t) = \beta(t,X)+\eta(t) , \quad (2)$$

where (x,y) and (x_P,y_P) respectively are the target and platform horizontal positions, η the zero-mean Gaussian process noise with known covariance σ_β^2; X designating the complete unknown state vector relatively to the target for a reference time t^*

$$X = [x(t^*), y(t^*), \dot{x}, \dot{y}, z]' . \quad (3)$$

Due to the non-observability of the situation (even estimating only (x,y) from a single bearing), the target is assumed to move along a horizontal straight line with a constant speed. This weak hypothesis explicits the form of the state vector X which becomes now observable (excepted z) from a bearing time history, provided the platform maneuvers sufficiently (ie. in a non ambiguous way) [9]. Nevertheless one easily admits that a minimum few minutes is necessary to satisfy such requirements. And that explains the resort to additive measurements such as multipath. Let us introduce the model equations for the differential multipath time-delays and their doppler shifts. Denoting by T the time-delay for an eigenray path to go exactly from the source to the platform and by τ the differential time-delay between such two paths say i and j, we have for a constant SSP ($c(z)=c_0$)

$$T(t,X) = \frac{1}{c_0}\sqrt{R^2+Z^2} , \quad (4)$$

$$\tau(t,X) = T_i(t,R,z)-T_j(t,R,z) , \quad (5)$$

where $R=\sqrt{(x-x_P)^2+(y-y_P)^2}$ is the horizontal range between the target and the platform, and Z the differential depth between the source and the acoustical image of the platform according to the actual number of surface and/or bottom reflections. Furthermore, in the constant SSP case this differential depth is easily expandable using a few parameters (ε, n_B, n_S) characterizing each ray

$$Z = 2\varepsilon n_B z_B + (-1)^{n_B+n_S} z_P - z , \quad (6)$$

 0-7803-2431-5/95 $4.00

with z_B and z_P being the bottom and platform depths, ϵ = +1 (resp. -1) if the ray goes down (resp. up) starting from the source, n_B (resp. n_S) being the number of bottom (resp. surface) reflections.

For a bilinear SSP, when the propagation is no longer rectilinear, the eigenrays time-delays are much more complicated [10] but can be expressed in terms of the target depth z and the elevation angle θ at the source

$$T = H(\theta, z) , \quad (7)$$

$$\theta = G^{-1}(R, z) , \quad (8)$$

the detailed expressions for the function G and H are given in appendix, being understood that a set of parameters omitted here (see appendix) enables to distinguish specific functions G and H for each existing eigenray. In the same manner, the differential doppler shift attached to one preceeding τ measurement is

$$\dot{\tau}(t, X) = \frac{\partial T_i(t, X)}{\partial t} - \frac{\partial T_j(t, X)}{\partial t} , \quad (9)$$

where $\dot{T} = \frac{\partial T}{\partial t}$ is

$$\dot{T}(t, X) = \frac{1}{c_0} \frac{R\dot{R}}{\sqrt{R^2 + Z^2}} , \quad (10)$$

omitting t dependance in the right part of the equation for the sake of simplicity, and with $R\dot{R} = (x - x_P)(\dot{x} - \dot{x}_P) + (y - y_P)(\dot{y} - \dot{y}_P)$, $\dot{R}$ being the relative horizontal range rate. Finally considering additive noise processes ζ and ξ, the measurement equations are

$$\tau_m(t) = \tau(t, X) + \zeta(t) , \quad (11)$$

$$\dot{\tau}_m(t) = \dot{\tau}(t, X) + \xi(t) , \quad (12)$$

ζ and ξ are independant [11] with known covariances σ_τ^2 and $\sigma_{\dot{\tau}}^2$. These time-delays and doppler shifts measurements result from an autocorrelation of the signal at the beam output in the target direction .

3. TARGET PARAMETERS ESTIMATION IN THE PRESENCE OF MULTIPATH

As being introduced before, a collected batch of measurements $\{\beta_m(t), \tau_m(t), \dot{\tau}_m(t); t = t_1, \ldots, t_N\}$ serves as a $3N \times 1$ vector of observations M_m in the maximum likelihood estimation (MLE) of the 5×1 state vector X. In fact assuming the Gaussian nature of noise, the MLE $\widehat{X}$ reduces to a nonlinear least squares estimate by classically considering the log-likelihood of the measurements

$$\widehat{X} = \arg\min_X Q(X) , \quad (13)$$

where $Q(X)$ is the following quadratic criterion

$$Q(X) = \|M_m - M(X)\|_\Sigma^2 , \quad (14)$$

Σ denoting the diagonal covariance matrix of the measurement vector, made of the $\sigma_\beta(t_i)$, $\sigma_\tau(t_i)$, and $\sigma_{\dot{\tau}}(t_i)$, and $M(X)$ the noiseless observation vector. Rather than producing detailed explanations about $\widehat{X}$, which can be easily computed from usual optimization routines, it is worth to discuss much about the Cramer-Rao lower bound (CRLB) connected to this estimation problem.

Recalling the definition of the Fisher information matrix (FIM), one have

$$CRLB(X) = \left[\frac{\partial M}{\partial X}^t \Sigma^{-1} \frac{\partial M}{\partial X} \right]^{-1} . \quad (15)$$

So the CRLB computation consists mainly in expliciting the various first order partial derivatives of the observation vector versus the 5 unknown state vector components, in order to elaborate the Jacobian matrix $\frac{\partial M}{\partial X}$.

3.1. The constant SSP case

Considering the definition of the CRLB, we give the following expresions for the first order partial derivatives which compose the Jacobian matrix, ie. relatively to x, y, $\dot{x}$, $\dot{y}$ and z taken at each sample time, with $D = \sqrt{R^2 + Z^2}$ designating the actual distance between the source and the platform

$$\frac{\partial \beta}{\partial x} = \frac{y}{R^2} \quad \frac{\partial \beta}{\partial y} = -\frac{x}{R^2} \quad \frac{\partial \beta}{\partial z} = 0 \quad (16)$$

$$\frac{\partial \beta}{\partial \dot{x}} = t \frac{y}{R^2} \quad \frac{\partial \beta}{\partial \dot{y}} = -t \frac{x}{R^2} \quad (17)$$

$$\frac{\partial \tau}{\partial x} = \frac{1}{c_0} x [D]_j^i \quad \frac{\partial \tau}{\partial y} = \frac{1}{c_0} y [D]_j^i \quad \frac{\partial \tau}{\partial z} = \frac{1}{c_0} [Z D^{-1}]_j^i \quad (18)$$

$$\frac{\partial \tau}{\partial \dot{x}} = t \frac{\partial \tau}{\partial x} \quad \frac{\partial \tau}{\partial \dot{y}} = t \frac{\partial \tau}{\partial y} \quad (19)$$

$$\frac{\partial \dot{\tau}}{\partial x} = \frac{1}{c_0} \left[\dot{x} D^{-1} - x R\dot{R} D^{-3} \right]_j^i \quad (20)$$

$$\frac{\partial \dot{\tau}}{\partial y} = \frac{1}{c_0} \left[\dot{y} D^{-1} - y R\dot{R} D^{-3} \right]_j^i \quad \frac{\partial \dot{\tau}}{\partial z} = \frac{R\dot{R}}{c} [Z D^{-3}]_j^i \quad (21)$$

$$\frac{\partial \dot{\tau}}{\partial \dot{x}} = t \frac{\partial \dot{\tau}}{\partial x} + \frac{x}{c_0} [D^{-1}]_j^i \quad \frac{\partial \dot{\tau}}{\partial \dot{y}} = t \frac{\partial \dot{\tau}}{\partial y} + \frac{y}{c_0} [D^{-1}]_j^i \quad (22)$$

Notice that $[u]_j^i$ stands for $u_i - u_j$, and also that we use t, x, y, $\dot{x}$, $\dot{y}$ instead of $t - t^*$, $x - x_P$, $\dot{x} - \dot{x}_P$, $\dot{y} - \dot{y}_P$ respectively for simplicity in the right part of the previous equations.

3.2. The bilinear SSP case

Here the computation of the CRLB turns out to be feasible, even if the time-delay is an implicit function of the unknown X: it does not require any numerical inversion since only the function G occurs (and not the inverse G^{-1}). Furthermore software enables to safely calculate the following expressions in a symbolic way [12]. As for the constant SSP we can explicit now the first order partial derivatives for the time-delays and the doppler shifts, with the same conventions as taken previously (the bearings derivatives being unchanged)

$$\frac{\partial \tau}{\partial x} = \frac{x}{R} \left[H_\theta G_\theta^{-1} \right]_j^i \quad \frac{\partial \tau}{\partial y} = \frac{y}{R} [H_\theta G_\theta^{-1}]_j^i \quad (23)$$

$$\frac{\partial \tau}{\partial z} = \left[H_z - H_\theta \frac{G_z}{G_\theta} \right]_j^i \quad \frac{\partial \tau}{\partial \dot{x}} = t \frac{\partial \tau}{\partial x} \quad \frac{\partial \tau}{\partial \dot{y}} = t \frac{\partial \tau}{\partial y} \quad (24)$$

$$\frac{\partial \dot{\tau}}{\partial x} = \left[\frac{x\dot{R}}{R} \left(\frac{H_{\theta\theta}}{G_\theta^2} - \frac{H_\theta G_{\theta\theta}}{G_\theta^3} \right) + \frac{y}{R^3} (\dot{x} y - x \dot{y}) \frac{H_\theta}{G_\theta} \right]_j^i \quad (25)$$

$$\frac{\partial \dot{\tau}}{\partial y} = \left[\frac{y\dot{R}}{R}\left(\frac{H_{\theta\theta}}{G_\theta^2} - \frac{H_\theta G_{\theta\theta}}{G_\theta^3}\right) - \frac{x}{R^3}(\dot{x}y - x\dot{y})\frac{H_\theta}{G_\theta}\right]_j^i \quad (26)$$

$$\frac{\partial \dot{\tau}}{\partial z} = \left[(H_{\theta z} - \frac{H_{\theta\theta}G_z}{G_\theta})\frac{\dot{R}}{G_\theta} - \frac{\dot{R}H_\theta}{G_\theta^2}(G_{\theta z} - \frac{G_{\theta\theta}G_z}{G_\theta})\right]_j^i \quad (27)$$

$$\frac{\partial \dot{\tau}}{\partial \dot{x}} = t\frac{\partial \dot{\tau}}{\partial x} + \frac{x}{R}[H_\theta G_\theta^{-1}]_j^i \; \frac{\partial \dot{\tau}}{\partial \dot{y}} = t\frac{\partial \dot{\tau}}{\partial y} + \frac{y}{R}[H_\theta G_\theta^{-1}]_j^i \quad (28)$$

where the symbols A_u and A_{uv} denote respectively the first order partial derivative of the function A versus u and the second order partial derivative versus u and v.

4. RESULTS

The contribution of the differential doppler shifts has been analyzed on a short duration for a 40 knots, 60 m depth target going South and firstly detected in the azimuth 45 deg for 5 km. The platform is moving at 5 knots on course 300 deg, at 120 m depth. The sample time is 8 sec, with $\sigma_\beta = 0.1$ deg, $\sigma_\tau = 0.5\ 10^{-4}$ sec, and $\sigma_{\dot{\tau}} = 2.2\ 10^{-5}$. Two cases are studied: a constant SSP (1500 m/s) and a bilinear SSP ($g_1 = -0.5$ and $g_2 = 0.0174\ ms^{-1}/m$). Three eigenrays are considered: the direct path (D), the bottom reflected path (B), and the doubly bottom single surface reflected path (BSB). These rays constitute two time-delays denoted D/B and D/BSB. Figs. 1 to 4 present the 1-sigma CRLB with doppler shifts and Figs. 5 to 8 the CRLB ratios (without/with doppler shifts) for the four interesting components during time starting from 8 sec up to 64 sec: the target depth relatively to the bottom depth ($z_B = 2400$ m), the relative range, the course and the relative speed. The improvement naturally appears on the very beginning of the TMA (being infinite for a single time) and identically for both SSP's. Even when the performance is poor on the location parameters, the CRB for the target course is sufficient enough to determine whether it comes towards the ownship or not.

5. CONCLUSION

This paper has contributed firstly to establish the Cramer-Rao bounds for TMA using jointly bearings, multipath differential time-delays and their doppler shifts, showing here the improvement due to the doppler shifts specially for short scenarios and fast targets, and secondly to introduce a bilinear sound speed profile into such computation without using ray-tracing programs.

ACKNOWLEDGMENT

This work was partly supported by DRET (French MOD, Paris, France).

APPENDIX: EIGENRAYS IN A BILINEAR SSP

We detail the analytical computation of the two functions G and H introduced in Sec. 3.2, the following equations and notations being largely borrowed from [10]. Denoting by c, c_P, c_1, c_2, c_3 the sound speed respectively at the source depth z, the platform depth z_P, the sea surface depth z_1, the minimum of celerity depth z_2, and at the bottom depth z_3 (assuming $c_1 > c_3 > c_2$, otherwise exchange the bottom and the surface, and z positive downwards), and by g_1, g_2 the algebraic gradients of celerity in the surface layer and in the deep layer, one recalls the expressions obtained in [10] for the eigenrays

$$\begin{aligned} R = \frac{\sec\theta}{2c_P}\{&\frac{1}{g_1}[(-1-\alpha)\sin\theta - 2\nu(2L-\gamma+\delta)\sin\theta_1 \\ &+(1+\beta)\sin\theta_P] - \frac{1}{g_2}[(1-\alpha)\sin\theta + (\beta-1)\sin\theta_P \\ &-4\mu L\sin\theta_3] + (\frac{1}{g_1} - \frac{1}{g_2})(4L-2+\sigma)\sin\theta_2\}, \end{aligned} \quad (29)$$

where R is the horizontal range between the source (emitter) and the array (receiver), and $\theta, \theta_P, \theta_1, \theta_2, \theta_3$ the elevation angles of the eigenray respectively at the source, at the observer, and when intersecting the surface, the minimum of speed and the bottom interfaces. These elevation angles (positive downwards) can be expressed in function of θ and of the SSP (from Snell's law)

$$\sin\theta_P = \delta\frac{c_P}{c}\sqrt{\left(\frac{c}{c_P}\right)^2 - 1 + \sin^2\theta}\ , \quad (30)$$

$$\sin\theta_i = \frac{c_i}{c}\sqrt{\left(\frac{c}{c_i}\right)^2 - 1 + \sin^2\theta}, \text{ for } i = 1,2,3\ . \quad (31)$$

The other parameters, which control the type of eigenray, are now explained for the SOFAR rays, the surface-reflected-bottom-reflected rays (SRBR), the refracted-bottom-reflected rays (RBR), and the refracted-surface-reflected rays (RSR)

$$\alpha = \pm 1 \text{ if } z \gtrless z_2,\ \beta = \pm 1 \text{ if } z_P \gtrless z_2\ , \quad (32)$$

$$\gamma = \text{sign}(\theta),\ \delta = \text{sign}(\theta_P), \quad (33)$$

$$\sigma = |\alpha+\gamma| + |\beta-\delta| - |\gamma+\delta|, \quad (34)$$

$$(\mu,\nu)_{SOFAR} = (0,0),\ (\mu,\nu)_{RBR} = (1,0), \quad (35)$$

$$(\mu,\nu)_{RSR} = (0,1),\ (\mu,\nu)_{SRBR} = (1,1), \quad (36)$$

where L is the number of bottom reflections or bottom turning points (refractions). The right part of (29) constitutes the function G as used in the main sections (Eq. 8). The second important function which characterizes the eigenrays is the propagation travel time (the companion function H in (7)) as

$$\begin{aligned} T = \ &\frac{1}{g_1}\ln\left[\Psi(\theta)^{-\frac{1+\alpha}{2}}\Psi(\theta_P)^{\frac{1+\beta}{2}}\Psi(\theta_1)^{-\nu(2L-\gamma+\delta)}\right] \\ &-\frac{1}{g_2}\ln\left[\Psi(\theta)^{\frac{1-\alpha}{2}}\Psi(\theta_P)^{\frac{\beta-1}{2}}\Psi(\theta_3)^{-2\mu L}\right] \\ &+(\frac{1}{g_1} - \frac{1}{g_2})\ln\left[\Psi(\theta_2)^{2P-1+\frac{\sigma}{2}}\right]\ , \end{aligned} \quad (37)$$

where $\Psi(x) \stackrel{\text{def}}{=} \tan(\frac{x}{2} + \frac{\pi}{4})$.

REFERENCES

[1] S.C. Nardone, A.G. Lindgren, and K.F. Gong, "Fundamental properties and performance of conventional bearings-only tracking", *IEEE trans. on Autom. Control*, vol. 29, 775-787 (Sept. 1984).

[2] S.E. Hammel, P.T. Liu, E.J. Hilliard, K.F. Gong, "Optimal observer motion for localization with bearings measurements", *Computers and Mathematics with applications*, Vol-18, 1989, 171-180.

[3] J.M. Passerieux, D. Pillon, P. Blanc-Benon, and C. Jauffret, "Target motion analysis with bearing and frequency measurement", In *Proc. of the 22nd Asilomar Conf. on SSC.* (Pacific Grove, CA, Nov.1988).

[4] Y. Chocheyras, "Time-delay and differential doppler tracking on real data", In *Proc. of the Underwater and Defence Technology Conf.* (London, UK, Feb. 1990).

[5] J.F. Arnold, Y. Bar Shalom, R. Estrada, and R.A. Mucci, "Target parameter estimation using measurements acquired with a small number of sensors", *IEEE J. Ocean. Eng.*, vol. 8, no. 3 (July 1983).

[6] B. Friedlander, "Accuracy of source location using multipath delays", *IEEE trans. on Aerosp. Electron. Syst.*, vol. 24 no. 4, 346-359 (1988).

[7] M.J.D. Rendas, and M.F. Moura, "Cramer-Rao bound for location systems in multipath environments", *IEEE trans. on Signal Processing*, vol. 39, 2593-2610 (Dec. 1991).

[8] P. Blanc-Benon, "Passive target motion analysis mixing conventional bearing measurements and multipath parameters", In *Proc. of the Underwater and Defence Technology Conf.*, London, UK, July 1994, 428-432.

[9] S.C. Nardone, V.J. Aidala, "Observability criteria for bearings-only target motion analysis", *IEEE trans. on AES*, vol. 17, No 2, March 1981, 162-166.

[10] R.N. Baer, and M.J. Jacobson, "Sound transmission in a channel with bilinear sound speed and environmental variations", *J. Acoust. Soc. Am.* 54, 80-91 (1973).

[11] P.M. Schultheiss, E. Weinstein, "Lower bounds of the localization errors of a moving source observed by a passive array", *IEEE trans. on ASSP*, vol. 29, No 3, June 1981, 600-607.

[12] MAPLE User's guide, Waterloo Maple Software (Ontario, Canada, 1991).

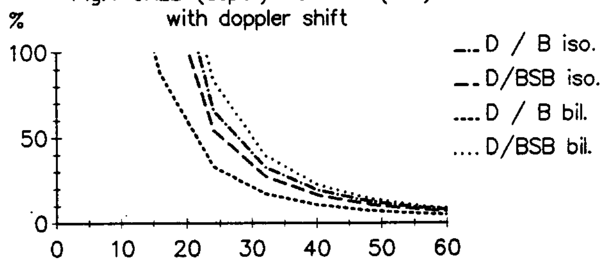

Fig.1 CRLB (depth) .vs. time (sec) with doppler shift

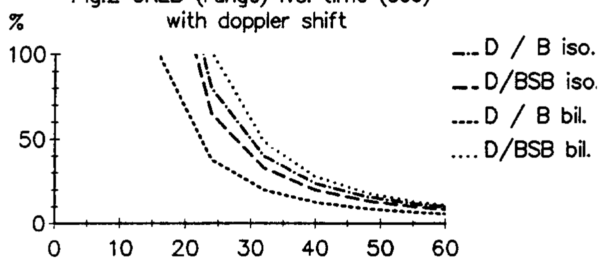

Fig.2 CRLB (range) .vs. time (sec) with doppler shift

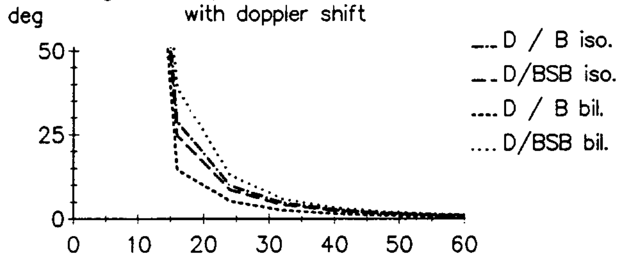

Fig.3 CRLB (course) .vs. time (sec) with doppler shift

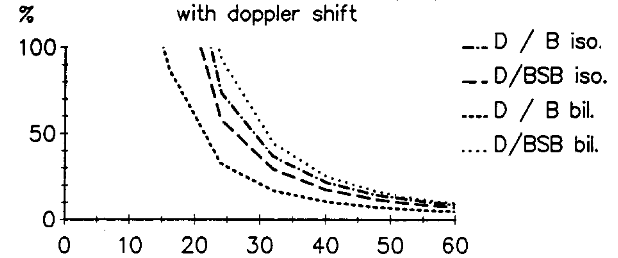

Fig.4 CRLB (speed) .vs. time (sec) with doppler shift

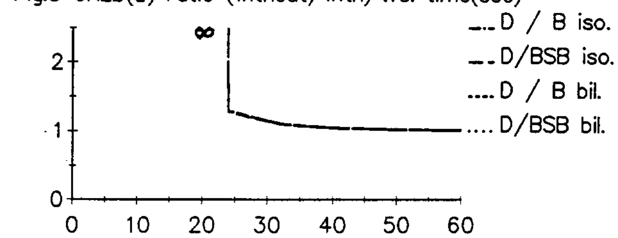

Fig.5 CRLB(z) ratio (without/with) .vs. time(sec)

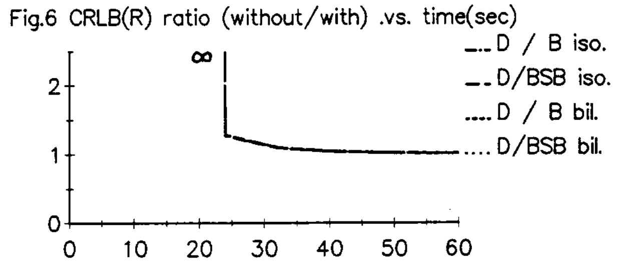

Fig.6 CRLB(R) ratio (without/with) .vs. time(sec)

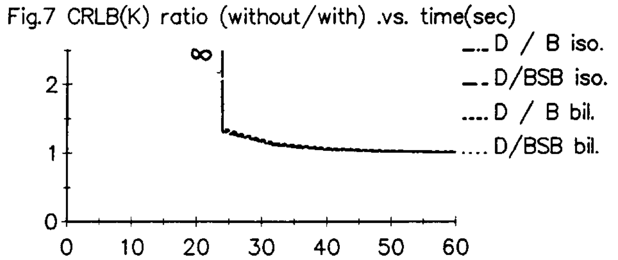

Fig.7 CRLB(K) ratio (without/with) .vs. time(sec)

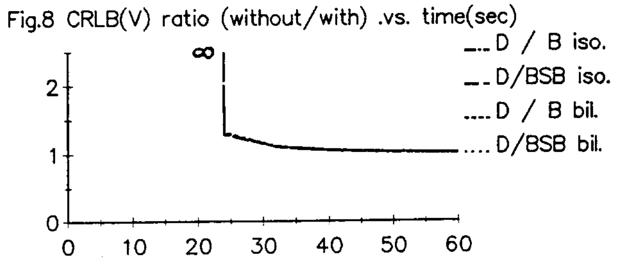

Fig.8 CRLB(V) ratio (without/with) .vs. time(sec)