MAXIMUM *A POSTERIORI* PROBABILITY ESTIMATION OF SEAFLOOR MICROROUGHNESS PARAMETERS FROM BACKSCATTER SPATIAL COHERENCE

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ABSTRACT

A technique is presented for the estimation of a set of parameters associated with a geologically motivated model for seafloor microroughness due to Goff and Jordan [1]. The method seeks to connect the spatial covariance of the backscattered acoustic field with the correlation properties of the seafloor by constructing the *a posteriori* probability density function (pdf) of the parameters that define the seafloor microroughness wavenumber spectrum. The processor maximizes the joint *a posteriori* probability density of the model parameter set. Due to the complexity of the probability surface, the method of simulated annealing is used to search for the globally optimum solution vector.

1. INTRODUCTION

High frequency acoustic remote sensing of the statistical variability in small-scale seafloor relief represents an important mechanism for improving our understanding of marine geological relief forming processes such as sediment deposition, abyssal circulation, and bioturbation. In this work, a technique is presented for the estimation of a set of five parameters associated with a geologically motivated model for seafloor microroughness due to Goff and Jordan [1]. The approach operates on full field (amplitude and phase) observations of the bottom interacting acoustic field and is based on the premise that statistical fluctuations of the amplitude and phase of bottom reverberation will be influenced mainly by bottom features on the order of the wavelength of the acoustic carrier signal. In effect, the method attempts to connect the spatial covariance of the backscattered acoustic field with the correlation properties of the seafloor by constructing the *a posteriori* probability density function (pdf) of a set of parameters that define the seafloor microroughness wavenumber spectrum. Observations of acoustic backscatter received at a horizontal array are simulated using full-field acoustic modeling derived from the 3-D Kirchhoff approximation to the Helmholtz integral equation. The optimum processor maximizes the joint *a posteriori* probability density of the 5-dimensional surface roughness parameter set. Due to the complexity of the objective function, the method of simulated annealing is used to search for the globally optimum solution vector. The MAP estimation of anisotropic Goff-Jordan seafloor roughness spectrum parameters from acoustic backscatter was first proposed by Premus and Alexandrou [2]. In that work, the authors used observations of backscattering strength angular dependence to infer seafloor roughness statistics. The work described herein extends the earlier treatment of [2] by exploiting spatial coherence information contained in the backscattered acoustic field.

2. THE GOFF-JORDAN SURFACE MODEL

The surface parameterization employed in this work, due to Goff and Jordan [1], is an anisotropic fractal-based description of seafloor geomorphology with five free parameters to control the correlation properties of the surface roughness: rms height H, cross-lineation characteristic wavenumber k_n, along-lineation characteristic wavenumber k_s, lineation direction ζ_s, and fractal dimension D. The 2-D wavenumber spectrum assumes an anisotropic power-law form given by

$$P_h(\mathbf{k}) = 4\pi\nu H^2 \mid \mathbf{Q} \mid^{-\frac{1}{2}} [u^2(\mathbf{k}) + 1]^{-(\nu+1)} \quad (1)$$

where ν is related to the fractal dimension, $\mathbf{Q}$ is the scale matrix related to the characteristic wavenumbers k_n and k_s, and $u(\mathbf{k})$ represents the dimensionless norm of the wavenumber $\mathbf{k}$.

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The capability of the model to simulate naturally occurring seafloor microroughness is illustrated in Figure 1. This figure depicts a sedimented seafloor province with a highly lineated, rippled appearance attributable to the influence of abyssal currents on the distribution of sediments [3]. This surface realization is characterized by the parameter set $H = .0125\ m$, $k_n = 2.5\ cycles/m$, $k_s = 0.5\ cycles/m$, $D = 1.6$, and $\zeta_s = 45°$, measured clockwise from the y-axis. Note that the axes of the plot are normalized with respect to an acoustic wavelength. This scaling convention serves to emphasize the fact that the surface roughness to which the Bayesian processor will be sensitive is of the order of the sonar carrier wavelength.

3. THE HELMHOLTZ-KIRCHOFF ACOUSTIC MODEL

Physical modeling of the scattered acoustic field is based on the 3-dimensional Kirchhoff approximation to the Helmholtz integral equation. In the Kirchhoff formulation, the boundary condition at the interface is approximated by equating the scattered field at each element of the rough interface by that which would exist if the scattering element were part of an infinite plane tangent to the surface at the given point. From Beckmann and Spizzichino [4], the Kirchhoff solution for the field scattered from a randomly rough interface is given by

$$\begin{aligned} p(\mathbf{r}) &= \frac{ike^{ik(r'+r_1)}}{2\pi r'r_1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D(x,y)\Re(x,y) \\ &\times \quad (\alpha\xi_x + \beta\xi_y + \gamma)\exp[ik(\alpha x + \beta y - \gamma\xi)]dxdy \end{aligned} \tag{2}$$

where ξ, ξ_x, and ξ_y represent the surface height, x-gradient, and y-gradient, respectively; α, β, and γ represent the x, y, and z components of the unit difference between the incident and scattered wavevectors; D is a Gaussian beampattern introduced to suppress edge effects; and $\Re$ is the local plane-wave reflection coefficient. Full field realizations of acoustic backscatter are simulated by applying (2) to independent realizations of Goff-Jordan type seafloor microroughness derived from (1).

4. *MAP* ESTIMATION OF SURFACE ROUGHNESS PARAMETERS VIA SIMULATED ANNEALING

From Bayes' Theorem, the *a posteriori* probability density function of the surface roughness parameter set $\mathbf{\Psi}$ can be written as

$$p_{\mathbf{\Psi}|\mathbf{X}}(\mathbf{\Psi} \mid \mathbf{X}) = \frac{p_{\mathbf{X}|\mathbf{\Psi}}(\mathbf{X} \mid \mathbf{\Psi})p_{\mathbf{\Psi}}(\mathbf{\Psi})}{\int_{\mathbf{\Psi}} p_{\mathbf{X},\mathbf{\Psi}}(\mathbf{X},\mathbf{\Psi})d\mathbf{\Psi}} \tag{3}$$

where $p_{\mathbf{X}|\mathbf{\Psi}}(\mathbf{X} \mid \mathbf{\Psi})$ represents the likelihood function of the observation vector $\mathbf{X}$, and $p_{\mathbf{X},\mathbf{\Psi}}(\mathbf{X},\mathbf{\Psi})$ is the joint pdf of the observation vector and the model parameters.

The likelihood function characterizes the stochastic nature of the scattered field due to interaction with the randomly rough interface. It is of central importance to the optimum processor as it is the mechanism through which the data $\mathbf{X}$ modifies the prior state of knowledge of $\mathbf{\Psi}$ [5]. If we assume the *a priori* distribution $p_{\mathbf{\Psi}}(\mathbf{\Psi})$ to be non-informative, that is uniform, corresponding to maximum uncertainty regarding the knowledge of $\mathbf{\Psi}$ in the absence of any observed data, then the *a posteriori* pdf of $\mathbf{\Psi}$ will be proportional to the likelihood function

$$p_{\mathbf{\Psi}|\mathbf{X}}(\mathbf{\Psi} \mid \mathbf{X}) = Cp_{\mathbf{X}|\mathbf{\Psi}}(\mathbf{X} \mid \mathbf{\Psi}). \tag{4}$$

From point scattering theory, the likelihood function of the complex pressure field observation vector $\mathbf{r}$ given the surface model parameters $\mathbf{\Psi}$ is N-dimensional, and jointly Gaussian

$$p(\mathbf{r} \mid \mathbf{\Psi}) = [(2\pi)^{N/2} \mid \mathbf{Q}(\mathbf{\Psi}) \mid^{\frac{1}{2}}]^{-1}\exp[-\frac{1}{2}\mathbf{r}^*\mathbf{Q}^{-1}(\mathbf{\Psi})\mathbf{r}] \tag{5}$$

where $\mathbf{\Psi} = \{H,\ k_n,\ k_s,\ \zeta_s,\ D\}$ and $\mathbf{Q}(\mathbf{\Psi})$ represents the spatial covariance of the scattered field under the Helmholtz-Kirchhoff formulation [6]. In the event of uniform *a priori* knowledge of $\mathbf{\Psi}$, equation (5) represents the objective function to be maximized.

The energy function defined in (5) is a multi-modal probability surface defined over a five-dimensional space. As a result, standard gradient-based search techniques techniques cannot be used to perform the maximization, as they would invariably get trapped in local maxima. Instead, the method of simulated annealing is used to search (5) for the globally optimum solution vector. The annealing procedure is an iterative process which involves the random sequential perturbation of the parameter set and repeated evaluation of the energy function. The power of the method lies in the finite probability of acceptance of decreased energy states, which decreases with each iteration, thereby permitting the escape from sub-optimal local maxima.

The SA process is initialized by randomly sampling the parameter space using a uniform pdf. At each subsequent temperature step, the parameter set is perturbed according to

$$\mathbf{\Psi}_i = \mathbf{\Psi}_{i-1} + \beta^{i-1}\zeta\mathbf{\Delta_i} \tag{6}$$

where ζ is uniformly distributed over $[-1,1]$, β is the perturbation relaxation parameter, and Δ_i is the maximum perturbation allowed for the parameter vector.

The parameter β is chosen to be less than unity to gradually decrease the perturbation magnitude with temperature [7].

After the parameter set is perturbed, the energy change due to the perturbation

$$\Delta E = p_{\mathbf{X}|\mathbf{\Psi}}(\mathbf{X} \mid \mathbf{\Psi})_i - p_{\mathbf{X}|\mathbf{\Psi}}(\mathbf{X} \mid \mathbf{\Psi})_{i-1} \quad (7)$$

is calculated. If $\Delta E > 0$, the perturbation is always accepted. If $\Delta E < 0$, the perturbation is accepted if

$$P_{Boltz} = \frac{1}{1 + \exp(-\Delta E/T_i)} > \psi_i \quad (8)$$

where ψ_i is uniformly distributed on $[0, 1]$ and T_i, the temperature at the i-th step, is given by $T_i = \alpha_T^i T_0$, where $0 < \alpha_T < 1$.

5. RESULTS

The experimental geometry employed in this simulation study is depicted in Figure 2. A horizontal array is used to sample the azimuthal dependence of the scattered field covariance induced by the anisotropic bottom microroughness correlation structure. The dimensions associated with the array geometry, e.g. beamprint area, range to insonified seafloor patch, maximum roughness correlation length, etc., were selected such that the far field criterion is satisfied. The size of the insonified surface patch was selected to span a large number of surface correlation lengths, generally between 10 and 50, depending on the surface parameters.

The acoustic data set is comprised of 100 independent simulated array observations. The observations were obtained by applying the Kirchhoff scattered field representation in (2) to independent realizations of Goff-Jordan surface relief characterized by the surface parameters $H = .0175\ m$, $k_n = 2.5\ cycles/m$, $k_s = .5\ cycles/m$, $\zeta_s = 30°$, and $D = 1.6$. Each realization was insonified at an incidence angle of 20° and an acoustic frequency of 12 kHz. The startup temperature, T_0, was initialized to 500. The temperature scaling parameter, α_T, was .976. The perturbation relaxation parameter, β, was .99, with one perturbation per temperature step. Uniform prior knowledge of all 5 surface parameters was assumed, with upper and lower bounds summarized in Table 1. A representative result of the MAP estimation procedure based on backscatter spatial coherence using the simulated annealing technique is illustrated in Fig. 3. Convergence performance for all five parameters is excellent.

6. CONCLUSIONS

To the best of our knowledge, this result represents the first attempt at the optimum estimation of anisotropic surface roughness statistics based on the physical modeling of acoustic backscatter spatial coherence. It is believed that this estimation approach represents a significant advance over an earlier estimation technique based on observations of backscattering strength angular dependence [2]. While the estimation results obtained with each method are equally good, the sonar configuration presented herein is much more practical to implement with regard to the nature of the ship track required to obtain adequate azimuthal sampling of the scattered field horizontal wavenumber space. In the case of MAP estimation of Goff-Jordan surface parameters from backscatter angular dependence [2], 360° azimuthal sampling would require a ship to traverse a spiral path as a function of range. For estimation from backscatter spatial coherence, the horizontal array samples the azimuthal correlation structure in the reverberation field as the ship traverses a straight line path over a spatially homogeneous seafloor province. Further, the estimation based on backscatter spatial coherence does not involve any preprocessing of the data (e.g. summing squares of quadrature components) and thus does not discard phase information.

The application of the parameter estimation theory framework to the characterization of the properties of the rough seabed is by no means limited to statistical inference of the microroughness wavenumber spectrum parameters. The next step will be to consider the optimum estimation of subbottom properties based on the statistics of the backscattered acoustic field. In particular, it is believed that optimum estimation techniques will be particularly suited to the estimation of correlation parameters associated with physical models for randomly inhomogeneous ocean subbottom sediment layers.

7. REFERENCES

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Goff-Jordan Parameter	Lower Bound	Upper Bound
H	$.001\ m$	$.03125\ m$
k_n	$.25\ cyc/m$	$2.5\ cyc/m$
k_s	$.25\ cyc/m$	$2.5\ cyc/m$
ζ_s	0°	180°
D	1.5	2.4

Table 1: Lower and upper bounds on simulated annealing search space.

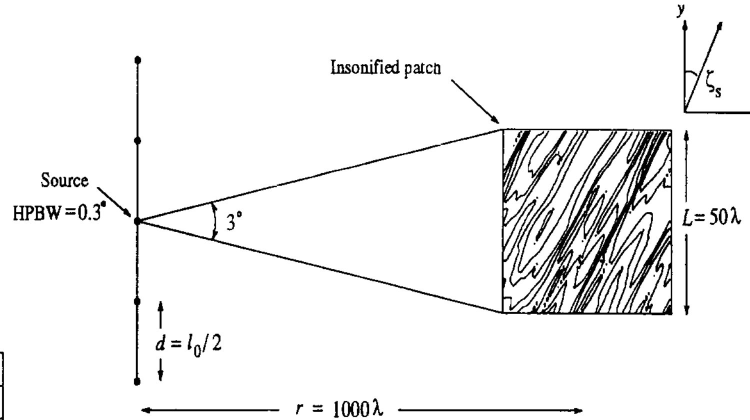

Figure 2: Receiver array geometry (plan view).

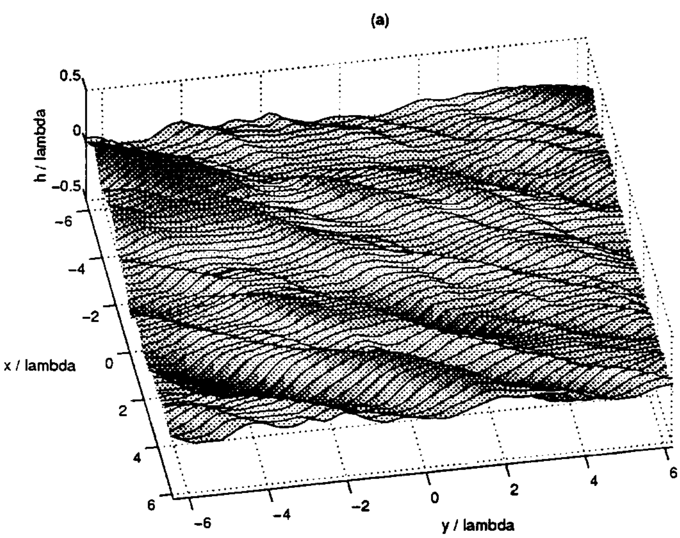

Figure 1: Simulated microroughness realization characteristic of a rippled sediment field.

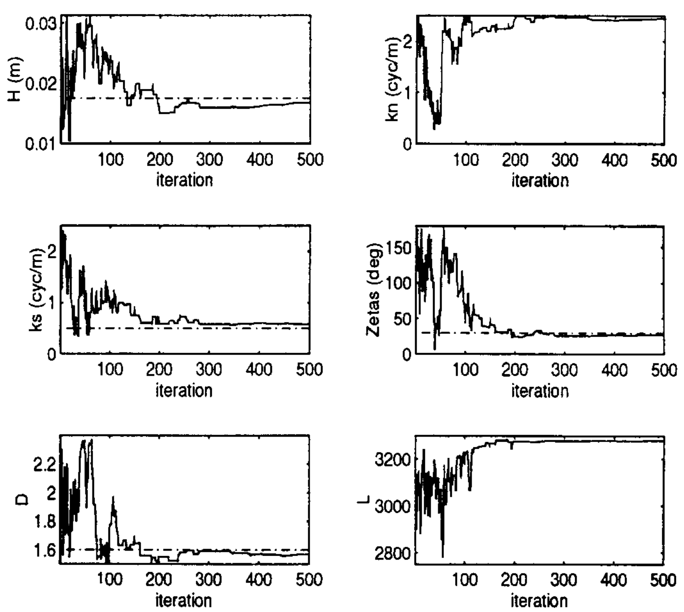

Figure 3: MAP estimation performance via simulated annealing. Actual parameter values are $H = .0175\ m$, $k_n = 2.5\ cyc/m$, $k_s = 0.5\ cyc/m$, $\zeta_s = 30°$, and $D = 1.6$. L denotes the energy function.