SOURCE LOCALIZATION IN AN ACOUSTIC WAVEGUIDE WITH UNCERTAIN SOUND SPEED

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ABSTRACT

Matched-field source localization methods attempt to estimate the range and depth of a source in an acoustic waveguide. These methods give good results when the waveguide parameters are known precisely; however, matched-field methods have been shown to be very sensitive to model mismatch resulting from errors in the assumed environmental parameters. In this paper, we describe an approach which minimizes model mismatch by estimating the environmental parameters and source location together. We propose an efficient way to initialize the maximum-likelihood search by projecting the received data onto subspaces corresponding to regions in parameter space.

1. INTRODUCTION

Matched-field source localization methods attempt to estimate the range and depth of a source in an acoustic waveguide. The measured sound field is compared, over the set of possible source locations, to the sound field model (called the replica field in the matched-field literature) which is based on the environmental parameters of the waveguide. The source location producing the best match between the measured and modeled sound fields is assumed to be the true source location. These methods have been shown to be very sensitive to model mismatch resulting from errors in the assumed environmental parameters [1],[2]. In this paper, we describe an approach which minimizes model mismatch by estimating the environmental parameters and source location together. Initial environmental parameter values are obtained from nominal, a priori information.

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2. SIGNAL MODEL

The signal received by the l*th* sensor of a vertical line array of M omni-directional sensors in an acoustic waveguide can be modeled mathematically as [3]

$$y_l(t) = \sum_{k=1}^{P} s_k(t) * g(t, z_l|\Theta_k, \Psi) + n_l(t), \qquad (1)$$

where $s_k(t)$ is the signal emitted by the k*th* source, $g(t, z_l|\Theta_k, \Psi)$ is the Green's function of the medium, $n_l(t)$ is the additive noise at the l*th* sensor, and $*$ denotes convolution. The l*th* receiver depth is denoted z_l, the vector Θ_k denotes the unknown source location parameters of the k*th* source, and Ψ is the vector of parameters used to describe the medium (e.g., parameters of the sound velocity profile or channel depth). In this case, the parameters of interest are range and depth, therefore the estimation problem is to determine $\Theta_k = [r_k, z_k^s]^T$.

The transfer function between source k and the receiver located at depth z_l can be obtained by Fourier transforming the Green's function of the medium,

$$G(\omega, z_l|\Theta_k, \Psi) = \mathcal{F}\{g(t, z_l|\Theta_k, \Psi)\}.$$

At large horizontal distances between the source and receiver in an acoustic waveguide, $G(\omega, z_l|\Theta_k, \Psi)$ can be approximated by a normal mode expansion [4]

$$G(\omega, z_l|\Theta_k, \Psi) = \sum_{m=1}^{Q} \alpha_m(\Theta_k, \Psi)\phi_m(z_l, \Psi) \qquad (2)$$

where Q is the number of propagating modes. The modal functions are found by solving the separated wave equation [5].

If we sample the signals received at each sensor over an interval of T seconds, we can obtain a frequency domain representation of the output of each sensor by discrete Fourier transforming (DFT) the samples. When

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T is much larger than the difference in propagation times of the fastest and slowest modes traversing the entire distance between the source and array, a frequency domain approximation to (1) can be written [6]

$$y_l(\omega_n) = \sum_{k=1}^{P} s_k(\omega_n) G(\omega_n, z_l|\Theta_k, \Psi) + n_l(\omega_n), \quad (3)$$

where $y_l(\omega_n)$ is the n*th* DFT coefficient of the l*th* sensor, $s_k(\omega_n)$ is the n*th* DFT coefficient of the k*th* source, and $n_l(\omega_n)$ is the n*th* DFT coefficient of the noise at the l*th* sensor.

Equation (3) can be written in matrix form as

$$\mathbf{y}(\omega_n) = \mathbf{A}(\omega_n, \Theta, \Psi)\mathbf{s}(\omega_n) + \mathbf{n}(\omega_n), \quad (4)$$

where

$$\begin{aligned}
\mathbf{y}(\omega_n) &= [y_1(\omega_n) \cdots y_M(\omega_n)]^T, \\
\mathbf{A}(\omega_n, \Theta, \Psi) &= [\mathbf{a}(\omega_n, \Theta_1, \Psi) \cdots \mathbf{a}(\omega_n, \Theta_P, \Psi)], \\
\mathbf{a}(\omega_n, \Theta_k, \Psi) &= [G(\omega_n, z_1|\Theta_k, \Psi) \cdots G(\omega_n, z_M|\Theta_k, \Psi)]^T, \\
\Theta &= [\Theta_1^T \cdots \Theta_P^T]^T, \\
\mathbf{s}(\omega_n) &= [s_1(\omega_n) \cdots s_P(\omega_n)]^T, \\
\mathbf{n}(\omega_n) &= [n_1(\omega_n) \cdots n_M(\omega_n)]^T.
\end{aligned} \quad (5)$$

3. MAXIMUM-LIKELIHOOD ESTIMATOR

3.1. Known Environment

Consider the case of a narrow band signal that can be represented by a single DFT coefficient. The likelihood function for N independent observations of the sensor array is $p(\{\mathbf{y}_i\}_{i=1}^N|\Theta,\{\mathbf{s}_i\}_{i=1}^N)$ which equals

$$K \exp\left\{-\sum_{i=1}^{N} \|\mathbf{y}_i - \mathbf{A}(\Theta, \Psi)\mathbf{s}_i\|^2_{\mathbf{R}_n^{-1}}\right\}, \quad (6)$$

where K is a constant and $\mathbf{R}_n^{-1}$ is the inverse of the noise cross-spectral matrix, $\mathbf{R}_n = E\{\mathbf{n}\mathbf{n}^H\}$, which is assumed known. Maximization of (6) is equivalent to minimizing the argument of its exponential. Therefore, the maximum-likelihood (ML) estimates of $\mathbf{A}(\Theta)$ and $\mathbf{s}_i$ result by solving

$$\hat{\Theta}, \hat{\mathbf{s}}_i = \arg \min_{\Theta, \mathbf{s}_i} \sum_{i=1}^{N} \|\mathbf{y}_i - \mathbf{A}(\Theta, \Psi)\mathbf{s}_i\|^2_{\mathbf{R}_n^{-1}}. \quad (7)$$

Substituting the optimal values of $\mathbf{s}_i$ into (7) results in a simpler optimization problem involving only Θ,

$$\hat{\Theta} = \arg \min_{\Theta} \sum_{i=1}^{N} \left\|(\mathbf{I} - \mathbf{A}(\Theta, \Psi)\mathbf{A}^{\#}(\Theta, \Psi))\mathbf{y}_i\right\|^2_{\mathbf{R}_n^{-1}}, \quad (8)$$

where $\mathbf{A}^{\#} = (\mathbf{A}^H\mathbf{R}_n^{-1}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{R}_n^{-1}$, or equivalently,

$$\hat{\Theta} = \arg \max_{\Theta} \sum_{i=1}^{N} \left\|\mathbf{A}(\Theta, \Psi)\mathbf{A}^{\#}(\Theta, \Psi)\mathbf{y}_i\right\|^2_{\mathbf{R}_n^{-1}}. \quad (9)$$

3.2. Uncertain Environment

The ML estimator of (9) assumes precise knowledge of the environmental parameters of the channel. Errors in the assumed environmental parameters will cause the maximum of (9) to occur at a value of $\hat{\Theta}$ away from the true source locations. In this case, the uncertain environmental parameters need to be estimated also and the ML estimator for the source locations and the environmental parameters is

$$\hat{\Phi} = \arg \max_{\Phi} \sum_{i=1}^{N} \left\|\mathbf{A}(\Phi)\mathbf{A}^{\#}(\Phi)\mathbf{y}_i\right\|^2_{\mathbf{R}_n^{-1}}, \quad (10)$$

where the vector $\Phi = [\Theta^T \; \Psi^T]^T$ is a concatenation of all the parameters to be estimated.

Consider an environment in which the sound velocity profile (SVP) is uncertain. The SVP can be modeled as the weighted sum of empirical orthogonal functions (EOF), $\varphi_l(z)$, [7] as

$$c(z) = \sum_{l=1}^{L} g_l \varphi_l(z) + c_0(z), \quad (11)$$

where $c_0(z)$ is the mean sound speed and $-1 \le g_l \le 1$. The EOF's can be computed from a historical database of measured SVP files by performing a singular value decomposition of the estimated SVP covariance matrix [8]. The environmental parameter vector Ψ contains the EOF coefficients g_l in (11).

4. ML ESTIMATOR IMPLEMENTATION

4.1. Conventional Implementation

The implementation of the ML estimator in (10) requires a nonlinear multivariate search over all the source locations and uncertain environmental parameters. An efficient means of searching the likelihood surface for its peak is required for the estimator to be useful in practical applications. Generally, the search is conducted in two steps. First, a coarse grid search over the parameters to be estimated is performed on the surface. This coarse estimate is then used as the initial estimate for a gradient-based search in step two. Unfortunately, poor performance can result if the grid spacing is selected too coarsely and the peak is narrow or if the likelihood surface is very oscillatory and the initial estimate is not near the main lobe of the peak.

As an example, we will look at the log-likelihood surface (the norm in (10)) for a typical acoustic waveguide scenario. The waveguide consists of a 110 meter water layer over a 3.5 meter sediment layer above basement rock. Layer depths are considered range invariant. The SVP is modeled using a single EOF in (11), i.e. $L = 1$, with $g = 0.4$. The receive array consists of 11 evenly spaced sensors at a separation of 5 meters with the shallowest sensor at a depth of 25 meters. We consider the case of a single source of frequency 100 Hz located at 10000 meters in range and 50 meters in depth. In this case, the norm in (10) is the norm of the data vector projected onto a replica vector which is a function of source location and environmental parameters. The noise cross-spectral matrix is assumed to be the identity.

Fig. 1 shows a cross section through the log-likelihood surface parallel to the range axis with z^s and r fixed at their correct values. A cross section through the log-likelihood surface with g and r fixed at their correct values is shown in Fig. 2 and Fig. 3 shows a cross section with g and z^s fixed. Notice the oscillatory nature of the cross sections and the sharpness of the peaks. At the cost of increased computation, more grid points could be computed to broaden the peaks and decrease the possibility of missing them. The main lobe widths and side lobe heights of the cross sections vary significantly with frequency, source location, and environment. Therefore, without complete a priori knowledge of the environment and the source and its location, selection of the most efficient grid spacing is difficult.

4.2. Replica Subspace Projection

In the case of a single source, we propose replacing the projection onto a replica vector with a projection onto a replica subspace. The subspaces are generated by sets of replica vectors computed at evenly spaced points over fixed intervals in both source location and environmental parameters for all possible combinations of the points. The projection matrix onto each subspace is computed from the singular value decomposition (SVD) of the matrix whose columns are the set of replica vectors for that subspace. The subspaces can be abutted or overlapped. The surface obtained by projecting onto replica subspaces will be called the *smoothed log-likelihood surface.* The smoothed surface can be obtained with significantly less evaluations of (10). However, generating the sets of replica vectors along with the SVD computed for each set increases the amount of computation to the level of a replica vector fine grid calculation.

Using the same scenario as before, cross sections through the smoothed log-likelihood surface using the replica subspace projections were calculated. Fig. 1 shows a cross section using subspaces with r fixed at range values of 9960 – 10060 meters and z^s fixed at depth values of 48 – 52 meters. Although difficult to see on the plot, the peak of the smoothed log-likelihood surface is at the correct value. In Fig. 2, a cross section is shown using subspaces with g fixed at 0.4, r fixed at range values of 9960 – 10060 meters, and each z^s was 5 meters wide and overlapped the adjacent subspace by 2.5 meters. The values are plotted at the center of each subspace. Fig. 3 shows a cross section with g fixed at 0.4, z^s fixed at depth values of 48 – 52 meters, and each r was 100 meters wide and overlapped the adjacent subspace by 80 meters.

Figures 1 and 2 show that, with respect to source depth and EOF coefficient, the smoothed log-likelihood surface is amenable to a search technique that does not require evaluation of a complete grid of subspaces. The cross section of the surface with respect to range shown in Fig. 3 is not as smooth as the previous two plots. Fig. 3 indicates that there is an approximate periodicity in range with a period of about 200 m. This suggests that a better way to form replica subspaces in range would be to use a group of intervals spaced at about 200 m to form a single subspace. Using a group of intervals in range should give a much smoother surface.

An alternative to the approach described in this section was proposed in [9]. This alternate approach is essentially replica vector grid searches over source location averaged over all possible environments. Therefore, it appears to be susceptible to the grid spacing difficulties discussed earlier and is also computationally expensive.

5. CONCLUSIONS

This paper presented a maximum-likelihood estimator for source location and environmental parameter estimation in an uncertain sound speed, acoustic waveguide. A method using replica subspace projections to search the log-likelihood surface was described. This method was seen to produce a much smoother log-likelihood surface. It was seen that a thorough search of the surface could be performed with much fewer function evaluations, but the increased computations required for generating the subspaces made the total computations equivalent to performing a replica vector fine grid search. Future work will be to develop an efficient search strategy on the smoothed log-likelihood surface.

6. REFERENCES

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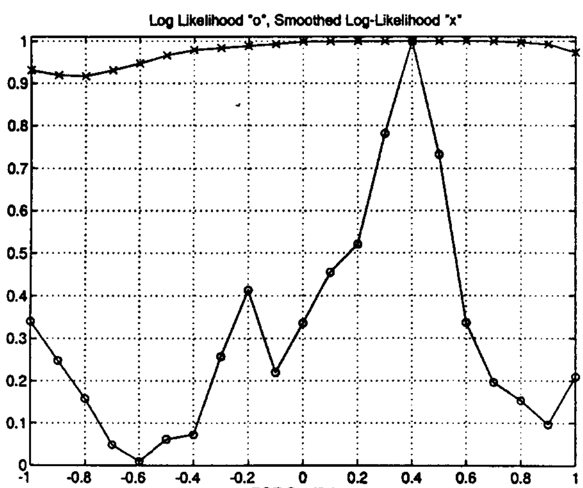

Figure 1: Cross-sections of exact and smoothed log-likelihood surfaces parallel to the EOF coefficient axis

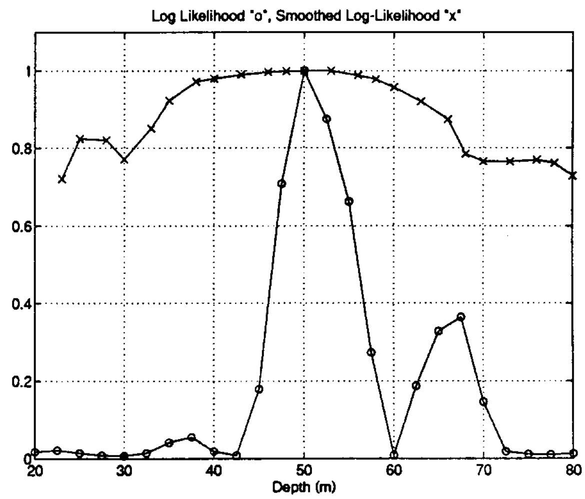

Figure 2: Cross-sections of exact and smoothed log-likelihood surfaces parallel to the depth axis

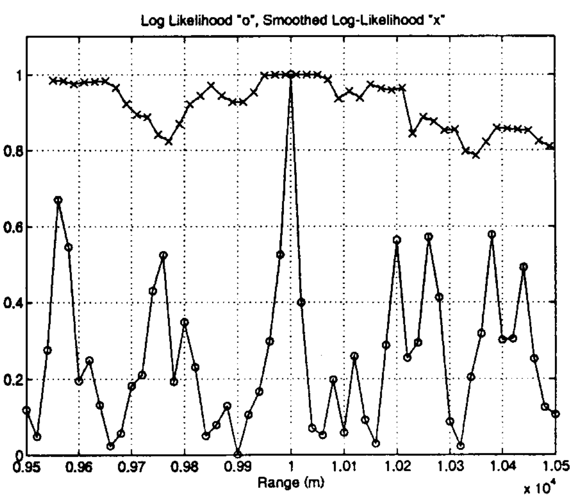

Figure 3: Cross-sections of exact and smoothed log-likelihood surfaces parallel to the range axis