SOURCE LOCALIZATION IN SHALLOW WATER USING POLYNOMIAL ROOTING

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ABSTRACT

Source localization in a waveguide involves a multi-dimensional search procedure. In this paper we propose a new algorithm, in which the search in the depth direction is replaced by polynomial rooting. The proposed algorithm decreases the search dimension to one for a 2D localization problem (range and depth) and to two for a 3D one (range, depth, and direction-of arrival (DOA)), independently of the number of sources. Consequently, the presented algorithm requires significantly less computation.

1. INTRODUCTION

In recent years, there has been a growing interest in passive source localization in an ocean waveguide. In particular, several Matched Field Processing (MFP) techniques have been developed. MFP techniques exploit the complex multipath propagation model. Many algorithms, which use the full wave acoustic propagation model of complex multipath conditions in an ocean waveguide have been developed (e.g. [2, 4]) to estimate source location parameters such as range, depth and DOA. All of these algorithms involve a multi dimensional search procedure, which should be performed on the space consisting of source location parameter, signal spectrum/cross-spectrum and the environmental parameters. For instance, in the maximum-likelihood (ML) approach, when the number of sources or the number of unknown parameters increases, the search dimension grows accordingly. A grid search procedure is necessary, since the ambiguity surface in most of the matched field estimators have many false (local) peaks and simple gradient search methods will therefore not find the global maximum.

A signal subspace decomposition approach for this problem has been developed in [4]. It was also shown that the performance of this method is very close to the ML one. Since the amount of computation does not increase when the number of sources grows, this approach does offer an advantage over the ML. Still, a multi-dimensional search over range, depth and DOA in addition to other nuisance parameters is required.

In this paper, we propose a localization method based on the generalization of the polynomial rooting approach to MFP applications. This approach was developed for planar wave field and linear equi-spaced array in [1] and it was extended to spherical wave field and an arbitrary array shape in [7].

As in eigenstructure-based methods, the search dimension of the algorithm is independent of the number of sources. The method involves a search procedure over the range and other unknown environmental parameters combined with polynomial rooting which replaces the search in the depth direction. Due to the excellent polynomial rooting routines that are available, it is generally accepted that polynomial rooting is preferable other methods of searching, specially when the ambiguity surface is not smooth, such that a high density grid should be chosen.

2. PROBLEM FORMULATION

We consider L point sources, located in a waveguide. Each source radiates a narrowband stochastic signal $\{s_l(t)\}_{l=1}^{L}$ centered at ω_o. The waveguide is assumed to be deterministic and time invariant. The excited field is sampled by a general three dimensional array of N sensors, satisfying $N > L$. The sensors are located at $(\underline{x}_i, z_{ai}), i = 1, \cdots, N$, where $\underline{x}_i$ is the position of sensor i in the horizontal plane and z_i denotes its depth. The sensor locations are assumed to be known. The location of source l is defined by (θ_l, r_l, z_l), denoting its direction, range and depth (see figures 1 and 2).

Under the far field assumption, where the range r_l is large compared with the array aperture, the range r_{il} from source l to sensor i is: $r_{il} = r_l + \underline{\hat{k}}^t(\theta_l)\underline{x}_i$, and by using the complex (analytic) signal representation, the field measured by sensor i can be expressed by (see e.g. [6]):

$$y_i(t) = \sum_{l=1}^{L}\sum_{m=1}^{M} \phi_m(z_l)\phi_m(z_{ai})\frac{e^{j\kappa_m(r_l+\underline{\hat{k}}^t(\theta_l)\underline{x}_i)+j\frac{\pi}{4}}}{\sqrt{\kappa_m r_l}} s_l(t) + n_i$$

$$i = 1, \cdots, N \ , \quad -\frac{T}{2} \le t \le \frac{T}{2} \ ,$$

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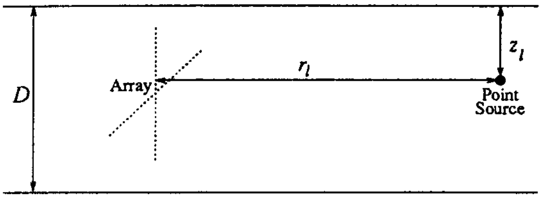

Figure 1: Problem geometry

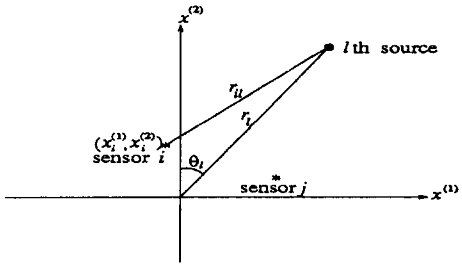

Figure 2: Problem geometry - horizontal plane (top view)

where $\{\phi_m(\cdot)\}_{m=1}^M$ are the modal depth eigenfunctions and $\{\kappa_m\}_{m=1}^M$ are the modal horizontal wavenumbers. M is the number of propagating modes in the channel and $n_i(t)$ denotes the additive noise at sensor i. We assume that the signals and the noise are stationary and ergodic complex valued random processes with zero mean.

The Fourier coefficient of (1) at a single frequency ω_o in matrix notation is:

$$\mathbf{y} = \sum_{l=1}^{L} \mathbf{T}(\theta_l, r_l)\mathbf{\Phi}(z_l)\tilde{s}_l + \mathbf{n} \triangleq \mathbf{A}\mathbf{s} + \mathbf{n}, \qquad (2)$$

where

$$\mathbf{A} = [\mathbf{a}(\theta_1, r_1, z_1), \cdots\cdots, \mathbf{a}(\theta_L, r_L, z_L)] \ ,$$

$$\mathbf{a}(\theta_l, r_l, z_l) \triangleq \mathbf{T}(\theta_l, r_l)\mathbf{\Phi}(z_l), \ \ l = 1, \cdots, L \ .$$

In (2) $[\mathbf{y}]_i$ and $[\mathbf{n}]_i$ denote the Fourier coefficients at the frequency ω_o of the measurement and the noise, respectively, in sensor i, and $[\mathbf{s}]_l = \tilde{s}_l$ stands for the corresponding Fourier coefficients of the lth signal.

The elements of the matrix $\mathbf{T}(\theta_l, r_l)$ and the vector $\mathbf{\Phi}(z_l)$ are given by:

$$[\mathbf{T}(\theta_l, r_l)]_{im} = e^{j\frac{\pi}{4}}\phi_m(z_{ai})\frac{e^{j\kappa_m(r_l + \hat{\underline{k}}^t(\theta_l)\underline{x}_i)}}{\sqrt{\kappa_m r_l}} \ , \qquad (3)$$

$$[\mathbf{\Phi}(z_l)]_m = \phi_m(z_l) \ . \qquad (4)$$

We collect K snapshots and rewrite equation (2) for each snapshot:

$$\mathbf{y}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k), \quad k = 1, \cdots, K \ . \qquad (5)$$

Our goal is to estimate the source location parameters, $\{\theta_l, r_l, z_l\}_{l=1}^L$, from the measurements $\{\mathbf{y}(k)\}_{k=1}^K$.

3. HOMOGENEOUS WAVEGUIDE

In this section, we present the polynomial rooting approach for source depth estimation in a homogeneous waveguide. It follows from our assumptions that:

$$\mathbf{R_y} \triangleq E\{\mathbf{y}(k)\mathbf{y}(k)^H\} = \mathbf{A}\mathbf{R_s}\mathbf{A}^H + \mathbf{R_n} \ , \qquad (6)$$

$$\mathbf{R_s} \triangleq E\{\mathbf{s}(k)\mathbf{s}(k)^H\} \ , \qquad (7)$$

$$\mathbf{R_n} \triangleq E\{\mathbf{n}(k)\mathbf{n}(k)^H\} \ . \qquad (8)$$

Assuming that the vectors $\mathbf{a}(\theta_l, r_l, z_l)$ and $\mathbf{a}(\theta_j, r_j, z_j)$ are linearly independent for every $l, j = 1, \cdots, L, l \neq j$, it follows that $\mathbf{V}^H\mathbf{A} = 0$, where $\mathbf{V}$ is the matrix of the $N-L$ eigenvectors corresponding to the $N-L$ smallest eigenvalues of the matrix pencil $(\mathbf{R_y}, \mathbf{R_n})$. The MUSIC algorithm exploits these properties of the covariance matrix $\mathbf{R_y}$. It suggests the following procedure:

1. Estimate the covariance matrix $\mathbf{R_y}$ by: $\hat{\mathbf{R}}_\mathbf{y} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{y}(k)\mathbf{y}^H(k)$,
2. Compute $\hat{\mathbf{V}}$, the noise subspace eigenvectors of $\hat{\mathbf{R}}_\mathbf{y}$ (i.e the eigenvectors associated with the $N-L$ smallest eigenvalues).
3. Find the L lowest minima of the function:

$$\hat{Q}(\theta, r, z) = \mathbf{a}^H(\theta, r, z)\hat{\mathbf{V}}\hat{\mathbf{V}}^H\mathbf{a}(\theta, r, z) \ . \qquad (9)$$

The minima of $\hat{Q}(\theta, r, z)$ can be found by performing a three dimensional search. Due to the existence of local minima, this procedure is computationally expensive, since it requires a three dimensional search over a high density grid.

In a homogeneous waveguide, with depth D (see fig. 1) and boundary conditions: $\phi_m(z)|_{z=0} = 0$ and $\left.\frac{d\phi_m}{dz}\right|_{z=D} = 0$, the mth mode eigenfunction is $\phi_m(z) = \sqrt{2/D}\sin\gamma_m z$, where $\gamma_m = \left(m - \frac{1}{2}\right)\frac{\pi}{D}$ is the vertical wavenumber of the mth mode. Thus, the elements of the array manifold, $\mathbf{a}(\theta, r, z)$, are:

$$a_i(\theta, r, z) = \frac{\sqrt{2/D}}{2j}\left[\sum_{m=1}^{M} T_{im}e^{j\frac{\pi}{D}(m-\frac{1}{2})z} - \sum_{m=1}^{M} T_{im}e^{-j\frac{\pi}{D}(m-\frac{1}{2})z}\right] . \qquad (10)$$

Let define w as: $\mathrm{w} \triangleq e^{j\frac{\pi}{D}z}$, so that equation (10) can be written as:

$$a_i(\theta, r, z) = \frac{\sqrt{2/D}}{2j}\mathrm{w}^{(-M+\frac{1}{2})}\sum_{m=0}^{2M-1} F_{i,m+1}\mathrm{w}^m \ . \qquad (11)$$

The ith row of $\mathbf{F}$ is given by:

$$[\mathbf{F}]_i \triangleq [-T_{iM}, \cdots - T_{i1}, T_{i1}, \cdots, T_{iM}] \ . \quad (12)$$

Using (11) for the array manifold expression, equation (9) can be written by a different function $Q(\theta, r, \mathrm{w})$:

$$Q(\theta, r, \mathrm{w}) = \frac{1}{2D}\mathbf{g}(1/\mathrm{w})\mathbf{F}^H(\theta, r)\mathbf{V}\mathbf{V}^H\mathbf{F}(\theta, r)\mathbf{g}(\mathrm{w}), \quad (13)$$

where $\mathbf{g} \triangleq [1, \mathrm{w}, \cdots\cdots, \mathrm{w}^{2M-1}]^T$.

The expression $Q(\theta, r, \mathrm{w})$ in equation (13) is a polynomial in w of order $4M - 2$. For true values of θ and r, the roots of $Q(\theta, r, \mathrm{w})$ are located on the unit circle and they correspond to $\mathrm{w}_l = e^{j\frac{\pi}{D}z_l}$, where z_l is the depth of the lth source. Due to the estimation errors of the covariance matrix $\mathbf{R_y}$, these roots are located near the unit circle and they minimize the expression $Q(\theta, r, \mathrm{w})$. We propose to compute the roots of the polynomial $Q(\theta, r, \mathrm{w})$ for every θ and r, instead of the search over θ, r, z. By search over θ and r, we generate the localization function:

$$G(\theta, r) = \frac{1}{1 - |\mathrm{w}_o(\theta, r)|} \ , \quad (14)$$

on which the maximization will be carried out. For given θ and r, $\mathrm{w}_o(\theta, r)$ denotes the closest root to the unit circle. The L highest peaks of $G(\theta, r)$ corresponds to the positions of the L sources. In this way, we reduced the search dimension by one.

Note, that the coefficients of the polynomial $Q(\theta, r, \mathrm{w})$ are real. Therefore, for each complex root w_o, there exists another root at w_o^*. In addition, (13) shows that for each root w_o, there exists a root at $1/\mathrm{w}_o$. That is, each complex root w_o is related to three additional roots: w_o^*, $1/\mathrm{w}_o$ and $1/\mathrm{w}_o^*$.

4. NON-HOMOGENEOUS WAVEGUIDE

Now, we extend the algorithm to non-homogeneous waveguides. Consider a variable propagation velocity profile in the waveguide, with the following assumptions:

- The propagation velocity satisfies: $c(\underline{x}, z) = c(z)$, $\forall\ \underline{x}$ and $z \in [0, D]$
- The channel depth is constant: $D(\underline{x}) = D$, $\forall\ \underline{x}$.

The eigenfunction of the mth mode, satisfies the following normal mode equation:

$$\frac{d^2\psi_m(z)}{dz} + \frac{\omega_o^2}{c^2}[1 + \epsilon(z)]\psi_m(z) = \mu_m^2\psi_m(z) \ , \quad (15)$$

with the boundary conditions of the homogeneous problem. Therefore, $\psi_m(z)$ can be expressed by linear combination of the homogeneous eigenfunctions:

$$\psi_m(z) = \sum_{j=1}^{\infty} P_{mj}\phi_j(z) \ , m = 1, \cdots, M \ . \quad (16)$$

Assume smooth propagation velocity profile. Then $\psi_m(z)$ are smooth, and they can be approximated by a linear combination of ***finite*** spectral components, enabling truncation of the series:

$$\psi_m(z) \cong \sum_{j=1}^{J} P_{mj}\phi_j(z) \ , \quad J > m \ , \quad m = 1, \cdots, M \ , \quad (17)$$

and in a vector notation: $\mathbf{\Psi}(z) = \mathbf{P\Phi}(z)$. For smooth propagation velocity profile, fast decay of the coefficients P_{mj} is guaranteed. The matrix $\mathbf{P}$ and the horizontal wavenumbers μ_m can be found according to the procedure described in [4]. If the propagation velocity profile is available, then the matrix $\mathbf{P}$ is known. Now $\hat{Q}(\theta, r, z)$ from equation (9) can be expressed by:

$$\hat{Q}(\theta, r, z) = \quad (18)$$
$$\mathbf{\Phi}^H(z)\left(\tilde{\mathbf{T}}(\theta, r)\mathbf{P}\right)^H \hat{\mathbf{V}}\hat{\mathbf{V}}^H \left(\tilde{\mathbf{T}}(\theta, r)\mathbf{P}\right)\mathbf{\Phi}(z) \ ,$$

where $\tilde{\mathbf{T}}(\theta, r)$ is given by (3) with the new eigenfunctions $\psi_m(z)$ and horizontal wavenumbers μ_m.

This interpretation enables to perform the polynomial rooting operation as with the homogeneous waveguide on $\hat{Q}(\theta, r, z)$, where the matrix $\mathbf{T}(\theta, r)$ is replaced by $\tilde{\mathbf{T}}(\theta, r)\mathbf{P}$.

This approach can be used in any other generalized model (e.g. the adiabatic normal mode model used in [3]), in which the boundary conditions are identical to those of the homogeneous problem.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm and compare it to the ML estimator performance. The following scenario describes a typical problem of source localization in an underwater acoustic waveguide, but for the sake of simplicity matter we assume a homogeneous waveguide.

The field was generated by a point source, located at a depth of $z_o = 78m$ below the upper surface of a homogeneous waveguide consisting of two parallel infinite plates, with a depth of $D = 100m$. The source radiates a stochastic and narrow-band signal centered at frequency of $f = 100Hz$. The propagation velocity in the waveguide is $c = 1500m/sec$. These conditions enable propagation of 13 modes in the waveguide.

The generated field is sampled by a vertical array, located at a distance of $10km$ from the source. The array consists of 30 sensors with 2m spacing. The center of the array was located 49 meters below the upper surface. The additive noise at the sensors is zero-mean, Gaussian and i.i.d. The SNR at each sensor depends on its depth. Here, we defined SNR as the average SNR per sensor.

Figure 3 depicts the function $G(r)$ from equation (14) versus the range with an SNR of $0dB$. This figure shows that the nearest root to the unit circle is at the true range. The angle of this root corresponds to the estimated depth ($78.02m$).

The performance of this algorithm is evaluated using 100 Monte Carlo trials. The results are shown in figure 4. For comparison, we evaluated the Cramer-Rao bound. The results of the algorithm achieves the CRB at high SNR, indicating the efficiency of the algorithm, i.e. $\lim_{SNR-\infty} cov(\Delta r, \Delta z) = CRB(r, z)$. This conclusion is supported by also performance analysis results of this algorithm derived in [5].

At SNR of $\sim -12dB$ a threshold phenomenon is observed. This threshold is caused by convergence of the algorithm in local maxima. In order to compare the location of this threshold, a corresponding Monte Carlo simulation on the ML estimator was carried out. The threshold for the ML is at $\sim -17dB$, about $5dB$ below the polynomial rooting algorithm. In [5], the performance of this algorithm was studied analytically

6. SUMMARY

In this paper a new algorithm for source localization in a waveguide is presented. The performance of the algorithm is studied numerically and it is compared to the CRB and ML simulations. The algorithm was shown to be efficient, while being computationally simpler than other existing algorithms, such as ML or MUSIC. This method generalizes the polynomial rooting approach to non-planar wavefronts, as encountered in non-homogeneous and/or bounded environments.

7. REFERENCES

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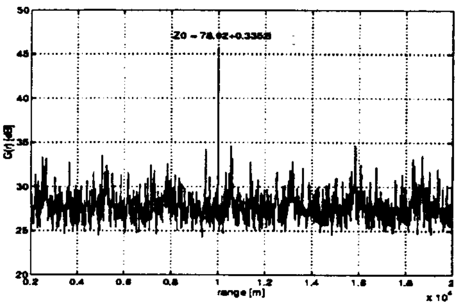

Figure 3: The localization function $G(r)$ for a vertical array with a source located at range of 10km and depth 78m.

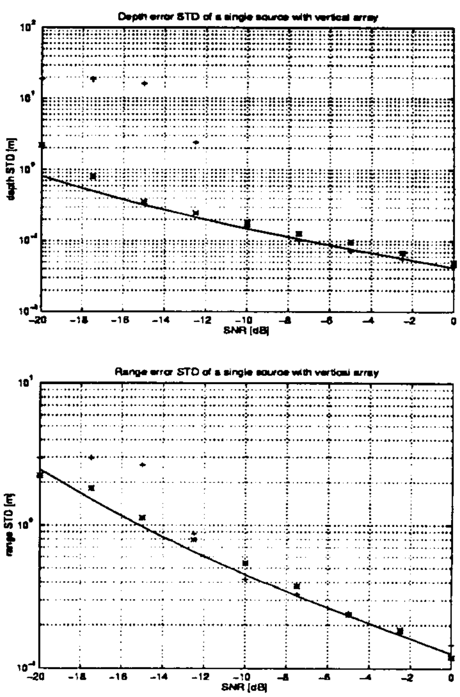

Figure 4: Range and depth error standard deviation versus SNR. Monte Carlo simulations based on 100 independent trials of the proposed estimator(+) compared to ML estimator results (*) and CRB (solid line).