ADAPTIVE FILTERING OF STABLE PROCESSES FOR ACTIVE ATTENUATION OF IMPULSIVE NOISE

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ABSTRACT

We describe a new class of algorithms for active noise control (ANC) for use in environments in which impulsive noise is present. The well known filtered-X and filtered-U ANC algorithms are designed to minimize the variance of a measured error signal. For impulsive noise, which can be modeled using non-Gaussian stable processes, these standard approaches are not appropriate since the second order moments do not exist. We propose a new class of adaptive algorithms for ANC that are based on the minimization of a fractional lower order moment, $p < 2$. By studying the effect of p on the convergence behavior of adaptive algorithms, we observe that superior performance is obtained by choosing $p \approx \alpha$ where $\alpha < 2$ is a parameter reflecting the degree of impulsiveness of the noise. Applications of this approach to noise cancellation in a duct are presented.

1. INTRODUCTION

In recent years, many systems have been proposed for active cancellation of acoustic noise. The most successful of these are feedforward systems in which a reference signal, measured using a reference sensor near the noise source, is fed forward via an adaptive filter to a secondary source so as to cancel the unwanted noise. The coefficients of the adaptive filter are chosen to minimize some function of the residual noise as measured at an error sensor. Almost all of the current methods for adapting the coefficients of the adaptive filter have been based on minimization of the variance of the residual noise at the error sensor. For example, the filtered-X LMS algorithm for FIR filters [2, 3], and the filtered-U RLMS algorithm for IIR filters [4] both attempt to minimize the mean squared residual error.

There is however an important class of random processes known as stable distributions, with parameter α ($0 < \alpha \leq 2$), for which the second moment does not exist for any $\alpha < 2$ (a stable distribution reduces to a Gaussian, when $\alpha = 2$). Compared with the Gaussian distribution, the stable distributions for $\alpha < 2$ exhibit heavier tails, so that outliers occur more frequently than in Gaussian processes. The importance of the stable distributions is that they are able to model a range of non-Gaussian signals including impulsive phenomena. The smaller α, the heavier the tails of the density function, and thus the more impulsive the process. Since the variance does not exist for these processes, it makes little sense to use traditional second order moment based adaptive algorithms to cancel noise of this type. In this paper we propose a new class of algorithms for active noise control for use in environments where impulsive noise is present. In place of the variance which is minimized in the standard algorithms, we minimize a fractional lower order moment ($p < \alpha$) that does exist.

Single channel algorithms are presented for applications to plane waves in a duct; however, they are directly extendable to multichannel problems for cancellation of more complex acoustic fields.

2. FRACTIONAL LOWER ORDER MOMENT BASED ADAPTIVE ALGORITHMS

A comprehensive tutorial on signal processing based on stable processes and fractional lower order moments is given in [1]. Here, we restrict our attention to the special class of symmetric stable distributions, called *symmetric α-stable or $S\alpha S$*, with $1 \leq \alpha \leq 2$ and characteristic function:

$$\phi(t) = \exp\{jat - \gamma|t|^{\alpha}\} \tag{1}$$

where $\gamma > 0$ is the dispersion and a is the location parameter. For an $S\alpha S$ process with $\alpha < 2$, only moments of order less than α are finite. Since in these cases the variance is not finite, the minimum mean squared error criterion is an inappropriate objective for adaptive filtering. Instead, the *minimum dispersion* criterion serves as a measure of optimality in stable signal processing. The dispersion is a parameter of the $S\alpha S$ process which plays a similar role to the variance in the Gaussian process. It is shown in [1] that minimizing dispersion is equivalent to minimizing a fractional lower order moment of the residual error: $E|e(n)|^p$ for $p < \alpha$. Although we can in theory use any fractional lower order moment with $p < \alpha$, the convergence behavior of adaptive algorithms based on lower order moments varies considerably with p. Here, we illustrate this with a simple adaptive experiment using a first-order AR process. Consider an i.i.d. stable process, $\{v(n)\}$, driving an AR model:

$$u(n) = au(n-1) + v(n) \tag{2}$$

where $a = 0.99$ is the parameter of the process. To estimate the parameter a from the AR process $\{u(n)\}$, we implemented (i) a theoretical steepest descent algorithm, and (ii) a stochastic LMP (least mean P-norm) algorithm [1].

For both algorithms, the objective is to solve the optimization problem:

$$\hat{w}(n) = argmin_{\hat{w}(n)} E[|e(n)|^p] \tag{3}$$

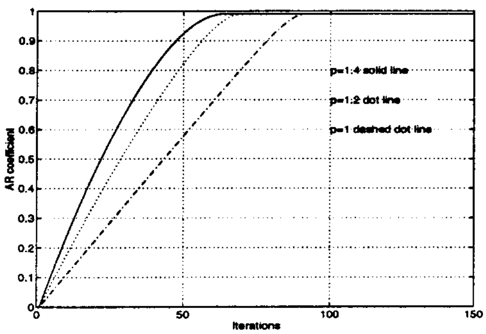


Figure 1: Convergence behavior of the steepest descent algorithm for minimization of the p^{th} moment of the residual error for $\alpha = 1.5$ and various values of p.

where

$$e(n) = u(n) - \hat{w}(n)u(n-1), \tag{4}$$

for $p < \alpha$. The theoretical steepest descent algorithm for optimizing this function has the form:

$$\hat{w}(n+1) = \hat{w}(n) - \mu\nabla_{\hat{w}(n)}E[|e(n)|^p] \tag{5}$$

$$= \hat{w}(n) + \mu p E[u(n-1)sign(e(n))|e(n)|^{p-1}] \tag{6}$$

$$= \hat{w}(n) + \mu p|\tilde{w}(n)|^{\alpha+1} sign(\tilde{w}(n))\gamma_u C(p,\alpha)\gamma_{e(n)}^{\frac{p-\alpha}{\alpha}}$$

where

$$\tilde{w}(n) = a - \hat{w}(n), \quad C(p,\alpha) = \frac{2^{p+1}\Gamma(\frac{p+1}{2})\Gamma(-\frac{p}{\alpha})}{\alpha\sqrt{\pi}\Gamma(-\frac{p}{2})}$$

$$\gamma_{e(n)} = |\tilde{w}(n)|^\alpha \gamma_u + \gamma_v, \quad \gamma_u = \frac{\gamma_v}{1-|a|^\alpha}$$

Since the true pth moment is unknown, it can be approximated using an instantaneous estimate:

$$E[|e(n)|^p] \approx |e(n)|^p \tag{7}$$

resulting in the stochastic LMP algorithm [1]:

$$\hat{w}(n+1) = \hat{w}(n) + \mu p u(n-1)|e(n)|^{p-1} sign(e(n)). \tag{8}$$

This is a generalization of the LMS algorithm from $p = 2$ to $p < 2$.

The steepest descent and LMP algorithms were implemented for the 1st order AR process described above. We then investigated the behavior of these methods for various values of p and α. Figure 1 shows the convergence of steepest descent for $\alpha = 1.5$ for several values of p. In each case, the step-size parameter was chosen as the largest constant value such that the estimated parameter did not overshoot the true parameter by more than 1%. This typical behavior indicates that choosing p as close as possible to α results in the fastest convergence. A natural upper bound is $p < \alpha$ since the moment does not exist for larger values.

Similar behavior was observed for the LMP algorithm. In this case 50 realizations of the AR $S\alpha S$ process were generated and ensemble statistics computed. Shown in Figure 2 are curves of the asymptotic mean absolute residual error ($E|\hat{w}(n=\infty)-a|$) versus convergence speed for various step sizes and for three different values of p. Again it is clear that using p close to α results in an optimal trade-off between residual error and convergence rate. For the LMP methods, it is possible to use a value of $p \geq \alpha$. However our experience is that in this case the algorithms eventually become unstable for convergence rates in the range shown. Since active noise control systems typically run continuously, it is very important that the adaptive filters remain stable, and hence only values of $p < \alpha$ are used in the following.

3. FRACTIONAL LOWER ORDER MOMENT BASED ACTIVE NOISE CONTROL ALGORITHMS

In the case of active noise control, we adapt the filter coefficients to minimize the dispersion of the residual error as measured at an error sensor. Consider an FIR based ANC system with an input consisting of a stationary $S\alpha S$ noise $\{u(n)\}$. The noise propagates through the plant transfer function producing the signal $\{d(n)\}$ at the location at which the filtered reference signal $\{y(n)\}$ is introduced. The residual error $e(n) = d(n) + y(n)$ is measured after passing through a secondary plant transfer function $\{c_0, c_1, \ldots, c_{N-1}\}$, i.e. $e_2(n) = \sum_{j=0}^{N-1} c_j e(n-j)$. We assume here that the transfer function $\{c_j\}$ has been identified off line. The problem is then to choose the tap weights of the feedforward FIR filter $\{w_0, w_1, \ldots, w_{M-1}\}$ such that the filter output matches the desired plant noise $\{d(n)\}$ as closely as possible in the sense that the pth moment of the error signal is minimized, for some $0 < p < \alpha$. The pth moment is given by:

$$J = E(|e_2(n)|^p) = E(|\sum_{j=0}^{N-1} c_j e(n-j)|^p)$$

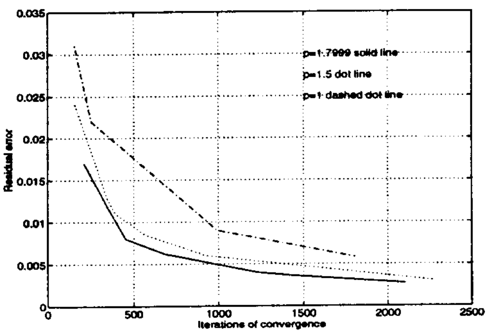
$$= E(|\sum_{j=0}^{N-1} c_j(d(n-j) + \sum_{k=0}^{M-1} w_k u(n-j-k))|^p)$$

We propose the following stochastic gradient method to update the coefficients:

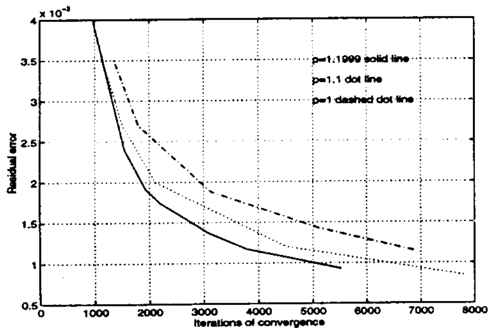
$$\hat{w}_k(n+1) = \hat{w}_k(n) - \mu|e_2(n)|^{p-1} sign(e_2(n)) \sum_{j=0}^{N-1} c_j u(n-j-k) \tag{9}$$

for $0 \leq k \leq M-1$, where $\mu > 0$ is the step size. This is an LMP generalization of the conventional filtered-X algorithm and will be referred to as LMP-filtered-X. It reduces to filtered-X when $p = 2$.

The minimum dispersion approach can also be applied when IIR filters are used to model feedback in ANC systems. Erikson [4] extended the filtered-X concept from FIR to IIR adaptive filters using Feintuch's recursive LMS (RLMS) algorithm. This is commonly referred to as the filtered-U algorithm. Replacing the minimum variance criterion of RLMS with minimization of the p^{th} moment, and applying this to the observed error $e_2(n)$, results in an LMP extension of the filtered-U algorithm. We refer to



$\alpha = 1.8$ for $v(n)$



$\alpha = 1.2$ for $v(n)$

Figure 2: Convergence behavior of the stochastic LMP algorithm for various values of p. Each curve represents a different value of p. The points on each curve correspond to different step sizes and show the coefficient error norm $E[|\hat{w} - a|]$, after convergence, vs. the number of iterations to convergence (defined as the iteration corresponding to 90% of the final residual).

this method as LMP-filtered-U. Specifically, the reference signal is filtered with an IIR filter as follows:

$$y(n) = \sum_{k=0}^{M-1} \hat{a}_k u(n-k) + \sum_{k=1}^{L} \hat{b}_k y(n-k) \qquad (10)$$

where the IIR coefficients, $\{a_k\}$ and $\{b_k\}$, are updated using

$$\begin{aligned}\hat{a}_k(n+1) &= \hat{a}_k(n) - \mu p |e_2(n)|^{p-1} sign(e_2(n)) \\ &\times \sum_j c_j u(n-j-k) \qquad 0 \le k \le M-1 \\ \hat{b}_k(n+1) &= \hat{b}_k(n) - \mu p |e_2(n)|^{p-1} sign(e_2(n)) \\ &\times \sum_j c_j y(n-j-k-1) \qquad 1 \le k \le L\end{aligned}$$

This reduces to the standard filtered-U algorithm for $p = 2$.

The adaptive LMP methods defined above for ANC were simulated in Matlab and also implemented in real time on a TMS320C30 DSP and applied to noise cancellation in an experimental acoustical duct. In order to compare convergence behavior, in both real time experiments and computer simulations we adjusted the adaptive step size so that both the conventional ($p = 2$) and new ($p < 2$) algorithms converge to similar levels of residual noise. Overall we found that with appropriately chosen step sizes and for relatively small values of α (corresponding to highly impulsive noise) our new algorithms achieved extremely low residual noise levels. These low levels could not be obtained with conventional algorithms, regardless of the step size used. As α is increased, the performance of the standard algorithms improved, but generally exhibited slower convergence than the LMP methods for similar levels of residual error.

In Figure 3, we show samples of our computer simulation results for the LMP-filtered-X and LMP-filtered-U algorithms. This figure compares the convergence behavior of both algorithms with the corresponding standard ANC algorithms. The two filtered-X algorithms ($p = 2$ and $p = 1.19$) were tested for stable noise with $\alpha = 1.2$. Convergence for $p = 1.19$ is clearly much faster than for $p = 2$. The two filtered-U algorithms ($p = 2$ and $p = 1.49$) were tested for stable noise with $\alpha = 1.5$. Again, convergence behavior for $p = 1.49$ is much faster than for $p = 2$. In Figure 4, we show the error signal collected for a real time active noise control experiment performed in an acoustic duct, where the noise was generated by passing a pseudo-random i.i.d. $S\alpha S$ sequence ($\alpha = 1.5$) through a 16 bit DAC. The plots show the transient behavior of the residual noise for LMP-filtered-U with $p = 1.4$ and $p = 2$, respectively. In this experiment, we use 200 forward coefficients and 150 feedback coefficients. The secondary path was identified using off line noise injection with a 250-tap FIR filter. The sampling frequency was 4 KHZ.

4. CONCLUSION

We conclude that in environments with impulsive noise that can be well modeled by a $S\alpha S$ distribution, active noise control algorithms based on fractional lower order moments offer the potential for significant improvements over standard algorithms. Our Monte-Carlo simulations indicate that accurate knowledge of α is important for optimal performance. Consequently, it would be appropriate to incorporate on-line estimation of α as part of the adaptive algorithm. The results presented here are preliminary - further theoretical, simulation and experimental studies are required to evaluate the potential and limitations of the LMP methods for active noise control

5. REFERENCES

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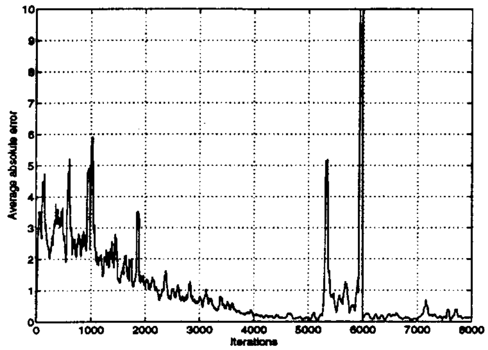
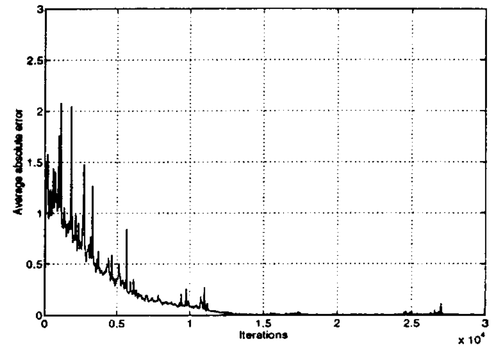
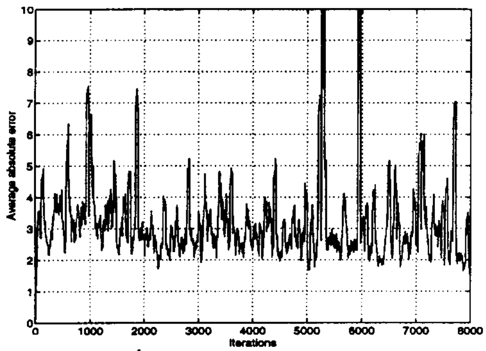


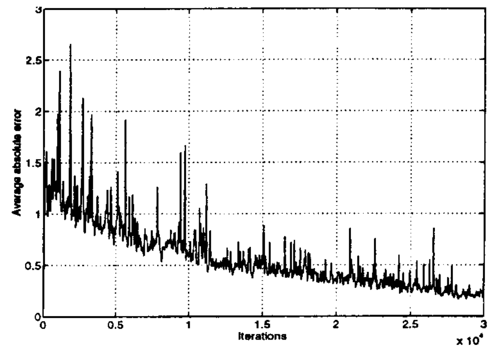
(a) Filtered-X LMP,$p = 1.19$



(b) Filtered-U, $p = 1.49$



(a) Filtered-X LMS



(b) Filtered-U LMS

Figure 3: Sample results from Matlab simulations: The plots show the convergence history of the ensemble average of the absolute error as measured by the error sensor. The ensemble average is computed across 10 realizations. The left column shows results for an FIR system for $\alpha = 1.2$ with $p = 1.19$ (upper) and $p = 2$ (lower). The right column is for the IIR system with $\alpha = 1.5$ for $p = 1.49$ (upper) and $p = 2$ (lower).

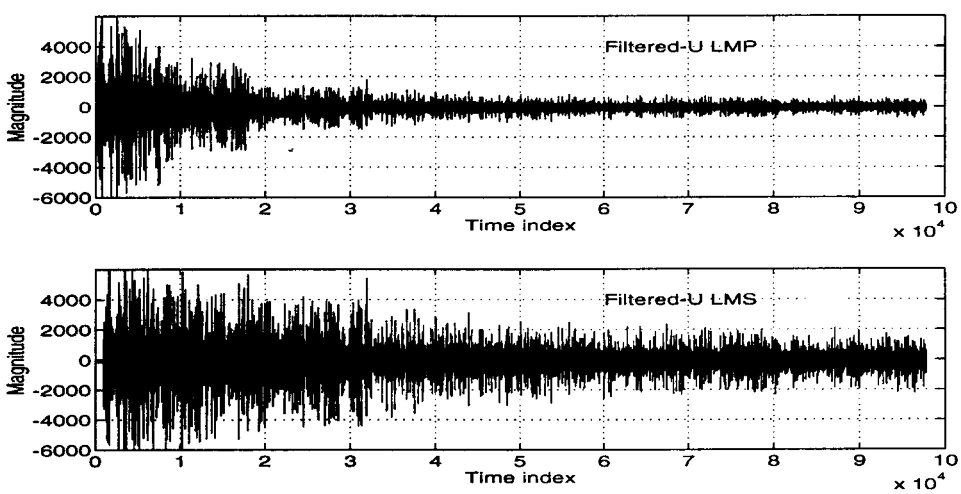


Figure 4: Sample results from real time experiments: The plots illustrate the transient behavior of residual errors when applying the LMP-filtered-U algorithm for $p = 1.4$ (upper) and $p = 2$ (lower). The injected impulsive noise was generated by passing $S\alpha S$ noise, $\alpha = 1.5$, through a 16-bit DAC.