

A COMPARATIVE STUDY OF MULTIPLE TESTS BASED TECHNIQUES FOR OPTIMAL SENSOR LOCATION FOR KNOCK DETECTION

A. M. Zoubir

Signal processing Research Centre, QUT
GPO Box 2434, Brisbane Q 4001, Australia
e-mail: a.zoubir@qut.edu.au

ABSTRACT

A comparative study of techniques for finding optimal sensor positions in a group of vibration sensors for knock detection in spark ignition engines is presented. The methods assume the transmission of the acoustical oscillations in the combustion chamber to the engine housing to be time invariant and linear. Based on this model, suitable multiple tests have been performed to reject sequentially irrelevant sensors from the sensor group under consideration. It was found in various simulations that two of the proposed methods that do not assume any probability distribution of the data lead to the expected results. In a real experiment performed on a test bed using a four cylinder spark ignition engine, the two methods reveal the same optimal sensor location for monitoring knock in two cylinders at three different speeds.

1. INTRODUCTION

An effective means for lowering fuel consumption of spark ignition engines has been to increase the compression ratio. Further increase in compression is limited by the occurrence of *knock* – an extremely fast combustion that generates a knocking or ringing sound [2]. Frequent occurrence of knock has to be avoided because of its damaging effect on the engine, especially when operating at high speed.

Knock can be controlled by adapting the angle of ignition. For an engine with a high compression ratio, the angle corresponding to minimum fuel consumption is located in a region where knock occurs, and thus knock can be avoided only at the cost of decreased efficiency. To attain safe operation at maximum efficiency, engine knock control systems have been developed to detect knock and adapt spark timing of each cylinder independently [1]. The performance of such systems, however, suffers from the low signal-to-noise ratio (SNR) of the structural vibration signal recorded by an accelerometer on the engine wall. In-cylinder pressure monitoring is preferred because of the high SNR of the signal but requires proper mounting of a pressure sensor in each cylinder. Corresponding costs restrict use of in-cylinder pressure to reference purposes in engine or fuel development.

Improvement can be obtained if one can find suitable sensor *positions* of accelerometers for knock detection. A suitably placed vibration sensor improves, independently of the knock detection scheme, the detectability of knocking cycles and permits safe operation with high efficiency.

The double-pulsed laser holography has been proposed for

imaging knock-centres on an engine wall [5], but its application is complicated and does not necessarily lead to a practical solution. Up to now, the sensor positions have been chosen heuristically. Alternative methods that use signal processing and statistical tests to find optimal sensor positions within a group of sensors distributed on the engine wall have been proposed in [3, 4, 5]. In this contribution, a comparative study of these techniques is presented.

The paper is organised as follows. We briefly illustrate in section 2 the methods proposed in [3, 4, 5]. We then, in section 3, compare their performance with simulations and present experimental results obtained on using a 1.8ℓ, four cylinder spark ignition engine, before we complete the paper with discussions.

2. SENSOR IRRELEVANCY TESTS

Let $S(t)$ and $Z_a(t)$, $t = 0, \pm 1, \pm 2, \dots$ be zero-mean stationary processes that model, respectively, the in-cylinder pressure signal recorded via a pressure transducer for reference purposes, and the vibration signal of sensor a , $a = 1, \dots, r$.

Optimal sensor location concerns the selection of sensors whose signals permit detection of knock with high power. Our idea is to select a subvector of $\mathbf{Z}(t) = (Z_1(t), Z_2(t), \dots, Z_r(t))'$ such that its transformation by time invariant and linear operations is near the reference signal $S(t)$ in some sense. This will lead to a minimum number of vibration sensors that are suitably placed for detecting knock in a spark ignition engine.

2.1. An Inverse Filter Approach

Let $\mathbf{Z}(t) = (\mathbf{X}(t)', Y(t))'$, where $\mathbf{X}(t)$ is an arbitrary $(r - 1)$ vector-valued series. Based on the contribution of $Y(t)$ to approximate $S(t)$ by means of time invariant and linear operations from $\mathbf{Z}(t)$, we develop tests for irrelevancy of the sensor with output signal $Y(t)$ [5].

Assume $C_{ZZ}(\omega)$ and $C_{SS}(\omega)$ to be known a priori or by estimation. We seek for suitable time invariant linear filters with impulse responses $\mathbf{g}_1(t)$ and $\mathbf{g}_2(t)$ to recover $S(t)$ from $\mathbf{Z}(t)$ and $\mathbf{X}(t)$, respectively. We construct estimates $\hat{S}_1(t) = \sum_{u=-\infty}^{\infty} \mathbf{g}_1(u)\mathbf{Z}(t-u)$ and $\hat{S}_2(t) = \sum_{u=-\infty}^{\infty} \mathbf{g}_2(u)\mathbf{X}(t-u)$ of $S(t)$ with errors $\mathcal{E}_1(t) = S(t) - \hat{S}_1(t)$ and $\mathcal{E}_2(t) = S(t) - \hat{S}_2(t)$, respectively. The minimisation of $\mathcal{E}_1(t)$, for example, with respect to $\mathbf{g}_1(t)$, in the mean square sense leads to a prediction filter whose frequency response is given by $\mathbf{G}_1(\omega) = C_{SZ}(\omega)C_{ZZ}(\omega)^{-1}$ and the error spectrum is given by

$$C_{\mathcal{E}_1\mathcal{E}_1}(\omega) = (1 - R_{SZ}^2(\omega))C_{SS}(\omega) \quad (1)$$

where

$$R_{SZ}^2(\omega) = \frac{C_{SZ}(\omega)C_{ZZ}(\omega)^{-1}C_{ZS}(\omega)}{C_{SS}(\omega)}, \quad C_{SS}(\omega) > 0 \quad (2)$$

is the multiple coherence of $S(t)$ with $Z(t)$ at frequency ω .

The approximation of $S(t)$ from $X(t)$ by linear time invariant filtering leads to an error spectrum similar to (1) except that $R_{SZ}^2(\omega)$ is substituted by $R_{SX}^2(\omega)$, which is obtained from (2) by replacing Z by X .

Removal of a sensor from the group and use of only $X(t)$ for approximating $S(t)$ leads to an increase of error spectrum that is proportional to $0 \leq R_{SZ}^2(\omega) - R_{SX}^2(\omega) \leq 1$, which can be seen as the coherence gain contributed by the sensor with output signal $Y(t)$. The lower this coherence gain, the less is the contribution of $Y(t)$ for approximating $S(t)$ from $Z(t) = (X(t)', Y(t)')'$.

Thus, based on the closeness to zero of $R_{SZ}^2(\omega) - R_{SX}^2(\omega)$ at some frequencies of interest, we shall decide the irrelevancy of the sensor with output signal $Y(t)$ by testing the hypothesis $H: R_{SZ}^2(\omega) - R_{SX}^2(\omega) \leq R_0^2(\omega)$ against the one-sided alternative, where $R_0^2(\omega)$ is a suitably chosen bound within the interval $(0, 1)$. Details are found in [5].

2.2. Linear Regression Based Techniques

Consider the linear prediction model

$$S(t) = \sum_{a=1}^r \sum_{u=-\infty}^{\infty} g_a(u) Z_a(t-u) + \mathcal{E}(t), \quad (3)$$

where $g_a(\cdot)$ is the a th impulse response of the prediction filter and $\mathcal{E}(t)$ is the prediction error series. Based on (3), we derive suitable tests to identify stepwise sensor signals $Z_a(t)$, $a = 1, \dots, r$, whose contribution for predicting the reference $S(t)$ at certain frequencies is negligible, which results in optimal sensor positions within the group of r sensors.

Given $S^{(l)}(t)$ and $Z_a^{(l)}(t)$ for $t = 0, \dots, T-1$, $a = 1, \dots, r$, $l = 1, \dots, L$, where L is the number of independent combustion cycles, we compute the finite Fourier transforms of the time series data, and (3) becomes

$$\mathbf{d}_S(\omega) \approx \mathbf{d}_Z(\omega)\mathbf{G}(\omega) + \mathbf{d}_\mathcal{E}(\omega), \quad (4)$$

where $\mathbf{d}_S(\omega) = (d_{S(1)}(\omega), \dots, d_{S(L)}(\omega))'$, $\mathbf{d}_Z(\omega) = (d_{Z_1}(\omega), \dots, d_{Z_r}(\omega))$, $\mathbf{d}_{Z_a}(\omega) = (d_{Z_a(1)}(\omega), \dots, d_{Z_a(L)}(\omega))'$, $a = 1, \dots, r$, and $\mathbf{G}(\omega) = (G_1(\omega), \dots, G_r(\omega))'$ being the vector of unknown transfer functions of the prediction filter. Equation (4) represents for each frequency ω a complex regression with the complex coefficients $G_a(\omega)$, $a = 1, \dots, r$.

2.2.1. The Backward Elimination

The principle of this procedure is to eliminate stepwise irrelevant sensors from a group of arbitrarily distributed sensors on the engine wall, based on the linear regression (4). The number of remaining sensors is determined by the accuracy of the regression. In the procedure, we identify stepwise the sensor l so that $\hat{R}_{SZ_l, Z^{(l)}}^2(\omega) = \min_a(\hat{R}_{SZ_a, Z^{(a)}}^2(\omega))$, $a = 1, \dots, q$, and test $H: R_{SZ_l, Z^{(l)}}^2(\omega) \leq R_0^2(\omega)$ against the one-sided alternative, where $R_0^2(\omega)$ is a suitably chosen bound, $R_{SZ_a, Z^{(a)}}^2(\omega)$ is the conditional coherence of $S(t)$

with $Z_a(t)$ at frequency ω after removing the linear effects of $Z^{(a)}(t)$, being a $q-1$ subvector of $Z(t)$ that excludes $Z_a(t)$, $a = 1, \dots, q$ and q is the number of sensors remaining in the group at this step. For a given level of significance α the hypothesis H is rejected if the statistic $\hat{R}_{SZ_l, Z^{(l)}}^2(\omega)$ is too small. In this case, the procedure stops and all q sensors are kept. Otherwise, the l th sensor is removed and the procedure is repeated with $q-1$ sensors [4].

2.2.2. Rank Based Tests

Assuming independence and identical distribution of the real and imaginary part of $\mathbf{d}_\mathcal{E}$ in (4), we obtain the real valued linear regression

$$\mathbf{Y}(\omega) = \mathbf{V}(\omega) \mathbf{B}(\omega) + \mathbf{E}(\omega). \quad (5)$$

Herein, $\mathbf{Y}(\omega) = (\text{Re}\{\mathbf{d}_S(\omega)'\}, \text{Im}\{\mathbf{d}_S(\omega)'\})'$ and $\mathbf{B}(\omega)$, $\mathbf{V}(\omega)$ contain the suitably arranged real and imaginary parts of the prediction filter frequency responses and the Fourier transformed sensor signals, respectively.

To find sensors that are optimally positioned for knock detection, we use a nonparametric test to first decide whether there is no sensor giving relevant information on the reference signal by testing the hypothesis $H: \mathbf{B}(\omega) = 0$ against the two-sided alternative. In the case of rejection, we then apply a test to decide the relevancy of additional sensors.

We use the quadratic rank test proposed by Adichie as discussed in [3]. We calculate (omitting ω)

$$M_L = \frac{(\mathbf{V}'\psi_L(\mathcal{R}_Y(\mathbf{Y})))'(\mathbf{V}'\mathbf{V})^{-1}(\mathbf{V}'\psi_L(\mathcal{R}_Y(\mathbf{Y})))}{A^2(\psi)}, \quad (6)$$

wherein $\psi_L(\mathcal{R}_Y(\mathbf{Y}))$ is a function of the ranks $\mathcal{R}_Y(\mathbf{Y})$ of \mathbf{Y} with $\psi_L(i) = \psi(i/(2L+1))$ and $A^2(\psi) = \int \psi(u)^2 du - \bar{\psi}^2$, $\bar{\psi} = \int \psi(u) du$. For example, for the Wilcoxon scores generated by $\psi(u) = u$, $A^2(\psi) = 1/12$ [3].

For the decision, whether additional sensors improve detection, it is necessary to test only some components of \mathbf{B} . For this, we re-write (5) in the form

$$\mathbf{Y}(\omega) = \mathbf{V}_1(\omega) \mathbf{B}_1(\omega) + \mathbf{V}_2(\omega) \mathbf{B}_2(\omega) + \mathbf{E}(\omega), \quad (7)$$

wherein $\mathbf{B}(\omega) = (\mathbf{B}_1(\omega)', \mathbf{B}_2(\omega)')'$ and $\mathbf{V}(\omega) = (\mathbf{V}_1(\omega), \mathbf{V}_2(\omega))$. The vectors $\mathbf{Y}(\omega)$ and $\mathbf{E}(\omega)$ are $2L$ vector-valued as before. \mathbf{B}_s is $2r_s$ vector-valued and the matrix $\mathbf{V}_s(\omega)$ has dimension $2L \times 2r_s$ for $s = 1, 2$. It is of interest to test $H_2: \mathbf{B}_2(\omega) = 0$, $\mathbf{B}_1(\omega)$ unspecified, against the alternative $K_2: \mathbf{B}_2(\omega) \neq 0$. Herein, $\mathbf{B}_1(\omega)$ corresponds to already chosen sensors and $\mathbf{B}_2(\omega)$ to further ones. A test similar to the one given above that uses a statistic \check{M}_L has been proposed by Adichie and is discussed in [3].

2.3. Multiple Tests

In the tests given above, we held the frequency ω fixed. In knock detection one has to consider P resonance frequencies and also frequencies in the neighbourhood of the resonance frequencies, because of the dampings. Thus, we test the multiple hypotheses $H_{1,-m}, \dots, H_{1,m}, \dots, H_{P,-m}, \dots, H_{P,m}$ and the global hypothesis $H_0 := \bigcap_{p=1}^P \bigcap_{k=-m}^m H_{p,k}$, where $H_{p,k}$ is any of the hypotheses discussed above at frequency $\omega_{p,k}$, $k =$

Table 1. SNRs and resonance modes of the simulated signals.

Sensor	S1	S2	S3	S4	S5	S6	S7	S8
SNR [dB]	-23	-28	-28	-28	-28	-28	-28	-28
Modes	1,2	1,3	1,4	2,3	2,4	3,4	4	3

$-m, \dots, m$, which is in the neighbourhood of the resonance frequency ω_p with $\omega_{p,0} = \omega_p$, $p = 1, \dots, P$. For testing the multiple hypotheses, we applied Holm's generalised sequentially rejective Bonferroni multiple test [4, 5], except for the rank based tests where a sum statistic has been used [3]. We also proposed a method for finding optimal sensor positions to control knock in more than one cylinder at various rotation speeds.

3. SIMULATION AND EXPERIMENTAL RESULTS

3.1. Simulation Results

To investigate the proposed selection procedures, we have simulated a pressure signal and various sensor signals. The process $S(t) = \sum_{p=1}^P A_p e^{-d_p t} \cos(\omega_p t + \Phi_p) + U(t)$ has been used to model the reference signal in knock analysis. Herein, A_p and Φ_p are independently and uniformly distributed random amplitudes and phases, respectively; d_p are dampings; ω_p are the cavity resonance frequencies for $p = 1, \dots, P$; and $U(t)$ is a white Gaussian noise process. We have fixed $P = 4$ and generated $L = 200$ records (except for the bootstrap tests where only 15 cycles were used) of length $T = 128$ each. We have then simulated eight sensor signals. For this, some of the bands centered at the four resonance frequencies have been filtered out. These are numerated in increasing order and given in Table 1. Noise has been added to generate a sensor signal. The estimated SNRs of the sensor signals are given in Table 1. Sensor S1 has the highest SNR and is the optimal one. Clearly, the method should be able to find the group (S1,S6) if one is interested in two sensors in the case of a six cylinder engine, for example. This is because the output signal of S6 contains the missing resonances in the signal of S1.

In the simulations, we have considered two neighbouring frequencies ($m = 1$) to each resonance frequency. The tests of the hypotheses were applied at a multiple level of significance $\alpha = 5\%$.

3.1.1. The Inverse Filter Approach

The irrelevancy of an arbitrary sensor was tested by interchanging the components of the vector $\mathbf{Z}(t)$ and each time performing a multiple bootstrap test based on Holm's generalised sequentially rejective Bonferroni procedure. In Table 2 the number of rejected hypotheses is noted for each tested sensor S_i , $i = 1, \dots, 8$, that is the sensor with output signal $Y(t)$. In step 1, the vector $\mathbf{Z}(t)$ was composed of all eight sensor signals ($r = 8$) and the bound R_0^2 was taken to be 0.04. H was retained for the sensors S2, S4, S6, S7 and S8. Sensor S7 was removed because its signal led to the lowest mean value of the sample coherence gain over the frequencies of interest. In step 2, the test was reconstructed with $r = 7$ and $R_0^2 = 0.05$. For the same reason as above, we removed sensor S4 and proceeded to the next step. In step 6, H was retained for sensor S3 leading thus

Table 2. Number of rejected hypotheses for each sensor test.

Sensor	S1	S2	S3	S4	S5	S6	S7	S8
Step								
1	3	0	2	0	2	0	0	0
2	3	0	2	0	1	0	-	0
3	4	0	2	-	2	0	-	0
4	3	-	2	-	1	0	-	0
5	2	-	1	-	0	2	-	-
6	2	-	1	-	-	1	-	-
7	4	-	-	-	-	0	-	-
Rank	1	-	-	-	-	2	-	-

Table 3. Number of rejected hypotheses for each sensor test.

Sensor	S1	S2	S3	S4	S5	S6	S7	S8
Step								
1	4	1	3	1	3	0	1	1
2	4	1	3	1	3	-	1	1
3	4	2	2	1	2	-	1	-
4	4	2	2	1	1	-	-	-
5	4	1	2	-	2	-	-	-
6	4	-	1	-	1	-	-	-
7	4	-	-	-	1	-	-	-
Rank	1	-	-	-	2	-	-	-

to the expected result. If one is interested in a group of two sensors, the group (S1,S6) is found to be optimal. If only one sensor is required, the tests in step 7 lead to sensor S1. In Table 2, the bound R_0^2 was varied from 0.04 to 0.1 in 0.01 increments.

3.1.2. The Backward Elimination

In Table 3, S_i , $i = 1, \dots, 8$ denotes the sensor whose irrelevancy is to be tested, that is, the sensor with output signal $Z_a(t)$. In step 1, the vector $\mathbf{Z}(t)$ was composed of all eight sensor signals ($q \equiv r = 8$) and the bound R_0^2 was taken to be 0.042. The global hypothesis H_0 is retained surprisingly for sensor S6. It should have been sensor S7 or S8. In step 2, the test was re-constructed with $q = 7$ and $R_0^2 = 0.045$. The lowest number of rejected hypotheses occurs with sensors S2, S4, S7 and S8. We calculated the mean values over all frequencies of the conditional coherences of the sensor signals and found that the lowest value was attained by sensor S8. Therefore, we removed the latter and proceeded to the next step. In step 6, the global hypothesis H_0 is rejected for all sensors. For the same reason as above, we removed sensor S3 from this group. If one is interested in a group of two sensors, the group (S1,S5) is found to be optimal, which is incorrect. If only one sensor is required, the tests in step 7 lead to sensor S1 which is the ideal. In the simulations, the bound R_0^2 varied from 0.042 to 0.084, which was based on $\frac{1}{q} \sum_{a=1}^q \frac{1}{P} \sum_{p=1}^P \sum_{k=-m}^m \hat{R}_{S_{Z_a}, Z^{(a)}}^2(\omega_{p,k})$.

3.1.3. Rank Based Tests

Under the assumption that the M_L statistics at different frequencies are independent, which holds for large T , we may test the irrelevancy of a sensor by considering $\mathcal{M}_L = \sum_{p=1}^P \sum_{k=-m}^m M_L(\omega_{p,k})$. Under the hypothesis *no regression possible*, i.e. the sensor is irrelevant, this sum is approximately chi-square distributed with $2r(2m+1)P$ degrees of freedom. We reject H_0 if, for a given level

Table 4. \mathcal{M}_L and $\check{\mathcal{M}}_L$ of the simulated signals. Selected sensors appear boldfaced.

Sensor	S1	S2	S3	S4	S5	S6	S7	S8
Step								
1	238	194	196	181	193	170	130	86
2	-	143	157	138	152	165	127	78
3	-	111	130	112	126	-	91	50
4	-	97	-	108	117	-	75	50
Rank	1	-	3	-	4	2	-	-

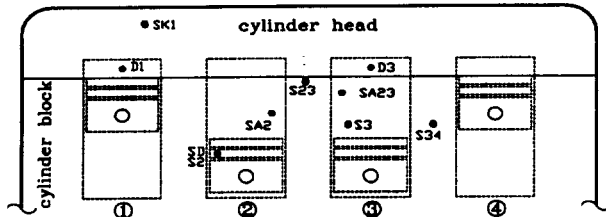


Figure 1. Position of sensors on the engine.

of significance α , the statistic \mathcal{M}_L is too large. Similarly we construct $\check{\mathcal{M}}_L = \sum_{p=1}^P \sum_{k=-m}^m \check{\mathcal{M}}_L(\omega_{p,k})$ to test $\check{H}_0 := \bigcap_{p=1}^P \bigcap_{k=-m}^m H_{2,p,k}$, $H_{2,p,k}: \mathbf{B}_{2,p,k} = 0$ against the two-sided alternative.

In order to find the optimally placed sensor group, we first test whether a single sensor is suitable to predict the reference signal. For this purpose, we compute \mathcal{M}_L . Sensors for which the hypothesis H_0 is not rejected are removed from the group. Among the remaining sensors, the one leading to the highest \mathcal{M}_L is chosen. In the following steps, the gain reached by adding a further sensor to the already determined one is tested. Using $\check{\mathcal{M}}_L$, we remove sensors for which the global hypothesis \check{H}_0 is not rejected and select the one with the highest $\check{\mathcal{M}}_L$. This procedure is repeated until enough sensors are determined or no sensor is left. Table 4 shows that sequentially sensor S1 and sensor then S6, whose signal contains the missing resonances 3 and 4 in the signal of sensor S1, are selected.

These results show that the bootstrap and the rank based tests are superior. They both lead to the expected results. It should be noted, however, that more simulations were performed. These were based on generating sensor signals with correlated noise but with all resonance modes present in the signals. In these situations the backward elimination procedure led to the expected results [4].

3.2. Experimental Results

Experiments were performed using a 1.8 l, 4 cylinder engine. The high-pass filtered pressure signals of the cylinders 1 and 3 (D1 and D3 in Figure 1) were recorded by an instrumentation tape recorder together with the output signals of eight acceleration sensors. Two of these sensors (sensor SD and S2 in Figure 1) had been mounted at the place suggested by the engine manufacturer, the other ones at heuristically chosen positions on the engine wall and on the cylinder head (sensor SK1). Two sensors, SA2 and SA23 had been mounted at the outlet side of the engine block, all others at the inlet side. Figure 1 schematically shows these positions.

Experiments were performed at 1750, 3500 and 5250 rpm

and at full load. Strong knock intensity was adjusted. Three thousand cycles were digitized with respect to cylinders 1 and 3 for the analyses. Approximately 200 strong knocking cycles for each rotation speed and each cylinder were chosen for the tests, except for the bootstrap tests where only the strongest 15 cycles were selected.

Four resonance frequencies were estimated by averaging periodograms of the knocking cycles of $S(t)$ and localising spectral peaks. Two neighbouring frequency bins to each resonance frequency have also been considered.

Multiple tests at a multiple level of 5% based on two in-cylinder pressure signals and three rotation speeds suggested the pair (S23, S34) in the case of the inverse filter approach and the pair (S23, SA23) with the rank based tests. This is a result if one is to use two sensors for detection. However, in our case where a four cylinder engine was investigated only one sensor is required. Both methods suggest the same single sensor S23.

4. DISCUSSION

We have discussed methods for finding stepwise (ir)relevant sensors from an arbitrary array of vibration sensors for use in detection of knock in spark ignition engines. This is based on a linear prediction of the in-cylinder pressure as a reference from vibration signals by linear time invariant operations. The techniques have been tested by simulations. It was found that rank based as well as bootstrap tests lead to the expected results. Both methods do not assume any distribution of the sensor data. Results obtained for engine data measured on a test bed suggest sensor positions that do not include the position proposed by the engine manufacturer. It should be, however, noted that the proposal made by the manufacturer was based on heuristical considerations. The methods developed can be easily applied to other problems encountered in vibration analysis, provided a suitable reference signal is available. Some applications of importance include the detection of wear or breakage of cutting tools in a milling process.

ACKNOWLEDGEMENTS

This work was performed while the author was with the signal theory division at Ruhr University Bochum in Germany.

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