HIDDEN PROCESS MODELING*

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ABSTRACT

In this paper, we present a method that is a generalization of hidden Markov modeling for the situations where elementary events cannot be clearly defined. A family of fuzzy sets, induced on a *temporal universe*, is used to model the dynamic trajectory of a physical system as a collection of hidden processes that coexist at the same time, but to different degrees. An algorithm based on unsupervised pattern recognition that estimates the prototypes and activities of the hidden processes is presented. The performance of the method is illustrated using experimental data obtained from electroencephalographic (EEG) signals recorded during sleep.

1. INTRODUCTION

Some real-world systems, biological and others, can be thought of as being *attracted* to specific modes of dynamic behavior during their existence. This property may be observed within small as well as large time scales. Unfortunately, the modes of dynamic activity cannot always be precisely defined [1]. Very often the relevant information about a physical process is unevenly distributed among a large number of signals, some of which are observable and some of which are not. Finding the regularities or patterns that repeat within measured data and quantifying the observed dynamic changes becomes an important step towards the better understanding of system functionality.

Hence, given a collection of signals measured from a physical process, we want to analyze the system behavior in an *unsupervised* manner. That is, we try to infer the dynamic structure of a physical system from the properties of its dynamic trajectory without forcing the use of specialized mathematical models that are both hard to interpret and may not relate to reality. Generally, the system inputs are unknown and prior knowledge about the internal system structure is not available. One basic assumption is that the physical process resulting in the observed system outputs can be characterized by a dynamic trajectory in a suitable feature space. Then, the trajectory is assumed to be traveling through several regions of attraction and connecting transient paths as shown in Fig. 1a.

In this paper we present a method that results in a reasonable, easy to interpret characterization of processes that are generated by complex physical systems. Our method provides the initial model based on simple output signal observations, application of unsupervised pattern recognition techniques, flexibility in the inclusion of heuristic knowledge at the later stages of development, and a final form suitable for further analysis.

The rest of the paper is organized as follows. In Sections 2 and 3 we briefly overview the basic concepts of hidden Markov modeling and temporal fuzzy sets, respectively. Building on this background the hidden process modeling is developed in Section 4. A practical realization of the method is presented in Section 5 and applied to the analysis of sleep dynamics. Finally, possible modifications and implementation problems are discussed in Section 6.

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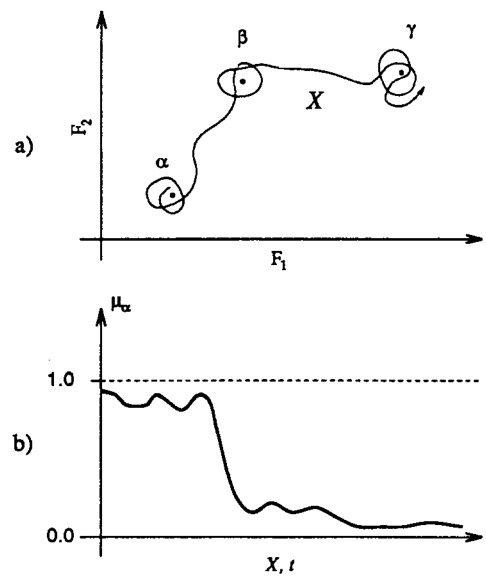

Figure 1. a) A dynamic trajectory X in two dimensional feature space, $\mathcal{F} = F_1 \times F_2$, with three regions of attraction centered at the poles α, β, and γ, that are indicated by bullets. b) A membership function, μ_α, indicates the level of influence the region of attraction around pole α has to the overall dynamics. It corresponds to a temporal fuzzy set which models that region of attraction.

2. HIDDEN MARKOV MODELS

If a real-world process produces a sequence of well defined symbols, hidden Markov modeling (HMM) may be used in building a signal model that explains and characterizes the occurrence of the observed symbols. If such a model is obtainable, it can be used later to identify or recognize other sequences of observations. The HMM, as defined in [2], "is a doubly stochastic process with an underlying stochastic process that is not observable (it is hidden), but can be observed through another set of stochastic processes that produce the sequence of observed symbols." Namely, a HMM breaks the physical process in two levels, as shown in Table 1. The upper level (UL) contains a sequence of symbols $\{O_i\}$ that is observable. The lower, or hidden level (LL) is based on the assumption that at each time unit, the process behavior is governed by a *single* hidden state, q_j. The elements of the output

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Table 1. A HMM with $c = 4$ hidden states, and a finite set of admissible observation symbols, $V = \{v_1, v_2, v_3\}$. At each time unit two random experiments are performed. One to determine the output symbol given the current state, and the other to decide about the possible state transition.

Observations (UL)	v_1	v_3	v_2	v_2	$\cdots$	v_3
Hidden states (LL)	q_4	q_4	q_1	q_3	$\cdots$	q_2
Clock time (i)	1	2	3	4	$\cdots$	T

sequence $\{O_i\}$ belong to a set of admissible observation values V that in general may be uncountable. A set of all admissible states is $Q = \{q_1, \ldots, q_c\}$, where $c \in N$ is the total number of states.

The success of hidden Markov modeling, particularly in speech processing, is attributed to a fact that many real-world processes seem to manifest a kind of sequentially changing behavior [2]. The properties of the process are usually held steady, except for minor fluctuations, for a certain period of time, and then, at certain instances, a gradual change to another set of properties occurs. Unfortunately, in situations where the fluctuations within steady, as well as transitional, periods are important for proper understanding of system dynamics, the HMM approach may not be appropriate. Moreover, in a number of practical situations, the steady periods cannot be clearly separated into disjunct classes using the available set of features which further obstructs the estimation of model parameters.

3. TEMPORAL FUZZY SETS

Temporal fuzzy sets [3, 4] are the fuzzy sets [5] constructed from a universe, elements of which are ordered in time. Every fuzzy set, A, constructed from a temporally ordered universe, X, belongs to this family. For example, consider a physical system governed by an ordinary differential equation

$$\frac{dx}{dt} = \mathbf{G}(t, x) \tag{1}$$

where $t \in \mathcal{T} = [t_0, \infty)$, $x \in R^p$, and $\mathbf{G}(\cdot,\cdot)$ is a real valued vector function that is Lipschitz continuous on a rectangle $\Gamma \subset \mathcal{T} \times R^p$, [6]. The unique solution of (1), given initial conditions (t_0, x_0) may be written as

$$x_t = \mathbf{X}(t) \tag{2}$$

and may not be obtainable in closed form. A vector function $\mathbf{X}(\cdot)$ in (2) represents the *state space trajectory* [6] of a physical process governed by (1). Considering the observation interval $\mathcal{T}_o = [t_0, t_1]$, and using the dynamic trajectory $\mathbf{X}(\cdot)$, one can generate the universe of objects $X^* = \mathbf{X}(\mathcal{T}_o)$ with elements x_t in a temporal order. Given an arbitrary fuzzy set A on X^*, the vector function $\mathbf{X}(\cdot)$ induces a fuzzy set B on $\mathcal{T}_o$ as suggested in [5]. That is, the membership function of B is defined to be

$$\mu_B(t) = \mu_A(y), \quad y \in X^* \tag{3}$$

for all $t \in \mathcal{T}_o$ that belong to the inverse image of y. Using '$\prec$' to denote a temporal order, we say that the resulting fuzzy set, B, induced from the dynamic trajectory $(X^*, \prec)$, is a *temporal fuzzy set* [4]. For clarity, $A^{\prec}$ is used to denote a temporal fuzzy set induced from a fuzzy set A on $(X^*, \prec)$. The influence of the properties modeled by a fuzzy set A in X^* to the overall dynamics may be quantified at each time instant, $t \in \mathcal{T}_o$, with the membership value $\mu_A^{\prec}(t) = \mu_A(x_t)$. The membership functions of temporal fuzzy sets are *functions of time* (see Fig. 1b). Whenever the universe of discourse is a dynamic trajectory, the temporal fuzzy sets correspond to the regions of attraction [3, 4]. If the trajectory corresponds to a quasi-stationary process, the temporal fuzzy sets (and regions of attraction) coincide with the stationary components [7]. Hence, the temporal fuzzy sets provide for the *dynamic profile* of a physical process [3]. The membership functions of temporal fuzzy sets may be estimated using the fuzzy partitioning algorithms [8] as described in [3].

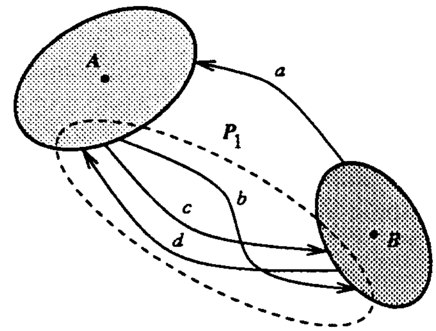

Figure 2. A single dynamic trajectory with two regions of attraction that are centered around the poles A and B. The parts of a trajectory within the regions of attraction are not shown. The transition links b-d are grouped into a transition pathway P_1, while the link a is isolated.

We construct temporal fuzzy sets in such a way that they characterize the regions of attraction in a space where the trajectory lies. In general, that is the feature space $\mathcal{F}$, and the feature space trajectory $(X, \prec)$. The *regions of attraction* are then considered the areas in space $\mathcal{F}$, "visited" by the feature vector during *prolonged* periods of time and in a specific pattern or sequence. The fragments of a trajectory that connect the regions of attraction are called *transition links*. If several links are "close" to one another they form a *transition pathway*. These concepts are illustrated in Fig. 2. In general, regions and pathways can overlap. Hence, it is often hard to distinguish one from the other. Pathways with a large number of links may "look" like the regions of attraction.

4. HIDDEN PROCESS MODELING

A collection of temporal fuzzy sets, that may be obtained using a fuzzy clustering algorithm [9], characterizes the dynamic activity of a system and is composed of the membership functions and cluster prototypes. Membership functions quantify *the activity*, while the prototypes provide quantitative physical characterization of *the activity*. Hence, one can think of *the activity* as a collection of *hidden processes* that coexist at the same time but to different degrees. That is the core of the *hidden process modeling* (HPM) concept.

While in HMM the observation symbol at any time instant is generated by a single *crisp* state, the HPM permits the same observation to be a joint product of concurrent dynamic tendencies. The HPM approaches the observed physical process in a different manner. Instead of forcing the process realization into a number of discrete states with a collection of probability distributions that describes the transitions and observations, we start by observing the regions of attraction in a feature space. Then, each region is characterized by a temporal fuzzy set, or a hidden process. At any time, the hidden processes coexist to the degrees quantified by their membership functions. Hence, the hidden states of a HMM are replaced by the hidden processes.

The set of admissible symbols V of a HMM is replaced by the set of admissible values that a dynamic trajectory $(X, \prec)$ can take during the observation. A natural question is where does the randomness go? There are at least three possible answers:

(i) the physical process may be considered deterministic;

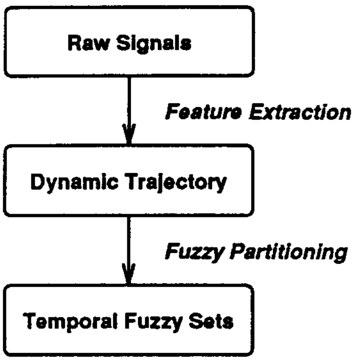

Figure 3. Raw signals are used to obtain the dynamic trajectory in a feature space. As a result of fuzzy partitioning the temporal fuzzy sets are induced in a form of dynamic profile with the membership functions that quantify the activity of hidden processes.

(ii) the membership values at each time constitute an outcome of a *fuzzy event*, [10]. Fuzzy events in this case may be modeled as fuzzy sets, [11], while the transitions between the fuzzy events are random; and,

(iii) the HPM is reduced to a classical HMM by letting only one hidden process exist at any given time, i.e. by restricting the membership functions of temporal fuzzy sets to a binary range $\{0, 1\}$ and forcing them to be crisp.

Assumption (i) is especially useful when prediction is not at issue, that is when considering only the system *analysis*. While the third possibility has been heavily explored under the name of hidden Markov modeling, the prediction that follows from (ii) is still an open research problem. One possible way of solving this problem is suggested in [12]. In summary, given the relationship between probability and fuzziness [13], and the way the temporal fuzzy sets are constructed, one can consider the hidden process modeling as a generalized hidden Markov modeling.

5. PRACTICAL IMPLEMENTATION

The hidden process modeling may be used most readily in analysis of dynamic systems and their related signals. A block diagram in Fig. 3 outlines a general template for implementing the HPM. Although a number of fuzzy partitioning algorithms can be used in estimating the membership functions [3], the fuzzy c-means (FCM) clustering [9] is the easiest to implement, the best understood, and the most studied in the literature.

The FCM algorithm minimizes the least-squares functional that is given by a generalized within-groups sum of square errors function [14]:

$$J_m(U, z) = \sum_{k=1}^{n} \sum_{i=1}^{c} u_{ik}^m d_{ik}^2, \qquad (4)$$

where U is a fuzzy c-partition of X with the elements $u_{ik} \in [0, 1]$, each column summing to unity; $z = (z_1, z_2, \ldots, z_c) \in R^{cp}$, with $z_i \in R^p$ as the cluster center or prototype of the i^{th} class; $d_{ik}^2 = \|x_k - z_i\|^2$, with $\|\cdot\|$ being any inner product induced norm on R^p; and *weighting* or *fuzzy exponent* $m \in (1, \infty)$. The optimum is reached when the fuzzy partition matrix U^* and a collection of prototypes z^* are found such that J_m is minimized. That is, when the weighted within-groups sum of distances between the samples and the prototypes is the smallest possible. The necessary

```
begin
  Fix c, 2 ≤ c < n;
  Choose any inner product norm metric for R^p;
  Fix m, 1 ≤ m < ∞; Initialize U;
  for l := 0 step 1 until maxiter − 1 do begin
    Calculate the c cluster centers {z_i} with (6) and U;
    Using (5) and {z_i} obtain U_new;
    if ||U_new − U||_∞ ≤ ε then stop;
    U := U_new;
  end
end.
```

Figure 4. Pseudo-code for the fuzzy c-means algorithm.

conditions for minimization of J_m can be written as

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d_{ik}}{d_{jk}} \right)^{\frac{2}{m-1}}}, \qquad (5)$$

and

$$z_i = \frac{\sum_{k=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m}, \quad \forall i. \qquad (6)$$

The singularities in (5) can be resolved as suggested in [14]. The pseudo-code is shown in Fig. 4. When the samples originate from a feature space trajectory, the membership functions of temporal fuzzy sets are calculated as $\mu_i^{\prec}[k] = u_{ik}$, while the poles of attraction are characterized by the cluster prototypes z_i, [3]. The clustering validity issues for the case of temporal fuzzy sets are discussed in [3].

In Fig. 5 we applied the HPM to model sleep dynamics from electroencephalographic (EEG) signals. It has been observed that the HPM captures the changes in sleep dynamics that cannot be detected using traditional analysis [15]. The prototypes of the temporal fuzzy sets (Fig. 5b) are calculated as the cluster centers using (6) and a time-frequency based feature space [7].

6. CONCLUSION

In the hidden Markov models, by forcing system states to be crisp, the information about actual dynamic activity is lost. The HPM approach treats the physical systems differently. An important advantage of the HPM comes from an observation that complex dynamic processes are very likely to be composed of a large number of concurrent processes *all of which are active at all times but to different degrees*. This is particularly true for physiological processes, where often several systems act together to produce a summary process. In other words, a HPM appears to be a natural tool in such situations. Another advantage of the HPM is that data fusion may be easily accomplished by extracting the features from different signals and then estimating temporal fuzzy sets from the fused feature space. Expert knowledge about the problem can be embedded within the fuzzy partitioning scheme to constrain the way the temporal fuzzy sets are estimated. Furthermore, the HPM is a basic step towards a more general methodology of *signal analysis in fuzzy information space* [12], where the membership functions of temporal fuzzy sets are used as a time-series that extracts the most significant information about the dynamic behavior of a physical process.

In practical applications, however, the estimated hidden process may not always correspond to a *single* physical process. It may stand for a combination of processes or may capture only the most

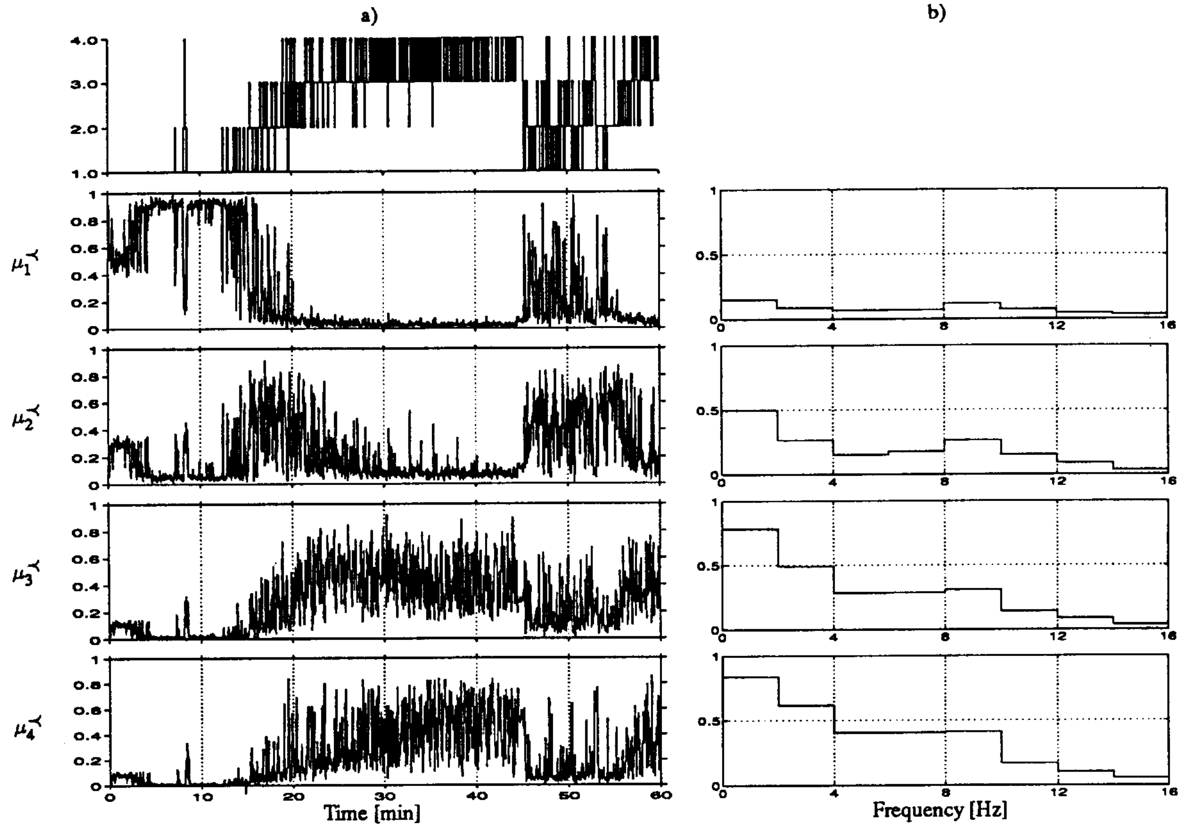

Figure 5. A typical dynamic profile for the first hour of normal sleep ($\Delta t = 3$s). a) Top graph contains the traditional (crisp) sleep onset obtained when the nearest maximum membership (NMM) operator [14] is applied on the fuzzy partition matrix. Bottom four graphs show the estimated membership functions of temporal fuzzy sets (hidden processes). The change in dynamics within the first seven minutes of sleep could not be detected with the crisp solution, but was completely captured with hidden process model (e.g. $\mu_1^{\prec}$ and $\mu_2^{\prec}$). b) Cluster prototypes for the temporal fuzzy sets (hidden processes). These prototypes estimate the normalized spectra corresponding to the poles of attraction.

prominent features of a more complex process. How well the hidden processes fit the real physical activities within complex systems will mostly depend on a selection of the feature variables and the way membership functions are estimated [3].

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