JOINT BEST BASES FOR FAST ENCODING IN MAGNETIC RESONANCE IMAGING

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ABSTRACT

We discuss the advantages and disadvantages of using a Karhunen-Loeve (K-L) expansion of a training set of images to reduce the number of encodes required for a Magnetic Resonance (MR) image of a new object. One form of this technique has been proposed [1] and another implemented [2]. We evaluate the error likely to be achieved as a function of the number of encodes and some two technical problems: reduced SNR in the images and smoothing of the K-L functions in practice.

As an alternative, we propose the use of joint best bases [3] derived from the local trigonometric library as an approximation to the K-L basis. These bases approach the rate-distortion characteristic achieved by the K-L basis, but they are easier to use in MRI and can be applied with existing methods for fast acquisition.

1. INTRODUCTION

Magnetic resonance (MR) imaging has become an essential tool in clinical medicine, producing exquisite contrast between soft tissue structures without the use of contrast agents. The major drawback is the time required to acquire the images. Depending on the contrast in the images, acquisition requires from a second or two to half an hour. An enormous amount of effort has been put into acquiring images faster. Faster imaging has opened many new and exciting applications such as cardiac imaging and functional imaging of the brain. The latter permits one to study evolving activity in areas of the brain during memory and motor tasks.

Techniques such as gradient echo imaging, echo planar imaging, RARE, and BURST produce images faster by increasing the rate at which data is acquired. There are limitations to these techniques: the image contrast can be adversely affected and expensive hardware modifications are required for some.There have also been a few techniques that attempt to reduce imaging times by using a priori information to reduce the amount of data required [1, 2].

This paper describes and analyses fast imaging methods of the latter type, based on reducing the number of encode steps required to image objects from a certain class. This reduction is obtained by encoding with waveforms adapted to the covariance structure of that class.

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2. BACKGROUND

We now review MRI, then synopsize some of the basic properties of joint best basis algorithm. For more on MRI one might refer to [4]; for details of the joint best basis algorithm and local trigonometric bases one might consult the recent book [3] and the references within.

2.1. Magnetic Resonance Imaging

In an MR scanner, information about an object is obtained in radio frequency (RF) signals generated by the stimulated precession of nuclear magnetic moments in a portion of the object. In common imaging techniques, a collection of such signals are used to build a map of the hydrogen density in a two-dimensional slice of the object. By careful control of the magnetic environment of the object, samples taken at intervals throughout the duration of a signal can be interpreted as measurements of the spatial Fourier transform of the object's hydrogen density taken along a curve in the spatial frequency plane. Commonly, many signals are obtained in order to obtain a dense enough sampling of the Fourier transform to enable construction of an image of the object.

The usual approach to imaging, spin warp imaging, begins by stimulating the nuclei in a restricted region of the sample, usually a thin planar slice. After this selective excitation, the variation of density of the excited nuclei within the slice is mapped by a sequence of manipulations of the magnetic environment. This sequence must be repeated many times to gain enough information to image the density. The signals produced look like

$$S_l(t) = \int dx \int dy\ \rho(x,y)\, e^{i\alpha x t}\, e^{il\,\beta y} \qquad 0 \le t \le T, \quad (1)$$

with $\rho(x,y)$ the hydrogen density, α, β are constants determined by the magnetic environment, and the integral is taken over the coordinates of the slice being imaged.

Each signal gives us one line of the slice's spatial Fourier transform, $\hat{\rho}\,(\omega_x = \alpha t, \omega_y = \beta)$ $0 \le t \le T$, parallel to the ω_x spatial frequency axis. A technique called *phase encoding* permits us to determine which line we measure, i.e. the value of $l\beta$. In many imaging situations, the spins in the sample must be allowed to "rest" for time after line is measured. The time between measurements can be long, resulting in long imaging time.

In this paper, we describe a technique which speeds imaging by encoding with a basis of spatial amplitude profiles on the spin system. These are chosen to exploit the known correlations among the members of a given class of images. In this method, phase encoding is replaced by a second selective RF pulse that excites a specially shaped excitation profile along the encoded axis; the general idea has been described in [4]. The signal produced is the inner product of the excited profile and the spin density along the encoded axis:

$$S_j(t) = \int dx \int dy \; \rho(x,y)\psi_j(y)e^{i\alpha x t} \, . \qquad (2)$$

Here we have chosen to encode the y axis with an amplitude profile given by the $j'th$ element of a basis $\{\psi_j\}_{j=1}^{N}$. Any axis could be encoded in the same way. We can then reconstruct the the projection of the spin density onto the encoded axis from these measurements in the usual fashion.

A Hadamard basis was the first used [5]. Since then, wavelet [4] and wavelet packet bases [6] have been proposed and implemented [7, 8]. There is a trade off between the signal to noise ratio (SNR) in the image and the speed with which the image can be acquired. If all the spins are excited by each RF pulse, the SNR is improved but the acquisition speed for given image contrast is reduced and artifacts are more pronounced [6].

MR Encoding with Joint Best Bases

In this paper, we will need to find a basis of profiles adapted to the statistical regularities of a particular class of images. The goal is to find a basis which captures most of the variability of the images within the first few basis elements. More specifically, we are interested in parsimonious representations in this basis, in the sense that truncated image expansions in the basis should have minimal expected mean square error. A natural choice would be the Karhunen-Loeve basis, which has precisely this characteristic property.

An alternative to the K-L basis has been proposed by Wickerhauser. In this paradigm, the K-L basis for a given class of signals on an interval is approximated from a large collectiom of simple bases, such as the local trigonometric library associated with a family of partitions of the interval[3]. The approach suggested utilizes a fast search over a tree structured library of possible bases to quickly find the best approximate K-L basis.

As a library of bases for a fast approximate Karhunen-Loéve transform, one might choose a wavelet packet library or a local trigonometric function library. Each basis of one of these libraries corresponds to a particular partition of the associated domain, time for the local trigonometric and frequency for the wavelet packet. Given a specified partition (the search partition) of the domain into a power of two number of subintervals, a fast binary tree searching strategy exists for finding a best partition (basis) in a collection of subpartitions of the search partition. We use a more general dynamic programming tree searching strategy to find a best partition over the entire collection of subpartitions of the search partition. [9]

For the approximate KL transform, we seek a basis which minimizes the volume of the variance ellipsoid for the ensemble. The volume is equal to the product of the diagonal elements in the autocovariance matrix. The best basis search itself uses the variance of the coefficients to evaluate a suitable cost function, namely the sum of the logs of the variances. The search complexity is $O(N^3)$. By eliminating some of the choices for where the left-most partition point may occur, faster searches are possible. As an example, one may limit the length of the left-most interval to be a power of two. In this case, the search complexity reduces to $O(N^2)$. One may further reduce this to the binary tree method by deleting futher intervals.

3. FAST ENCODING WITH K-L AND APPROXIMATE K-L BASES

Selective excitation in the y-direction, as in equation (2) above, can be used to reduce the number of excitations and therefore the acquisition time required to obtain an image from a known class of images. We study excitation profiles from the K-L basis associated with the image class, as well as approximations of these from local trigonometric libraries.

Cao and Levin studied the problem of finding an optimum set of phase encodes to estimate the first elements of the Karhunen-Loeve (K-L) basis from a training set of similar images. This reduced set of phase encodes was used to acquire an approximate image with a reduced data set [1]. Cao and Levin also suggest that the direct measurement of the K-L coefficients will improve the performance of their technique. The first elements of the K-L basis has also been used to estimate changes in the repeated acquisition of the same image [2]. The original image is used to generate the K-L basis so the reconstruction should be accurate with very few coefficients.

We studied direct measurement of K-L coefficients by selective excitation of K-L basis functions in the y-direction, evaluating performance on some typical image classes. K-L encoding has several technical limitations: the accuracy of the profiles excited by the selective RF pulses are limited by the length of the RF pulse. This makes excitation of the K-L basis functions difficult and of limited accuracy. In addition, the SNR of the images acquired with the K-L basis suffers compared to that in standard images.

To alleviate some of the technical problems, we suggest the direct measurement of the coefficients of an alternative, approximate K-L basis, obtained by searching a library of localized trigonometric bases, as described in the previous section. Localized trigonometric functions are much easier to use in MRI because the envelope is excited and the phase can be added accurately with gradients. Several coefficients can be obtained from one excitation by acquiring several phases with different phases across the profile.

3.1. Karhunen-Loeve basis from training images

We studied K-L encoding for a number of standard image classes; each class is a collection of trans-axial head images taken at a given height. All images came from standard scans in daily clinical studies. Only studies with no gross pathology were used. The T_2 weighted images from 8 studies were extracted and used as training set for the K-L expansion. A 9th was used as a test image to measure the error in the reconstruction.

The 8 images in the training set were used to generate K-L basis for the class. The variation in positioning and in anatomy likely to be seen in clinical practice was approximated with this data set, although a much larger data set would be required to generate a basis for the variety of pathology seen in clinical practice.

The training matrix, X, contains the 8 normalized training images as 256×256 submatrices, each image comprises a set of 256 rows in the 2048x256 image matrix. X can also be seen as the collection of 2048 vectors; each vector represents a line in the image that will be encoded with the K-L basis. The K-L basis vectors are the eigenvectors of $X X^t$. The eigenvectors are ordered so the associated eigenvalues decrease in size.

The K-L profiles $\{\psi_j\}_{j=1}^{256}$ obtained by this process are used as amplitude profiles in MRI. These profiles are excited by a selective RF pulse in the y-direction; the acquired data are frequency encoded in the x-direction to produce the signal described in equation (2) above. The x-direction inverse Fourier transform of the acquired data yield the K-L coefficients for each line in the image.

The form of the sample covariance matrix reflects the fact that two of the three dimensions are efficiently encoded in MRI, so we are only concerned with the covariance in the third dimension. We are essentially performing a one dimensional principal component analysis.

The error in the estimates of test images at multiple levels were computed as functions of the number of coefficients. Figure 1 plots the error energy as a percentage of the total energy in the image for the best and worst slices. For the test image at level 14, where the image plane is superior to the ventricles, the error energy is below 1% with 45 coefficients and below 0.5% with 62 coefficients. For all image planes, the error energy is below 1% with 100 coefficients and below 0.5% with 127 coefficients.

The SNR in images acquired with a K-L basis is lower than in conventionally acquired images, even if a complete set of encodes are used, by a similar analysis to that in [6]. The reduction in SNR we have seen for K-L bases averages to a factor of around three and a half. A second technical factor that should be considered is the RF pulse length required in exciting K-L basis functions. The profile excited by an RF pulse is approximately the Fourier transform of that RF pulse. Therefore, long RF pulses with wide bandwidths are required to excite profiles with sharp edges. The K-L basis functions we have seen have many sharp edges. In actual practice, the edges of these basis functions will not be present in the function actually excited, due to limitations on RF pulse length. These bandlimited profiles increase the error in the reconstructed test images significantly .

3.2. Approximate Karhunen-Loeve Encoding

We have explored using alternative, approximate K-L bases chosen from a library of localized trigonometric bases, as discussed in Section 2 above. Encoding with local trig bases sidesteps some of the technical limitations associated with K-L encoding. In particular, the basis functions have a simple envelope with a linear phase. This offers two big advantages. First, the basis functions do not have sharp discontinuities so they can be accurately excited with simple, short RF pulses. Secondly, the linear phase can be obtained with phase encoding gradients. This allows the same techniques developed to reduce the number of excitations in phase encoding to be applied. In particular, several phases across the same envelope can be acquired from one excitation with the RARE technique.

The joint best basis algorithms discussed previously can be used on the training images to obtain a localized trigonometric basis that approximates the mean squared error of the K-L basis. The joint best basis search is applied to the 2048 rows from the 8 images contained in the training matrix X. The interval to be encoded is initially partitioned into 16 intervals of equal length; the resulting collection of local cosine bases is searched by the dynamic programming algorithm outlined in section 2. The result is a local cosine basis associated with a particular of partition of the encoding axis. Figure 2 shows this partition superimposed on one of the test images. It also shows a sequence of reconstructions of that image from partial data.

For our purposes, the important property is that the best basis concentrates the variance of the training set quite well into its first few components. Figure 3 compares the best basis to the original representation (dirac basis) with respect to this property.

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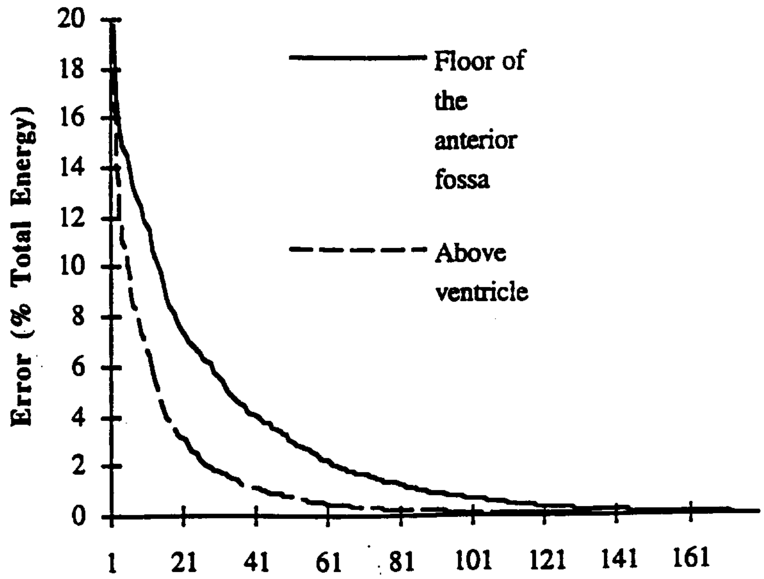

Figure 1. Error in reconstruction of test images as a function of K-L coefficients used. Results are given for the best and worst image classes.

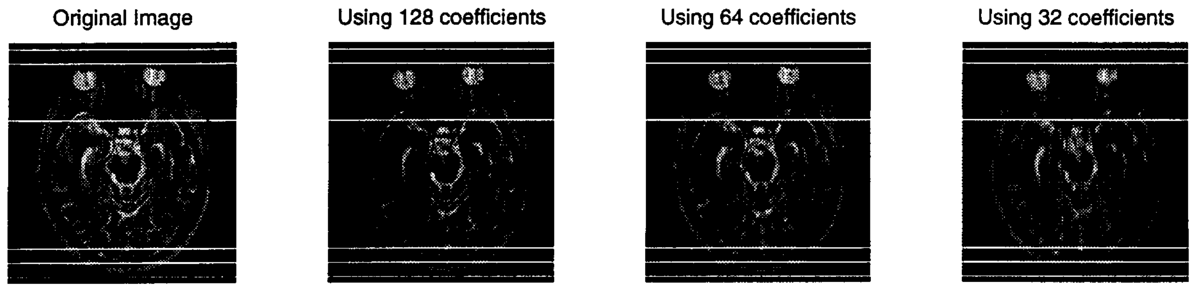

Figure 2. Reconstructions of a test image encoded with approximate K-L basis. White lines designate the partition corresponding to the approximate K-L local cosine basis.

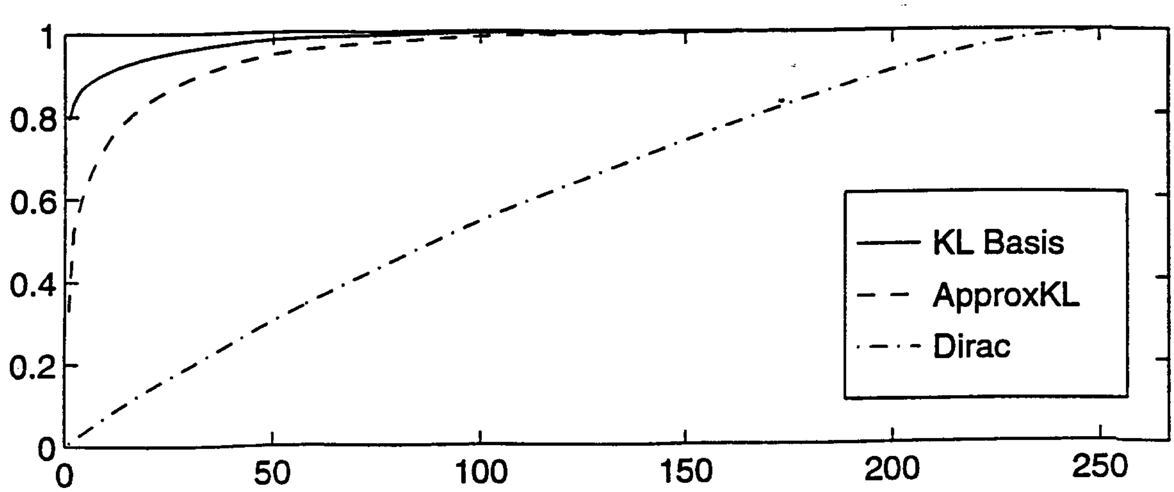

Figure 3. Variance captured as a function of coefficients measured in K-L, approximate K-L, and dirac bases.