HIGH RESOLUTION ASTRONOMICAL IMAGING BY POST-DETECTION IMAGE PROCESSING

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ABSTRACT

Advances in image-reconstruction techniques and computation capability have made it practical to use post-detection processing to overcome the resolution limits imposed on conventional astronomical observations by turbulence in the atmosphere. Techniques based on second and third-order spectra have proved to be very successful. The astronomical application requires special attention to the calibration of the measured atmospheric transfer function, the removal of photon-noise biases and the effects of finite detector size. A new area of application is the enhancement of images partially compensated by adaptive optics.

1. THE IMAGING PROBLEM

The angular resolution of large ground-based telescopes using traditional modes of observation is limited by the index-of-refraction fluctuations in the atmosphere to between 0.5 and 1 arcseconds at visible wavelengths [1,2]. This means, for example, that 4-m diameter telescopes, such as the Kitt Peak, AZ, and Canada-France-Hawaii telescopes, have an atmospherically-determined resolving angle about 30 times greater than their theoretical diffraction limit. In the past the motivation for building such large telescopes has been light-gathering area, but today with the computing power now available it has become possible and practical to recover much of the potential resolving capability of these large instruments by means of post-detection, digital image processing. With the new 8 and 10-m telescopes under construction or planned, the potential for improving resolution is even greater.

The nature and difficulty of the high-resolution, astronomical-imaging problem can be appreciated by considering that the wavefront distortions induced by atmospheric turbulence at visible wavelengths have a spatial coherence distance, r_0, of 10-20 cm., a time constant T of 10-50 ms, and departures from the unperturbed wavefront of up to several wavelengths. When the diameter D of the telescope is much greater than r_0, as is the case in high-resolution imaging, the point-source image consists of from 10^2 to over 10^3 bright patches in agitated motion within a cloud the size of the atmospheric resolution angle. The actual number of patches is proportional to $(D/r_0)^2$, and r_0 and T are both proportional to $\lambda^{6/5}$, where λ is wavelength. The individual patches, called speckles, have an angular size of λ/D, the diffraction-limited resolution of the telescope. Thus the instantaneous "speckle" image contains object information up to the telescope's diffraction limit, but in a highly distorted form. The averaging inherent in an exposure time greater than 50 ms effectively destroys all information in the Fourier spatial spectrum above a cut-off frequency, r_0/λ. That is why linear inversion methods are unable to improve the resolution in conventional images.

The key to imaging through turbulence is to record long sequences of 10^3 to 10^6 images, each with exposure time short enough to freeze the motion of the atmosphere. From these highly-distorted speckle images one estimates 2nd and 3rd order spatial spectra which have non-zero expected values up to the diffraction limit of the telescope. For example, the spatial power spectrum gives Fourier modulus information and Fourier phase information can be recovered from the 3rd order spatial bispectrum. Because the speckle phenomenon requires a narrow spectral band, 10-50 nm. and the exposure times are short, the individual speckle images contain less than 10^3 detected photon events for the objects of greatest interest. In summary the imaging problem is one of distortion so high that the detected images occupy 10 to 30 times the area of the true object, and signal-to-noise ratio much less than 1.

2. SPECKLE-IMAGE PROCESSING

Let $i_n(r)$ be the nth speckle image in a sequence of N, o(r) be the geometric projection of the object onto the image plane and $s_n(r)$ be the point spread function (PSF) or impulse response of the atmosphere-telescope combination when the nth image was taken. The variable r is position in the image plane measured as an angle from the optical axis. The corresponding spatial spectra are $I_n(u)$, O(u) and $S_n(u)$. Provided the object is contained within the 2-4 arcsec

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isoplanic area of the atmosphere, $i_n(r)$ is the convolution of $o(r)$ and $s_n(r)$, and

$$I_n(u) = O(u)S_n(u). \tag{1}$$

The triple-correlation (TC) function of the image is defined as

$$t_n(r,r_2) = \int_{image} i_n(r)\, i_n(r+r_1)\, i_n(r+r_2)\, dr, \tag{2}$$

and the bispectrum, the 4D Fourier transform of Eq (3), is given in terms of the image spectrum by [3,4]

$$B_n(u_1,u_2) = I_n(u_1) I_n(u_2) I_n^*(u_1+u_2). \tag{3}$$

Bispectra for the object and PSF can be defined in the same manner, from which it follows that

$$B_n(u_1,u_2) = B_o(u_1,u_2) B_{sn}(u_1,u_2) \tag{4}$$

In contrast to the expected value of the telescope-atmosphere transfer function $<S_n(u)>$, which is effectively zero for $u > r_o/\lambda$, the expected value of the bispectral transfer function, $<B_{sn}(u_1, u_2)>$, is non-zero up to the diffraction limit of the telescope. Furthermore, it is a real quantity so that the phase of the image bispectrum is the phase of the object bispectrum. This last statement assumes that the telescope has no aberrations. However, the bispectrum is inherently insensitive to even order aberrations and odd order aberrations can be corrected by calibration of the transfer function on a point, or unresolved, source. In practice, $<B_n(u_1, u_2)>$ is estimated by averaging the triple product of Eq. (3) over the N speckle images.

Because of the very noisy nature of the speckle images, 2nd and 3rd order correlations have noise bias terms that must be removed before phase estimates can be made [4]. The unbiased image bispectrum estimate is

$$\begin{aligned}\hat{B}_n(u_1 u_2) = {} & I_n(u_1) I_n(u_2) I_n^*(U_1+u_2) \\ & - |I_n(u_1)|^2 |I_n(u_2)|^2 |I_n(u_1+u_2)|^2 \\ & + 2I(o).\end{aligned} \tag{5}$$

Subsequent discussion assumes bispectra corrected for noise bias.

From Eq. (3) the bispectrum phase $\beta(u_1, u_2)$ is related to the object phase $\phi(u)$ by

$$\beta(u_1,u_2) = \phi(u_1) + \phi(u_2) - \phi(u_1+u_2). \tag{6}$$

Eq. (6) is the basis of a recursive algorithm for finding $\phi(u)$ in terms of the bispectrum phase and the object phases at two frequencies such that $u_1 + u_2 = u$. To start the recursive process it is necessary to know the object phase at frequencies $(0, \pm 1, 0)$. These phases are usually set equal to zero with the consequence that absolute position information is lost. In practice this is not a problem for the observer. Estimates of the object phase at one frequency can be obtained from the bispectral phases at different combinations of frequencies. These multiple estimates are combined in a weighted average with the weighting function determined from the signal-to-noise ratio (SNR) of the individual phase estimates [5]. The principal problem with the recursive method is that it does not use the available information optimally and the variances of the estimates accumulate as the recursions progress to higher frequencies. For these reasons it has been of interest to consider least-squares methods.

Equation (6) can be written in matrix notation as

$$\bar{\beta} = H\bar{\phi} \tag{7}$$

where $\bar{\beta}$ is an M-dimensional vector containing all the phases in the bispectrum and ϕ is an N-dimensional vector of object phases. A weighted least squares solution would be

$$\bar{\phi}_s = (H^T W H)^{-1} H^T W \bar{\beta}, \tag{8}$$

where W is a positive definite symmetric weighting matrix determined by the SNR of $\beta(u_1, u_2)$. Practical image sizes of 256 x 256 pixels make the H matrix extremely large, and because $M > N$ the system of equations is overdetermined. Furthermore, the real data is noisy and the bispectrum phases are known only modulo 2π. For these reasons direct evaluation of Eq. (8) is not practical. Meng et al [5] and Matson [6] describe and evaluate iterative phase recovery algorithms in which the phase relationships are carried in phasors in order to avoid the phase wrapping problem. Haniff [7] deals with this problem by minimizing the objective function

$$F = \sum_{i,j}^{N} \left\{ \frac{Mod[\beta_{ij} - (\hat{\phi}_i + \hat{\phi}_j - \hat{\phi}_{i,j})]}{\omega_{ij}} \right\}^2 . \qquad (9)$$

where the subscripts i and j define the discrete frequencies in the digitized image spectrum, ω_{ij} is a measure of the error on the unwrapped β, and the circumflexes denote quantities that are to be varied in order to minimize F.

Although $\langle B_{sn}(u_1, u_2) \rangle$ is non zero up to the diffraction limit, in the regions where $u_1, u_2 > r_0/\lambda$ it is depressed by a factor of $(r_0/D)^4$ compared to the transfer function in the absence of atmosphere. In the regions near the axes where either u_1 or u_2 is less than r_0/λ the transfer function is depressed by only $(r_0/D)^2$. Thus in the near-axis regions the SNR can be more than an order of magnitude higher than elsewhere in the bispectrum. Each value of $\mathbf{u_2}$ defines a plane in which $\mathbf{u_1}$ can take on values from 0 to the diffraction limit. The numbers of planes used in the computation is a convenient way to measure the amount of the bispectrum employed. Meng and Aitken [8] have studied this relationship over a wide range of photon rates, with objects of different degrees of complexity and atmospheric severity. The general guidelines are that there is a clear improvement in the quality of the images as the number of planes increases, but the improvement is greatest at photon rates midway between the low end where the recovered picture is only a field of noise and the level at which the atmospheric fluctuations become the dominant noise. Note that $\mathbf{u_1}$ and $\mathbf{u_2}$ are interchangeable. It was also found that the effectiveness of additional planes, or the return on the investment in computation, increased with increasing object complexity.

Although both the modulus and the phase of the stellar object's spectrum can, in principle, be recovered from the bispectrum, it is generally more convenient to obtain the modulus from speckle interferometry (SI) [4], which measures the object power spectrum. The modulus squared is attenuated by the factor $(r_0/D)^2$ at frequencies above r_0/λ. Calibration of the modulus is achieved by measuring the power spectrum of a neighbouring, unresolved star before, after or alternately with the object of interest [9]. Inverse Fourier transformation of the estimated spectrum gives the image.

3. IMAGE TRUNCATION

As described above, the short-exposure speckle image retains detail up to the telescope's diffraction limit, but the speckle cloud can be an order of magnitude or more larger. In order to adequately sample the speckle pattern, the speckle images are magnified to fill the detector aperture a(**r**), as completely as possible. This means that there can be some truncation of the wandering speckle cloud at the edges of the detector. Even small amounts of image truncation can cause severe artifacts and distortions in the reconstructed images [10]. This phenomenon, which usually has a negligible effect in conventional imaging, is quite pronounced in speckle-image reconstruction from higher-order spectra.

The effect of the finite detector size is to multiply the image i(**r**) by a(**r**). In the Fourier domain the image spectrum is convolved with the Fourier transform, A(**u**), of a(**r**). The cause of the distortion is the leakage of the very strong central "spike" of the bispectrum into the convolution integral through the sidelobes of the detector-aperture bispectrum, $A(\mathbf{u_1})A(\mathbf{u_2})A(\mathbf{u_1}+\mathbf{u_2})$.

Truncation of the long-exposure image is seen as strong negative artifacts and elongations of the stellar images in directions perpendicular to the edges causing the image truncation. Apodization may give a smoother image but cannot reverse the distortions. The problem must be avoided by correct design of the observation. An example illustrating the sensitivity of image reconstruction to this phenomenon is a 3-component object imaged onto a circular aperture such that about 3% of the long-exposure image energy was lost [10]. When D/r_0 takes the value 13.3, 12.9 and 12.5 the rms error between true and reconstructed images are 0.53, 0.043 and 0.032. Note the order of magnitude change between the first two cases. When the image is demagnified by a factor of 2 so that the long-exposure is well within the detector. the error was 0.00025.

4. FUTURE DEVELOPMENTS

Currently adaptive optics (AO), that is pre-detection compensation, is receiving a great deal of attention from the astronomical community. AO systems sense the wavefront deformations caused by the atmosphere and correct them in real time by means of a deformable mirror. Because at infra-red wavelengths both r_0 and T are larger than at visible wavelengths, AO systems are easier to implement in the infra-red. The success of the COME-ON project [11] have shown the potential of such systems. In the visible only partial correction can be achieved in practice. Despite these recent developments, post-detection image-reconstruction methods remain relevant. Roggerman et al [12] have shown that bispectral processes are effective in improving the quality of images that have been partially corrected by adaptive optics.

AO techniques at visible wavelengths are severely limited by the photon flux of the guide, or reference, star. However, it now appears technically feasible to produce guide stars by using the scattering of laser light in the sodium layer at 80 to 100 km above the earth's surface [13]. Such systems will still be limited by the need for natural reference objects brighter than 19th magnitude to correct for tilt fluctuation that are not correctable with a laser guide star. Bispectral processing, which is insensitive to wavefront tilt, is a potential solution. It seems certain that post-detection processing will always be required to extract the maximum amount of information from the AO system [14].

One problem with the bispectral method is that other prior information cannot be readily included. Information that would enhance the reconstruction are the fact that the object's intensity distribution must be positive and that good estimates of the objects support can be made from its spatial autocorrelation, the Fourier transform of the SI power spectrum. Ayers and Dainty [15], Christou [16] and Lane [17] have developed iterative deconvolution algorithms that alternately impose image domain constraints such as positivity and support, and frequency-domain constraints obtained from the measured higher-order spectra. Work in this direction has the potential to produce high-quality images using all possible known information about the object.

Schulz and Snyder [18] have proposed an iterative technique that recovers atmospherically degraded images from nth order correlations. It is straightforward to implement and allows prior information such as positivity and support constraints to be included. This approach is different from the previous bispectral and iterative deconvolution methods in that it produces the image directly through a sequence of image-domain iterations. The introduction of error measures other than the traditional least-squares metric opens up new avenues for exploration.

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