AN APPROACH FOR RANDOM MEDIA PARAMETER ESTIMATION USING SEISMIC REFLECTION DATA

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ABSTRACT

The traditional approach in seismic imaging assumes homogeneous layers with constant seismic velocities. In real earth, layers have numerous small-scale variations of properties (e.g., the seismic P-wave velocity) which are so irregularly distributed that they can no longer be described by deterministic models; statistical approaches have to be used instead. They can be described only by their stochastic properties. In this paper, every distinctive layer is described by its mean velocity, its variance, and a spatial autocorrelation function. This velocity model is transformed into a model for the autocorrelation of the corresponding impulse response. The parameters for this model are then estimated by the minimum least square method using stacked and migrated seismic sections as input. The method is applied to a real seismic data set. Additional borehole data is used to estimate one of the parameters independently and thus to test the approach.

1. INTRODUCTION

The reflection seismic experiment normally yields only the location of sharp changes of the velocity and density parameter. These changes are associated with structures and lithologies in the earth. However, few media are really homogeneous. Most have numerous small scale variations of the material properties. These inhomogeneities are so irregularly distributed that they can no longer be described by deterministic models. In addition, the size of structural or lithological units can be below the limit of resolution. Good examples are alluvial deposits in fluvial or deltaic regimes. Often it is not possible to resolve individual river channels, cross- or foreset beds, however it is possible to measure the average response of an ensemble of these features. Assuming a particular statistical model for the velocity or density, one can estimate the model parameters from a dataset. In this application, we use stochastic model parameters for the average dimensions and orientation of inhomogeneities. Estimating these parameters yields additional information about the underground. In the approach presented here, every distinctive unit is divided into a deterministic part and a stochastic field, described by its amplitude and second order statistics (autocorrelation function). The stochastic component of the medium parameter can be transformed into a model for the autocorrelation of a wavefield propagating through the medium. Thus we can estimate the stochastic parameters from a measured wavefield. To simplify the technique, the wavefield is replaced by stacked and migrated reflection seismic data. The procedure is tested with a real seismic data set. Additional borehole data can be used to estimate one of the parameters independently and thus to control the approach.

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2. VELOCITY AS A RANDOM FIELD

The data we have are completely processed seismic sections. The model we are looking for is the velocity model, and we must connect the data (wavefield) to the model (velocity). Dealing with random fields, we want to know only the statistics of the velocity model. But we cannot connect a deterministic model to the statistical model parameters. Therefore, we have to work with the statistics of the wavefield. This allows us to estimate the model parameters. The new task is to connect the statistics of the velocity model to the statistics of the wavefield. In the deterministic case we would do this with the Green's function (impulse response). We can still use a similar idea: we try to derive the statistics of the impulse response. First, we transform the statistical description of the velocity model into a description of the reflection coefficients. Then we convert the statistics of the reflection coefficients into a description of the impulse response. Finally, we account for the data processing applied to the data. These three steps allow us to derive a statistical description of the wavefield we expect from a velocity model of which we only know the statistics.

The velocity model $v(\underline{x})$ for some region of interest can then be decomposed into a deterministic part $\bar{v}(\underline{x})$ and a stochastic part $\delta v(\underline{x})$:

$$v(\underline{x}) = \bar{v}(\underline{x}) + \delta v(\underline{x}) \quad (1)$$

$$\delta v(\underline{x}) = v(\underline{x}) - \bar{v}(\underline{x}) = v(\underline{x}) - \mu_v \quad (2)$$

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The deterministic term $\bar{v}(\underline{x})$ describes large-scale velocity changes. In this paper it is associated with units such as lithology or structure. Within one unit it is constant and can be described by the mean velocity μ_v of the particular unit. This term is also called the background velocity. The wavefield corresponding to the deterministic part is of little interest in this study. It can be modeled with traditional techniques. The stochastic part $\delta v(\underline{x})$ is the difference between the actual velocity $v(\underline{x})$ and the deterministic part $\bar{v}(\underline{x})$. In fact, the deterministic and stochastic parts are not independent of each other. They are correlated by the decomposition into different units by size and geometry as well as the number of units and their associated length scales.

For the forward modeling we need stochastic models. The first order statistics of the stochastic velocity field is described by its probability density function *pdf*. Its first moment is the average μ_v, and the second moment the variance σ_v of the velocity field v.

$$\mu_v = \mathrm{E}[v(\underline{x})] = \int_{-\infty}^{+\infty} \zeta \, p_{V(\underline{x})}(\zeta) \, d\zeta \tag{3}$$

$$\sigma_v^2 = \mathrm{Var}[v(\underline{x})] = \int_{-\infty}^{+\infty} \zeta^2 \, p_{V(\underline{x})}(\zeta) \, d\zeta \tag{4}$$

$\mathrm{E}[x]$ and $\mathrm{Var}[x]$ denote the expectation operator and the variance operator of x, respectively. Due to the decomposition (2), the stochastic velocity field $\delta v(\underline{x})$ has a zero mean and thus $\mu_{\delta v} = 0$. For the sake of simplicity, we choose a Gaussian *pdf* with the mean $\mu_v (= 0)$ and the variance σ_v^2.

$$p_{v(\underline{x})}(\zeta) = \frac{1}{\sqrt{2\pi\sigma_v^2}} e^{-\frac{1}{2}(\frac{\zeta}{\sigma_v})^2} \tag{5}$$

We also need the second order statistics which relate the values at different locations to each other. This is described by the autocorrelation function *acf*.

$$\mathrm{R}_{vv}(\underline{x}_1; \underline{x}_2) = \mathrm{E}[v(\underline{x}_1) v^*(\underline{x}_2)] \tag{6}$$

We assume the earth to be locally *stationary*, and the statistical properties independent of their locations. The *acf* of a stationary field depends only on the spatial difference $\underline{\xi} = \mid \underline{x}_1 - \underline{x}_2 \mid$ of location. This allows the definition of *acf* as a convolutional product in a different way.

$$\mathrm{R}_{vv}(\underline{\xi}) = \mathrm{E}[v(\underline{\zeta} - \underline{\xi}) \, v^*(\underline{\zeta})] = v(\underline{\xi}) * v^*(-\underline{\xi}) \tag{7}$$

For simplicity we choose the *acf* to be Gaussian as well.

$$\mathrm{R}_{vv}(\xi) = e^{-\xi^2} \tag{8}$$

This yields isotropic velocity models of inhomogeneities. To allow for more general models, two correlation length parameters, a_1 and a_2, and an angular dependence ϕ are used.

$$\xi = \sqrt{\left(\frac{x\cos\phi + z\sin\phi}{a_1}\right)^2 + \left(\frac{z\cos\phi - x\sin\phi}{a_2}\right)^2} \tag{9}$$

With this definition for ξ, the *acf* defines a two-dimensional elliptical function whose semi-major-axis is aligned in the ϕ direction with length a_1. The length of the semi-minor-axis is a_2. For $a_1 = a_2$, the inhomogeneities are isotropic and the parameter ϕ is not defined.

For a constant density ρ, the reflection amplitudes yield the discontinuities in the velocity distribution. We can transform the *acf* for the velocities to a corresponding *acf* for the discontinuities. For normal incidence, we linearize the relation between the discontinuities and the velocities as follows:

$$\hat{c}_i = \frac{v_i - v_{i-1}}{2\mu} \tag{10}$$

Mathematically, we applied a linear filter $h(\underline{x}) = \frac{1}{2\mu}(-1, 1)$ to the velocity field. Due to linearity, the resulting field $\hat{c}_i$ is again a random field $\hat{c}(\underline{x})$ with the *acf* $\mathrm{R}_{\hat{c}\hat{c}}(\underline{\xi})$.

$$\begin{aligned} \mathrm{R}_{\hat{c}\hat{c}}(\xi_x, \xi_z) &= \frac{1}{4\mu^2} \cdot \{2\,\mathrm{R}_{vv}(\xi_x, \xi_z) \\ &- \mathrm{R}_{vv}(\xi_x, \xi_z - 1) - \mathrm{R}_{vv}(\xi_x, \xi_z + 1)\} \end{aligned} \tag{11}$$

The real seismic reflection data contain primary reflections as well as multiples and internal reflections. It is not just a reflection coefficient series, but the impulse response of the velocity field. To go from the reflection coefficients to the impulse response we need a transformation. In addition, we need an expression for the *acf* of this impulse response. To calculate the impulse response, we would need deterministic reflection coefficients, which we do not know. The only knowledge we have is the *acf* of these coefficients, which can be derived from our stochastic velocity model. As a first order approximation, we can replace the *acf* of the impulse response by the *acf* of the reflection series; the stochastic residual field $\delta v(\underline{x})$ varies smoothly. This means that the corresponding reflection coefficients are small. In this case the z-transform of the impulse response of a stack of layers can be written as [1]:

$$\hat{r}(z) = \frac{\hat{c}_0 + \hat{c}_1 \, z + \hat{c}_2 \, z^2 + \ldots}{1 + \mathrm{R}_{\hat{c}\hat{c}}(1) \, z + \mathrm{R}_{\hat{c}\hat{c}}(2) \, z^2 + \ldots} \tag{12}$$

If products of the third and higher order in the *acf* $\mathrm{R}_{\hat{r}\hat{r}}$ of the layered stack are neglected, it is easy to show that $\mathrm{R}_{\hat{r}\hat{r}} = \mathrm{R}_{\hat{c}\hat{c}}$. The main contribution of the *acf* of the impulse response equals the *acf* found from backscattering using the Born approximation or Rayleigh scattering [2].

It is not possible to take care of the influence of the data processing on the *acf*. An inherent assumption is, therefore, that a stacked and migrated section approximates a wavefield where source and receiver were at the same position and the wavefield propagated only in the vertical direction. The source wavelet is another problem. Most of the time the wavelet is unknown. If it were known, then it could either be removed from the data by a deconvolution or included in the model by convolving the *acf* of the wavelet with the *acf* of $\mathrm{R}_{\hat{c}\hat{c}}$.

3. PARAMETER ESTIMATION

At this point we have a model for the *acf* of a wavefield being scattered back from a random media. A Gauss-Newton approach is applied to estimate the model parameters $\hat{a}_1$,

$\hat{a}_2$, and $\hat{\phi}$ which fit the $acf$of the data best in the MMSE sense. The equation that has to be solved is

$$\mathrm{R}_{\hat{r}\hat{r}}(\hat{a}_1, \hat{a}_2, \hat{\phi}, \underline{\xi}_i) - \hat{\mathrm{R}}_{dd}(\underline{\xi}_i) = e_i \qquad (13)$$

under the constraint that $\sum e_i^2$ be minimal. The index i is used to denote all points in the data or the acfthereof. Let $\underline{\zeta} = (\zeta_1, \zeta_2, \zeta_3)^T$ denote $(\hat{a}_1, \hat{a}_2, \hat{\phi})^T$. Then the system can be linearized using $\underline{\zeta} = \underline{\zeta}^{(0)} + \delta\underline{\zeta}$.

$$\sum_{j=1}^{3} \frac{\partial \mathrm{R}_{\hat{e}\hat{e}}(\underline{\zeta}^{(0)}, \underline{\xi}_i)}{\partial \zeta_j} \delta\zeta_j + \mathrm{R}_{\hat{r}\hat{r}}(\underline{\zeta}^{(0)}, \underline{\xi}_i) - \hat{\mathrm{R}}_{dd}(\underline{\xi}_i) = \epsilon_i \qquad (14)$$

The resulting linear system of equations can be minimized very easily using QR decomposition. Due to the nonlinearity, we have to iterate until $\zeta_j^{(n)} = \zeta_j^{(n-1)} + \delta(\zeta_j)$ converges into a stable minimum. Unfortunately, the model space is rather large. There are many local minima where the estimation could be trapped. If the initial parameters were not close to the correct values, then wrong parameters would be estimated. Therefore, the Monte Carlo technique is used. The model space is searched randomly to find the best initial parameters. These are then used as initial values for the Gauss-Newton method.

It must be mentioned that the $acf\hat{\mathrm{R}}_{dd}$ of the data is unknown. The $acf$of the data is only an estimate based on a small rectangular window size $L \times L$ of real data. The choice of the acfestimator has some importance for this parameter estimation to work. Should the data wrap around in the acfestimation? How much padding should be chosen? Should a tapering be applied to correct for the decreasing number of overlapping datapoints? Generally, the larger the window the better. But there is also a trade-off for large windows. In reality the stochastic velocity component may not be stationary. On the contrary, a unit for which the random parameters have to be estimated has only a finite thickness, which limits the amount of data available. Also, the discretization introduces problems with aliasing and wraparounds. Fortunately, they can be neglected if the scale parameter is $a < \frac{L}{3}$.

4. APPLICATION TO REAL DATA

The field dataset is a three-dimensional survey with 37 * 39 cdp points with a spacing of 4m (obtained from Chevron Oil Company). Fortunately, there are different wells in the area which allow us to control the vertical parameter as estimated by the new technique. Figure 1 shows the P-wave sonic log, a median filtered P-wave sonic log and the natural gamma log for well CH4, as well as a simple geological interpretation of the area. The seismic dataset is used in two different ways. First, one stack of a seismic row in the middle is used to estimate the parameters as a function of depth. These results are then compared to the velocities from the nearby well. For this purpose the P-wave sonic log is resampled to have the same vertical resolution as the seismic section. The comparison is presented in Table 1. The vertical parameter estimated from the seismic shows a good agreement with the well-log.

In a second experiment, we produced a map of random media parameters using all rows and columns of the seismic dataset. For each row and column of the dataset, all three parameters $\hat{a}_1$, $\hat{a}_2$, and $\hat{\phi}$ plus the model misfit are calculated for a depth of 150 meters. The results are presented in Figures 2 and 3. In both column and row direction the horizontal parameter $\hat{a}_1$ is large, while the vertical $\hat{a}_2$ is rather small. The parameters change dramatically toward the end of the rows and columns. Also, the absolute error of the estimation decreases toward the edges. There are two possible explanations: (1) the processing applied was unable to take care of boundary effects such as the energy reflected from outside the area of the survey; and (2) there are fewer data points close to the edge, making it easier to fit our model. Finally, we created possible realizations of the velocity field using the parameters we estimated before (Figure 4). Due to the variability of the estimated parameters the random media parameters $\hat{a}_1$, $\hat{a}_2$ and $\hat{\phi}$ are averages of the different rows or column estimates which yield: $\hat{a}_r = 150$m, $\hat{a}_c = 140$m, $\hat{a}_z = 2.5$m and a dip $\hat{\phi} = -7°$.

Seismics					Log
Depth $[m]$	$\hat{a}_1$ $[m]$	$\hat{a}_2$ $[m]$	$\hat{\phi}$ [°]	$\sum \epsilon^2$	a_2 $[m]$
79	64.7	4.9	-2.1	7.9	-
115	51.5	4.6	-2.2	10.2	-
151	46.6	4.7	-3.2	7.4	5.4
187	49.7	4.8	-3.4	5.9	4.0
223	74.4	5.5	-4.2	8.3	1.4
259	82.5	6.3	-5.2	9.0	6.1
295	85.8	6.0	-5.9	12.8	4.7
331	65.5	5.7	-5.7	10.2	-
367	49.5	5.4	-6.2	10.0	-
403	42.9	6.1	-7.8	7.5	-
439	62.0	6.7	-10.8	10.4	-

Table 1: A comparison between the parameter estimated from the brute stack and from a well-log. The well-log data is resampled to have the same resolution as the seismic.

5. SUMMARY

We showed that it is possible to use surface seismics to estimate parameters for a simple random media model in two dimensions. The model has three parameters: two length-scales $\hat{a}_1$, $\hat{a}_2$ and an orientation $\hat{\phi}$. The method was applied to a real 3-D seismic dataset and the stochastic parameters were estimated as a function of position.

6. REFERENCES

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[2] K. Aki and P.G. Richards. *Quantitative Seismology. Theory and Methods*, volume 1+2. Freeman, San Francisco, 1980.

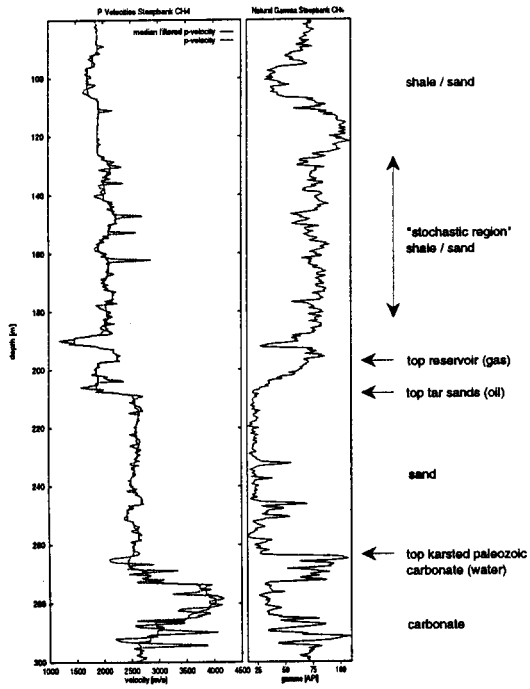

Figure 1: P-wave sonic log, median filtered P-wave sonic log and the gamma log of the well CH4. Also shown are some prominent features and a simple geological interpretation. The region between 130 and 180 m depth is the region used for the stochastic parameter estimation.

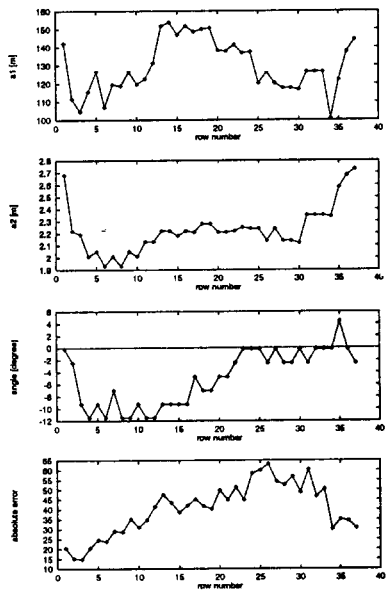

Figure 2: The random media parameter as estimated from the real data. Shown are the three parameters and the absolute error for each row of the dataset at a depth of 150 meters. The absolute error is the total error of the estimated parameters using the L_2 norm.

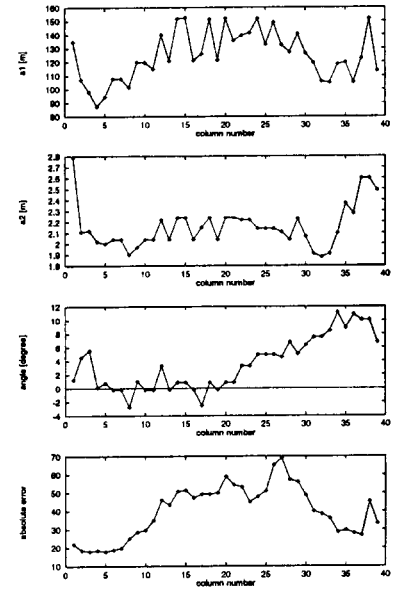

Figure 3: The random media parameter as estimated from the real data. Shown are the three parameters and the absolute error for each row of the dataset at a depth of 150 meters. The absolute error is the total error of the estimated parameters using the L_2 norm.

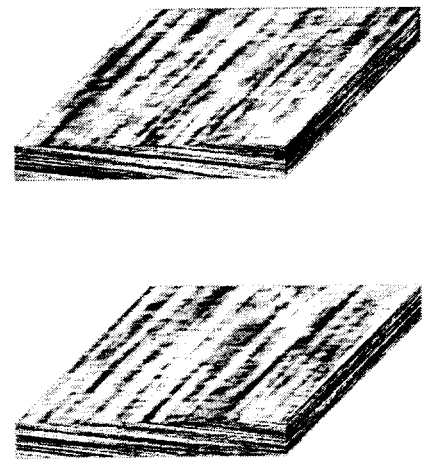

Figure 4: Two possible realizations of the velocity field as estimated from the real data. Shown are the velocity fields from a depth of 122m to 182m. The parameters used are: $\hat{a}_r = 150\text{m}$, $\hat{a}_c = 140\text{m}$, $\hat{a}_z = 2.5\text{m}$ and a dip $\hat{\phi} = -7°$.