SPACE-TIME ADAPTIVE PROCESSING FOR AIRBORNE RADAR

James Ward

Lincoln Laboratory
Massachusetts Institute of Technology
Lexington, MA 02173
jward@ll.mit.edu

ABSTRACT

Advanced airborne radar systems are required to detect targets in the presence of both clutter and jamming. Ground clutter is extended in both angle and range, and is spread in Doppler frequency because of the platform motion. Space-time adaptive processing (STAP) refers to the simultaneous processing of the signals from an array antenna during a multiple pulse coherent waveform. STAP can provide improved detection of targets obscured by mainlobe clutter, detection of targets obscured by sidelobe clutter,and detection in combined clutter and jamming environments. Fully adaptive STAP is impractical for reasons of computational complexity and estimation with limited data, so partially adaptive approaches are required. This paper presents a taxonomy of partially adaptive STAP approaches that are classified according to the type of preprocessor, or equivalently, by the domain in which adaptive weighting occurs. Analysis of the rank of the clutter covariance matrix in each domain provides insight and conditions for preprocessor design.

1. INTRODUCTION

Advanced airborne radar systems are required to detect targets in the presence of both clutter and jamming. The ground clutter observed by an airborne platform is extended in both angle and range, and is spread in Doppler frequency because of the platform motion. For low PRF radars, mainlobe and sidelobe clutter may completely fill the Doppler space, as shown in Figure 1. Noiselike jamming signals are localized in angle and spread over all Doppler frequency. Interference cancellation requires multidimensional filtering over the spatial and temporal domains. Uncertain knowledge of the clutter and jamming environment, as well as imprecise calibration, requires data-adaptive processing. Space-time adaptive processing (STAP) refers to the simultaneous processing of the spatial samples from an array antenna and the temporal samples provided by the echoes from multiple pulses of a radar coherent processing interval (CPI). STAP can provide improved detection of targets obscured by mainlobe clutter, detection of targets obscured by sidelobe clutter, and detection in combined clutter and jamming environments.

As with spatially adaptive processing, STAP requires estimation of the interference to compute an adaptive weight vector. Fully adaptive STAP, though optimum given perfect knowledge, is impractical for two reasons. First is the computational burden of solving large systems of equations in real-time. Secondly, the interference is unknown a-priori and must be estimated from the limited amount of data available during a radar dwell. Significant performance loss results with insufficient data. The inherent nonstationarity of radar clutter makes this estimation more difficult. Reduced-dimension or partially adaptive STAP algorithms are required to ease both computation and training support. A generic partially adaptive architecture that consists of a dimension-reducing preprocessor followed by adaptive weight computation is proposed. A taxonomy of approaches is developed where algorithms are classified according to the type of preprocessor. For example, beamspace algorithms utilize spatial preprocessing, while post-Doppler approaches perform temporal (Doppler) filtering before adaptive processing. After preprocessing, either sample-matrix-inversion (SMI) or subspace-based weight computation may be employed. Within each class, analysis of the rank of the clutter covariance matrix provides insight into the problem and conditions for minimum dimension preprocessors. The purpose of this paper is to provide a discussion of various approaches; the 'best' algorithm architecture is highly dependent on the radar system parameters and the training approaches that can be supported.

2. STAP FUNDAMENTALS

Consider a radar system utilizing an N-element uniform linear array with interelement spacing d. The radar transmits an M-pulse waveform at pulse repetition interval T. The received data for each range gate may be organized into an $MN \times 1$ *space-time snapshot* $\mathbf{x}$ formed by stacking the spatial snapshots from each pulse. When a target is present, the snapshot may be split into target and interference-plus-noise components

$$\mathbf{x} = \alpha \mathbf{v}(\phi, f) + \mathbf{x}_u \ . \tag{1}$$

Here α, ϕ, f are the target complex amplitude, angle, and Doppler frequency. The $MN \times 1$ *space-time steering vector* $\mathbf{v}$ is given by

$$\mathbf{v}(\phi, f) = \mathbf{b}(f) \otimes \mathbf{a}(\phi) \ . \tag{2}$$

where

$$\mathbf{a}(\phi) \ = \ \begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda}\sin\phi} & \dots & e^{j(N-1)\frac{2\pi d}{\lambda}\sin\phi} \end{bmatrix}^T \tag{3}$$

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$$\mathbf{b}(f) = \begin{bmatrix} 1 & e^{j2\pi fT} & \cdots & e^{j(M-1)2\pi fT} \end{bmatrix}^T \quad (4)$$

are the $N \times 1$ spatial steering vector and $M \times 1$ temporal steering vector.

The optimum processor [1] is the $MN \times 1$ weight vector

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{v} , \quad (5)$$

where $\mathbf{R} = E\{\mathbf{x}_u\mathbf{x}_u^H\}$ is the covariance matrix of the interference (clutter, jamming) plus noise component of the data. Although impractical to implement for the reasons mentioned above, analysis of the components of $\mathbf{R}$ is useful. We assume the clutter, jamming, and noise components are mutually uncorrelated

$$\mathbf{R} = \mathbf{R}_c + \mathbf{R}_j + \mathbf{R}_n . \quad (6)$$

where σ^2 is the receiver noise power per element per pulse. We shall assume that the radar instantaneous bandwidth is large relative to the PRF, so that noise and jamming signals decorrelate from pulse-to-pulse. With this assumption

$$\mathbf{R}_n = \sigma^2\mathbf{I}_{MN} , \quad (7)$$

and the jammer covariance matrix is block diagonal. With the additional assumption of a stationary jamming scenario,

$$\mathbf{R}_j = \mathbf{I}_M \otimes \mathbf{M}_j , \quad (8)$$

where $\mathbf{M}_j$ is the $N \times N$ jammer spatial covariance matrix.

Ground clutter is the primary source of interference for an airborne radar. The platform motion induces upon the clutter an angle-dependent Doppler frequency $f_c(\phi)$. The clutter covariance matrix may be written as

$$\mathbf{R}_c = \int_{-\pi}^{\pi} s_c(\phi)\mathbf{v}_c(\phi)\mathbf{v}_c^H(\phi)\, d\phi \quad (9)$$

where $\mathbf{v}_c(\phi) = \mathbf{v}(\phi, f_c(\phi))$ is the steering vector to a single clutter patch. The clutter power spectral density $s_c(\phi)$ depends on the radar system parameters, and is shaped by both the transmit pattern and the clutter distribution in angle. When the platform velocity vector is aligned with linear array axis, the clutter Doppler is a linear function of $\sin\phi$,

$$f_c(\theta, \phi) = \frac{2v}{\lambda}\cos\theta\sin\phi , \quad (10)$$

where v is the platform velocity. In terms of normalized spatial and Doppler frequency variables $\vartheta_c = d\lambda^{-1}\cos\theta\sin\phi$ and $\varpi = fT$, the clutter ridge has the simple form $\varpi_c = \beta\vartheta_c$, where $\beta = 2vT/d$. For $d = \lambda/2$, β describes the amount of Doppler ambiguity in the clutter ridge.

The rank of the clutter covariance matrix is a measure of the minimum number of adaptive degrees of freedom necessary for a space-time processor. An important result is the following theorem.

Theorem 1 *If β is an integer less than or equal to N,*

$$\operatorname{rank}(\mathbf{R}_c) = N + (M-1)\beta . \quad (11)$$

This result was suggested by Brennan and Staudaher [6], and a formal proof is given in [3]. For noninteger β (11) is an accurate predictor of the numerically significant portion of the rank. An intuitive explanation of Brennan's rule is that, in effect, clutter observations are repeated by different elements from different pulses, as the array moves. The rank is the number of distinct effective element positions during a CPI.

3. PARTIALLY ADAPTIVE STAP

We consider a generic partially adaptive processor consisting of an $MN \times D$ preprocessor $\mathbf{T}$ followed by a $D \times D$ adaptive weight computation. The preprocessor filters the input data set to a much smaller number of signals, thereby reducing both the computational and training requirements for the adaptive weight computation. The $D \times 1$ transform-domain snapshot is given by

$$\tilde{\mathbf{x}} = \mathbf{T}^H\mathbf{x} = \alpha\tilde{\mathbf{v}}_t + \tilde{\mathbf{x}}_u , \quad (12)$$

where $\tilde{\mathbf{v}}_t = \mathbf{T}^H\mathbf{v}_t$ is the transformed target steering vector and $\tilde{\mathbf{x}}_u = \mathbf{T}^H\mathbf{x}_u$ is the interference-plus-noise component. The adaptive weights are computed from

$$\tilde{\mathbf{w}} = \tilde{\mathbf{R}}_u^{-1}\tilde{\mathbf{g}}_t \quad (13)$$

where $\tilde{\mathbf{R}}_u = E\left\{\tilde{\mathbf{x}}_u\tilde{\mathbf{x}}_u^H\right\} = \mathbf{T}^H\mathbf{R}_u\mathbf{T}$ is the $D \times D$ covariance matrix of the transformed data. In (13), $\tilde{\mathbf{g}}_t$ is a $D \times 1$ desired response or target steering vector. Given a desired response $\mathbf{g}_t$ for a fully adaptive processor, the partially adaptive processor utilizes the desired response $\tilde{\mathbf{g}}_t = \mathbf{T}^H\mathbf{g}_t$. Applying the computed weights yields the final output

$$z = \tilde{\mathbf{w}}^H\tilde{\mathbf{x}} = (\mathbf{T}\tilde{\mathbf{w}})^H\mathbf{x} . \quad (14)$$

Typically a separate set of weights is computed for every angle and Doppler at which target presence is to be determined. In practice, data from multiple range gates are used to estimate the interference; either SMI or subspace-based adaptive weight computation may be used.

We shall classify STAP architectures based on the type of preprocessor, or equivalently, by the domain in which adaptive weighting occurs. Figure 2 divides STAP architectures into four basic types. Element space approaches retain full spatial adaptivity, but reduce the dimensionality through temporal preprocessing on each element. Temporal preprocessing may be simply selecting a small number of pulses, or it may be filtering the pulses on each element or beam. The former type of approach is termed pre-Doppler, as full CPI filtering occurs after adaptation. The next sections discuss each class in more detail.

3.1. Element-space Pre-Doppler

Define a *sub-CPI* to be a grouping of K_t successive pulses from the CPI. There are $M' = M - K_t + 1$ sub-CPIs, where the pth sub-CPI consists of pulses $p : p + K_t - 1$. Element-space pre-Doppler STAP solves a separate K_tN-dimensional adaptive problem for each sub-CPI. Typically, K_t need only be two or three, so that a dimensionality reduction of M/K_t is achieved. A fixed Doppler filter bank integrates the outputs of each adaptive problem to provide the full coherent gain and the means for velocity estimation. The preprocessor of each sub-CPI adaptive problem is given by

$$\mathbf{T}_p = \mathbf{J}_p \otimes \mathbf{I}_N \quad (15)$$

where $\mathbf{J}_p$ is a $M \times K_t$ selection matrix that chooses the K_t pulses for the pth sub-CPI on each element.

Pre-Doppler STAP provides weight updates every pulse. This is desirable for systems where the environment changes from pulse-to-pulse, such as with a rotating antenna. The clutter component of the sub-CPI covariance matrix has $\text{rank}(\tilde{\mathbf{R}}_c) = N+(K_t-1)\beta$, which is typically much less than K_tN. Full spatial adaptivity provides sufficient degrees of freedom to cancel jamming and clutter simultaneously.

3.2. Element-space Post-Doppler

Doppler filtering the pulses on each element is another way to reduce the STAP problem. A Doppler filter, with its potential for low sidelobes, can supress portions of the clutter ridge, thereby localizing competing clutter in angle. Element-space post-Doppler STAP utilizes a preprocessor consisting of a bank of K_t temporal filters on each element. The corresponding preprocessor is given by

$$\mathbf{T}_m = \mathbf{F}_m \otimes \mathbf{I}_N \ , \tag{16}$$

where $\mathbf{F}_m$ is an $M \times K_t$ matrix whose columns are the impulse responses of the filters applied to each element's pulses. Typically the preprocessor is tuned to particular Doppler frequency, and a separate adaptive problem is solved for each Doppler bin. The case $K_t = 1$ has been called factored post-Doppler, where separate spatial adaptive processing is done in each Doppler bin. Factored post-Doppler will not be considered here, as its performance is poor for systems with short CPIs and Doppler-ambiguous clutter.

The preprocessor can be designed to minimize the resultant clutter rank according to the following Theorem:

Theorem 2 *If the assumptions of Theorem 1 are satisfied, and if there exists a $K \times K$ nonsingular matrix $\mathbf{Q}$ and a length $M' = M - K + 1$ vector $\mathbf{f} = [f_0; f_1; \ldots; f_{M'-1}]$ such that*

$$\mathbf{F}_m\mathbf{Q} = \begin{bmatrix} f_0 & & \mathbf{0} \\ f_1 & \ddots & \\ \vdots & & f_0 \\ f_{M'-1} & & f_1 \\ & \ddots & \vdots \\ \mathbf{0} & & f_{M'-1} \end{bmatrix} . \tag{17}$$

then

$$\text{rank}(\tilde{\mathbf{R}}_c) = N + (K_t - 1)\beta \, . \tag{18}$$

The proof is a straightforward application of Theorem 1 to the filtered clutter covariance matrix. The case $\mathbf{Q} = \mathbf{I}_{K_t}$ results in a Toeplitz $\mathbf{F}_m$ that is equivalent to processing the M pulses on each element with an M'-pulse filter $\mathbf{f}$. This is Brennan's 'filter-then-adapt' [6] architecture. Theorem 2 can also be satisfied with adjacent bins from a standard DFT Doppler processor, as in DiPietro [5] provided that no tapering is employed.

Post-Doppler STAP typically provides slightly better Doppler space coverage than pre-Doppler STAP. There is some evidence that the ability to filter some clutter prior to adaptation frees up degrees of freedom for dealing with jamming. The weight update rate with post-Doppler STAP is once per CPI.

3.3. Beamspace Pre-Doppler

Element-space techniques become impractical for large arrays, in which case beamspace approaches provide additional dimensionality reduction. Beamspace pre-Doppler STAP refers to architectures where the signals from each pulse are beamformed, and then a sub-CPI of pulses from a selected set of beams are used for adaptation. As in Section 3.1, the outputs of each sub-CPI adaptation are coherently integrated in a fixed Doppler filter bank. The preprocessor for a beamspace pre-Doppler approach takes the form

$$\mathbf{T}_p = \mathbf{J}_p \otimes \mathbf{G} \tag{19}$$

where $\mathbf{J}_p$ is a selection matrix as before and $\mathbf{G}$ is an $N \times K_s$ beamformer matrix whose colums are the selected beamformers. Thus, each sub-CPI involves a K_sK_t size adaptive problem. Typically, K_t is two or three, and $K_s < N$ is small, so that a subtantial reduction in computation and training requirements results.

The clutter rank for beamspace pre-Doppler STAP depends on $\mathbf{G}$ and is minimized if the following theorem is satisfied.

Theorem 3 *If the assumptions of Theorem 1 are satisfied, and if there exists a $K_s \times K_s$ nonsingular matrix $\mathbf{Q}$ and a length $N' = N - K_s + 1$ vector $\mathbf{g} = [g_0; g_1; \ldots; g_{N'-1}]$ such that*

$$\mathbf{G}\mathbf{Q} = \begin{bmatrix} g_0 & & \mathbf{0} \\ g_1 & \ddots & \\ \vdots & & g_0 \\ g_{N'-1} & & g_1 \\ & \ddots & \vdots \\ \mathbf{0} & & g_{N'-1} \end{bmatrix} . \tag{20}$$

then

$$\text{rank}(\tilde{\mathbf{R}}_c) = K_s + (K_t - 1)\beta \, . \tag{21}$$

Displaced phase center antenna (DPCA) processing [2] is an example of non-adaptive beamspace pre-Doppler processing. The conditions for which Theorem 3 holds are equivalent to the conditions for which perfect DPCA clutter cancellation is possible. There is a clear duality between element-space post-Doppler and beamspace pre-Doppler. One approach combines displaced temporal subapertures from different elements, while the other combines displaced spatial subapertures from different pulses.

3.4. Beamspace Post-Doppler

Finally, beamspace processing can be combined with Doppler filtering on each element prior to adaptation. The combination of beamformer and Doppler filter sidelobes can produce significant nonadaptive suppression of portions of the clutter ridge, thereby leaving less to be adaptively cancelled. We shall restrict attention to separable $MN \times K_tK_s$ preprocessors of form

$$\mathbf{T}_p = \mathbf{F}_p \otimes \mathbf{G} \tag{22}$$

which may be thought of as the cascade of beamforming each pulse followed by temporal filtering each beam.

Theorem 4 *If the conditions for Theorem 1 hold, and if* $\mathbf{F}_p$ *and* $\mathbf{G}$ *satisfy (18) and (21), respectively, then the clutter rank is minimum and equal to*

$$\operatorname{rank}(\tilde{\mathbf{R}}_c) = K_s + (K_t - 1)\beta \,. \qquad (23)$$

One type of beamspace post-Doppler performs a 2-D filtering of the space-time snapshot with the $M'N' \times 1$ space-time filter $\mathbf{h} = \mathbf{f} \otimes \mathbf{g}$; the output $K_t K_s$ signals are then adaptively processed. A separate 2-D filter is used for each target angle and Doppler bin. Another beamspace post-Doppler architecture has been studied by Cai and and Wang [4]. Their preprocessor consists of a square block of adjacent output bins from a 2D-DFT, with a different subset of bins used for each target angle and Doppler. This approach satisfies Theorem 3 with no tapering. Other approaches to beam selection that result in nonseparable preprocessors (which cannot be expressed as (22)) are also possible.

Beamspace architectures can significantly reduce the adaptive problem size. However, the need to suppress combined jamming and clutter will impact the beamformer design. One approach is to include more spatial degrees of freedom in the preprocessor than is necessary for clutter cancellation alone. These may be additional directional beams that cover a sector or specially designed beams with broader coverage. A second approach is to deal with the interference in a two-step process [7], where the beamspace portion of the preprocessor is itself adaptive and removes jamming as part of the beamspace transform. This requires data that are free of mainlobe clutter for jammer training, such as clutter-free range gates or Doppler bins. The second step of adaptive processing then removes the clutter with either a pre or post-Doppler approach.

4. SUMMARY

Space-time adaptive processing can significantly improve airborne radar performance. Computational complexity and the need to estimate non-stationary interference with limited data forces consideration of partially adaptive architectures. A taxonomy of architectures classified according to preprocessor type was presented. Analysis of clutter rank in each domain provides conditions for preprocessor design, as well as insight into the relationships between different STAP architectures. The STAP computational complexity is driven not just be the size of a single adaptive problem, but also by the number of adaptive problems that must be solved per CPI. The best algorithm will depend on the specifics of the radar system and the training strategies that can be supported.

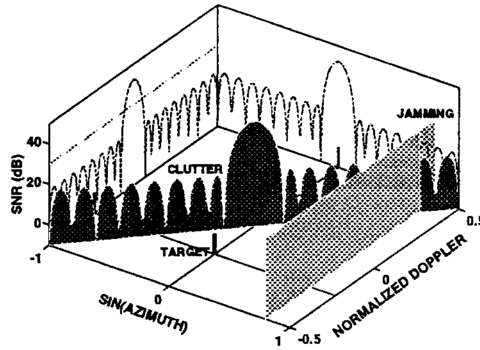

Figure 1: AEW radar interference environment.

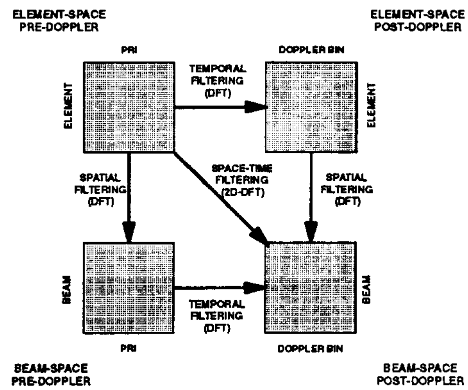

Figure 2: Taxonomy of STAP architectures.

5. REFERENCES

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