MULTIRESOLUTION STATISTICAL ANALYSIS AND ASSIMILATION OF LARGE OCEAN DATA SETS

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ABSTRACT

A recent significant problem in oceanographic remote sensing is the dense gridding or smoothing of sparsely sampled altimetric data. The smoothing of altimetric measurements has an application much broader than just the regular production of elevation maps for oceanographers, however. In particular, the ability to estimate ocean circulation patterns from altimetric data can serve as an important measure for the verification of ocean acoustic tomographic results.

In this paper we present a multiscale technique capable of extremely efficient interpolation of altimetric data: about 256000 estimates *and* estimation error variances are computed in one minute on a Sun Sparc-10. We also demonstrate how similar techniques may be used to directly estimate the surface gradient and biases in the geoid-model error.

1. INTRODUCTION

The need to perform sophisticated remote sensing of the ocean has increased substantially in recent decades. The role of the ocean on global warming and other meteorological trends has become increasingly apparent, and computer simulations of ocean circulation models meant to predict such events are running at increasing levels of precision and require correspondingly augmented quantities of reliable oceanographic data.

The cooperative international American/French Topex/Poseidon[5] mission has taken a significant step in providing this high quality data. Ocean surface height measurements, accurate to about 5cm, are available on densely sampled tracks spaced 300km apart; a complete coverage of the earth is obtained every ten days.

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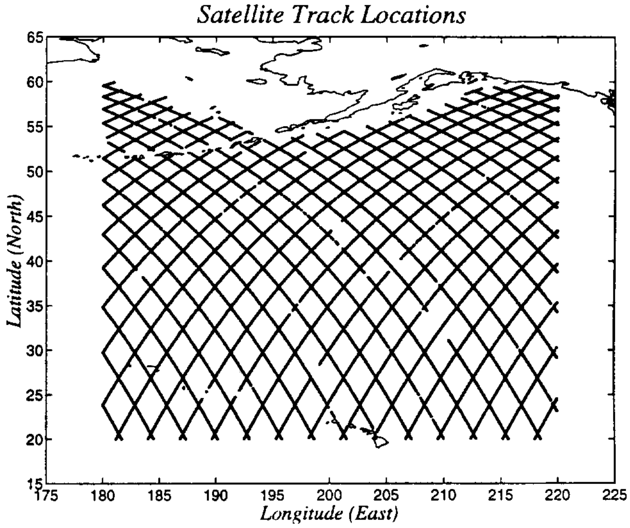

Figure 1: Set of TOPEX/POSEIDON measurement tracks in north Pacific

The goal of our research is to assimilate Topex/Poseidon altimetric measurements into forms that allow meaningful quantitative assessment by oceanographers and that can drive dynamic ocean circulation models. Although the altimeter makes measurements of the ocean surface only, it has a contribution to make to underwater oceanography: a careful assessment of the shape of the ocean surface permits the estimation of ocean currents below the surface. The estimation of such currents from surface data can provide an important verification of underwater acoustic tomography studies.

There are a number of aspects that make the development and application of a statistical model challenging in this specific problem:

- There is an enormous amount of data.
- The sampling pattern of the data is unusual.
- Temporal and spatial scales are coupled.

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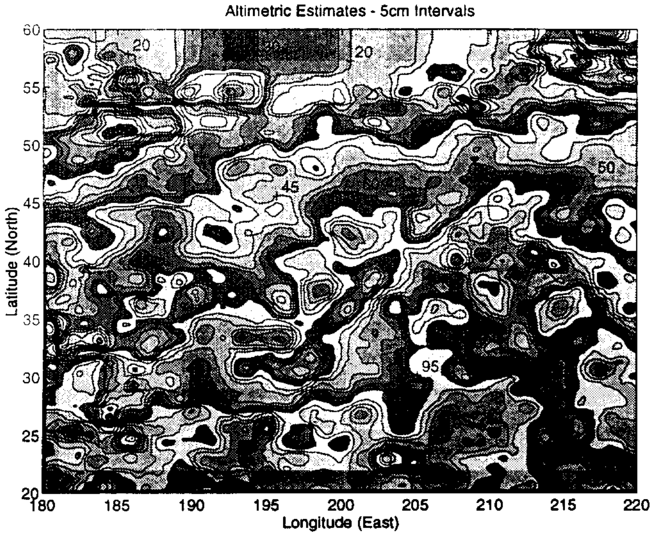

Figure 2: Estimation of ocean mean elevation based on a single ten day set of data

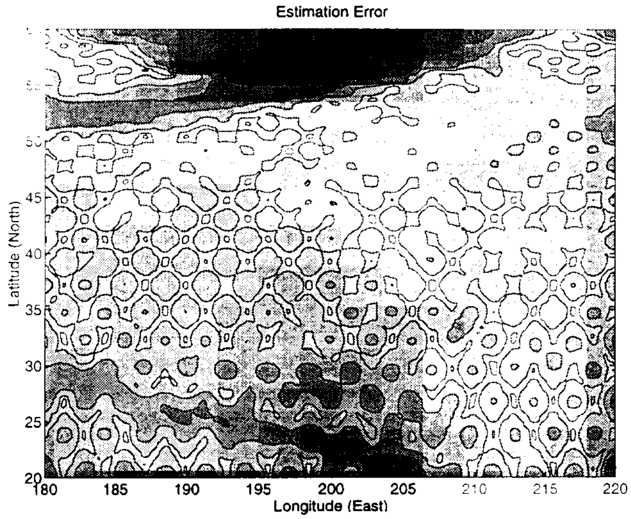

Figure 3: Estimation error variances based on one repeat cycle of data; darker regions represent greater uncertainty

- The prior statistics of the ocean surface are uncertain. Indeed, the determination of such statistics remains an important active research objective.
- Not only ocean surface elevation estimates, but also surface estimation *error* statistics are required.

We meet the above challenges by using a novel multiscale statistical estimation framework. The application of this multiscale framework to oceanographic altimetry data has been described elsewhere[2], but is briefly reviewed in the next section. The remaining sections of this paper will demonstrate the performance of the multiscale estimator applied to related problems in estimating the bias in the geoid (i.e., the gravitational equipotential surface) model, and in the estimation of ocean surface gradients.

2. MULTISCALE ESTIMATION OF THE OCEAN SURFACE

A multiscale estimation methodology has recently been developed[1, 4] that offers the promise of efficient optimal data assimilation and smoothing algorithms. The methodology effectively generalizes the Kalman filter and Rauch-Tung-Striebel smoother to operate on a tree structure (of arbitrary shape or size); that is, an explicitly multiscale system model is specified on the tree as

$$x(s) = A(s)x(s\bar{\gamma}) + B(s)w(s) \quad (1)$$
$$y(s) = C(s)x(s) + v(s) \quad (2)$$

where $x(s)$ represents the state at tree node s, $B(s)w(s)$ represents the process noise term from parent state $s\bar{\gamma}$ to state s, and $y(s)$ represents a possible measurement at s. The framework is very flexible and finds a natural application in multidimensional problems.

This multiscale methodology has a number of powerful features, several of which directly address the challenges listed earlier:

- Multiscale models are natural means of representing fractal or self-similar processes.
- The framework permits sparse, multiscale, and heterogeneous measurements.
- The resulting scale-recursive algorithm is extremely fast.
- Both estimation error covariances and *cross*-covariances may be computed between arbitrary states (even across scales), an impossibility with other approaches such as those using Markov random fields.
- Multiscale likelihood ratios can be computed, facilitating multiscale model identification.

A simple prior model of the ocean height field is a fractal or self-similar one (e.g., a $1/f^2$ spectrum), which is supported by empirical evidence. It has been shown[7] that the following multiscale model approximates the statistics of a $1/f^2$ process:

$$x(s) = x(s\bar{\gamma}) + B_s 2^{-m(s)/2} w(s) \quad (3)$$

where $w(s)$ is unit variance white noise, and $m(s)$ measures the scale of corresponding node s. By applying

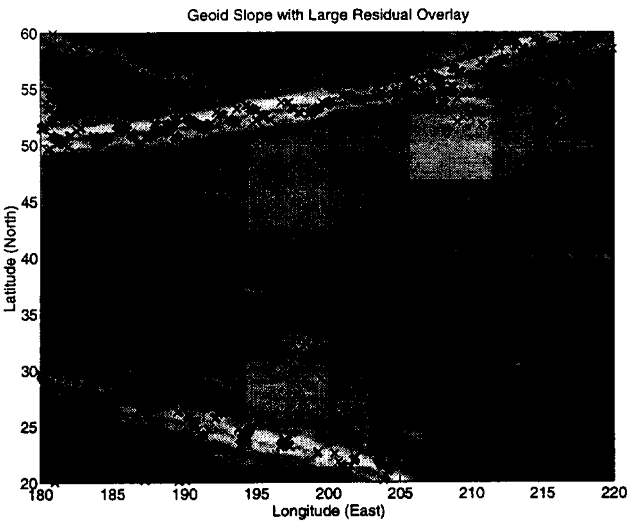

Figure 4: Overlay of geoid gradient map with the distribution of locations of large estimation residuals; regions of lighter shading represent a steeper geoid gradient.

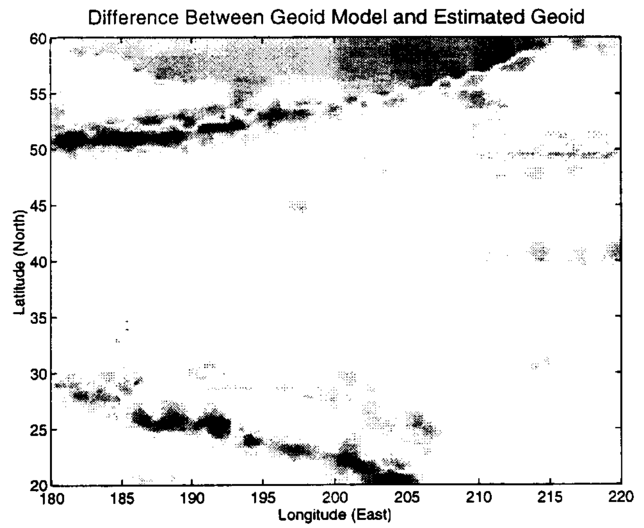

Figure 5: Estimated bias in the geoid model based on joint ocean-geoid estimation. Darker regions represent greater bias

this model to some period (from a few days to a year) of Topex/Poseidon data permits us to compute dense altimetry estimates and estimation error variances. Specifically, Figure 2 plots elevation estimates for the north Pacific, given ten days of data; Figure 3 plots the corresponding estimation error variances. The 512×512 maps of estimates *and* error variances required approximately one minute of computation time on a Sun Sparc-10.

3. GEOID BIAS ESTIMATION

The geoid, or the gravitational equipotential surface, is the baseline relative to which the ocean surface is measured; consequently errors in the geoid model, due to unknown or unmodeled gravitational anomalies, translate directly into ocean altimetry measurement errors. For the Topex/Poseidon data, a 360 degree spherical harmonic model is used to approximate the geoid, so certain small or abrupt features in the geoid may not be captured.

There is good reason to believe that biases in the geoid model may be estimatable using our framework: Figure 4, which shows the correlation between the gradient in the geoid model and the estimation residuals, motivates this belief.

We propose to *jointly* estimate the ocean surface elevation and the geoid. Once again, we find that a $1/f^2$ model approximates the empirical statistics of the geoid, leading to the following multiscale model:

$$\begin{bmatrix} x \\ n \end{bmatrix}(s) = \begin{bmatrix} x \\ n \end{bmatrix}(s\bar{\gamma}) + \begin{bmatrix} B_s & 0 \\ 0 & B_n \end{bmatrix} 2^{-\frac{m(s)}{2}} w(s) \quad (4)$$

where x and n denote the ocean elevation and the geoid, respectively, and where B_s and B_n control the relative variances of the $1/f^2$ models for the ocean surface and the geoid. The corresponding multiscale measurement model is

$$\begin{bmatrix} y_{\text{satellite}} \\ y_{\text{geoidmodel}} \end{bmatrix}(s) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ n \end{bmatrix}(s) + v(s) \quad (5)$$

$$v(s) \sim \mathcal{N}\left(0, \begin{bmatrix} (5cm)^2 & 0 \\ 0 & \phi(s) \end{bmatrix}\right) \quad (6)$$

where $\mathcal{N}$ denotes a Gaussian distribution. The measurement error covariance model (Equation 6)) asserts that the satellite measurements have a constant standard deviation of 5cm, and that the uncertainty in the geoid model varies spatially as $\phi(s)$; the spatial variation of the variance ϕ is a monotonic function of the geoid slope, determined empirically from the data of Figure 4. It is a significant feature of our multiscale framework that such space-varying models can be used with essentially no increase in computational effort.

Figure 5 plots the estimated bias, i.e., the difference between the geoid model at s and the estimate $n(s)$. The results are, of course, consistent with the distribution of large residuals from Figure 4, but also with our prior expectations: the predominant regions exhibiting a bias are the Aleutian archipelago in the north, the Hawaiian island chain in the south, and the bathymetric rifts in the east. Each of these features are abrupt and spatially localized, and thus unlikely to have been adequately captured by the geoid model. Further verification of the revised geoid estimates is an ongoing research project.

4. SURFACE GRADIENT ESTIMATION

We have demonstrated the ability to estimate ocean surface elevation. The corresponding ocean circulation, of considerable interest to oceanographers and in the acoustic tomography context, can in principle be deduced by computing synthetic gradients of the estimated surface. Such an approach has two obvious shortcomings:

- The error statistics for such a surface gradient are not immediately available.
- Estimating the gradient by differencing surface estimates may not yield an optimal (or even a reasonable) estimate of the gradient.

A solution to both of the above shortcomings is the development of a joint multiscale model for both the surface and its gradient.

In related research we have investigated the relationship between a certain class of surface reconstruction methods in computer vision, and our multiscale models. A simple surface model is that of a membrane (e.g., a rubber sheet); the variational cost function for a membrane has been shown to be similar to a $1/f^2$ constraint, which leads us to propose the following multiscale model:

$$\begin{bmatrix} z \\ z_x \\ z_y \end{bmatrix}(s) = \begin{bmatrix} 1 & A2^{-m(s)} & A2^{-m(s)} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ z_x \\ z_y \end{bmatrix}(s\bar{\gamma}) + \begin{bmatrix} B_s & 0 & 0 \\ 0 & B_g & 0 \\ 0 & 0 & B_g \end{bmatrix} 2^{-\frac{m(s)}{2}} w(s) \qquad (7)$$

that is, the gradient is modeled as a $1/f^2$ process, and the surface is modeled as the integral of the gradient, plus added noise.

Figure 6 shows a crude set of estimated gradients, chosen at some coarse scale, superimposed on an intensity plot of the estimated surface elevation. These results are somewhat preliminary, but promising. Deep questions remain, such as the identification of unknown parameters in the multiscale statistical models such as (7) and the reduction of "blocky" effects (visible in Figure 6 (both of which have been addressed with some success), and how to even talk about estimating the "gradient" of a surface such as the ocean, which has a fractal-like (or chaotic) nature.

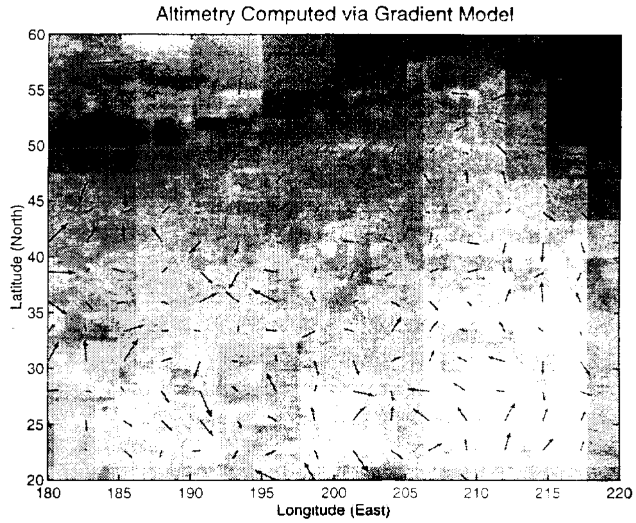

Figure 6: A preliminary set of multiscale estimates of the surface gradient.

5. REFERENCES

[1] K. Chou, A. Willsky, A. Benveniste, "Multiscale Recursive Estimation, Data Fusion, and Regularization", *IEEE Trans. on Automatic Control*, to appear.

[2] P. Fieguth, A. Willsky, W. Karl, "Multiresolution Stochastic Imaging of Satellite Oceanographic Altimetric", *IEEE First Int'l Conference on Image Processing - Special Session on Electronic Imaging and Model-Based Methods*, 1994

[3] P. Gaspar, C. Wunsch, "Estimates from Altimeter Data of Baryotropic Rossby Waves in the Northwestern Atlantic Ocean", *J. Physical Oceanography* (19)#12, pp.1821–1844, 1989

[4] M. Luettgen, W. Karl, A. Willsky, "Efficient Multiscale Regularization with Applications to the Computation of Optical Flow", *IEEE Trans. on Image Processing*, (3) #1, pp.41–64, 1994

[5] P. Marth et al., "Prelaunch Performance of the NASA Altimeter for the TOPEX/POSEIDON Project", *IEEE Trans. on Geoscience and Remote Sensing* (31) #2, pp.315–332, 1993

[6] R. Rapp, "Geoid Undulation Accuracy", *IEEE Trans. on Geoscience and Remote Sensing* (31) #2, pp.365–370, 1993

[7] G. Wornell, "Wavelet-Based Representation for the $1/f$ Family of Fractal Processes", *Proc. IEEE*, Sept. 1993