ITERATIVE LEAST SQUARES APPROACH TO THE MIXTURE MODELING PROBLEM

Murari Srinivasan

Dept. of Electrical Engineering
University of Maryland
College Park, MD 20742

Ruth DeFries

Dept. of Geography
University of Maryland
College Park, MD 20742

ABSTRACT

The mixture model is an attempt to accurately model the ground truth in the case of low-resolution remote-sensed imagery. The model assumes that a pixel in the image does not consist of a single class, but consists instead of the sum of fractions of various classes. We use an iterative least squares approach to estimate these fractions for every pixel. Results are provided on synthetic data as well as real AVHRR data from the African continent.

I.Introduction

This work is part of the NSF Grand Challenge Program on High Performance Computing for Land Cover Dynamics at the University of Maryland College Park campus. The program is highly interdisciplinary and involves physical,earth and computer scientists as well as electrical engineers. We are aiming at the development of scalable and parallel solutions to classification and mixture unmixing problems. This particular work is part of the ongoing effort to develop parallel estimation algorithms for the linear mixture model with an underlying correlation structure.

Image classification attempts to assign a class label corresponding to one of the cover classes to each pixel in the remotely sensed image. When the ground pixel is very large, as in the case of AVHRR (Advanced Very High Resolution Radiometer - which has a resolution of 8km to a pixel) data, this is not an accurate representation of the ground truth. A ground pixel will typically contain more than one cover class. The mixture model [1]-[3] assumes that the reflectance of a pixel in any spectral band is a linear combination of the reflectances of the different cover classes contained in that pixel. The weights of the linear combination are the proportions of the cover classes in that pixel.

A linear model is typically used since it facilitates the estimation of the proportions of each cover class over the pixels of the image. The unmixing problem involves obtaining the "best" estimates of the fractions of the different cover classes that are contained in the pixel. We use a least-squares criterion in our approach. The result of the mixture modeling approach is a set of fractional images that correspond to the variations in the proportions of the cover classes over the image. This is a quantitative result unlike the thematic map denoting a rigid assignment that is the end result of classification.

In the literature, one-shot least-squares methods [3] as well as iterated Weighted Least Squares (WLS) and constrained least squares algorithms [4] have been solve the unmixing problem. We implement a second-order, Newton-Raphson type of iterative least-squares algorithm with superimposed constraints to solve the mixture modeling problem. The algorithm is found to be more robust than the one-shot least-squares methods used in the literature. In experiments so far, it also seems to yield better estimates of the ground truth as compared to WLS approaches. The algorithm is tested on multispectral remotely sensed imagery of a large portion of the African continent.

II.The Linear Mixture Model and the Unmixing Algorithm

Each pixel is assumed to contain proportions of the various cover classes [3],[4]. Therefore, in the linear model, the spectral response of a pixel in a given spectral band

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is assumed to be generated by the linear combination of the responses of the various cover classes in that spectral band. Considering one pixel in the image, the vector of proportions is denoted by,

$$\mathbf{f} = [f_1\, f_2\, f_3 \ldots f_c]^t$$

where c is the number of classes and each f_i denotes the proportion (a fraction between 0.0 and 1.0) of class 'i' within that pixel. The multispectral observations for that pixel are denoted by

$$\mathbf{x} = [x_1\, x_2 \ldots\ldots x_n]^t$$

where n is the number of bands over which the observations are made and each x_j denotes the spectral response in band 'j'. The linear relation follows as :

$$x_i = m_{i1} f_1 + m_{i2} f_2 + \ldots + m_{ic} f_c$$

where m_{ji} denotes the spectral response that a pure pixel (proportion of 1.0) of cover class 'i' would produce in spectral band 'j'. In matrix notation,

$$\mathbf{x} = \mathbf{M}\mathbf{f}$$

The matrix M is conventionally known as the "endmember spectra". In addition to these mixing equations, we have the constraint that in any given pixel, the proportions must sum to one; i.e.,

$\mathbf{j}^t\mathbf{f} = 1$ where $\mathbf{j}$ is a vector containing ones.

The non-negativity of the f_i's is imposed by the algorithm during the iterations. Let the estimate of the proportions, $\mathbf{f}$, be $\widehat{\mathbf{f}}$. Then the squared estimation error is

$$e = (\mathbf{x} - \mathbf{M}\widehat{\mathbf{f}})^t(\mathbf{x} - \mathbf{M}\widehat{\mathbf{f}}).$$

We attempt to minimize the squared error subject to the constraint that the proportions must sum to one. A Lagrangian formulation follows directly and the unconstrained minimization problem may be cast as

Minimize

$$Q = (\mathbf{x} - \mathbf{M}\widehat{\mathbf{f}})^t(\mathbf{x} - \mathbf{M}\widehat{\mathbf{f}}) + \lambda(\mathbf{j}^t\widehat{\mathbf{f}} - 1)$$

Defining the variable $\mathbf{z}$ to be $\mathbf{z} = [\mathbf{x}^t\, \lambda]$, the Newton-Raphson approach to the minimization problem may be formulated as

$$\mathbf{z}(n+1) = \mathbf{z}(n) - \beta(\nabla^2_{\mathbf{z}}\mathbf{Q})^{-1}\nabla_{\mathbf{z}}\mathbf{Q}$$

where β is the step size, $\nabla^2_{\mathbf{z}}\mathbf{Q}$ is the Hessian matrix and $\nabla_{\mathbf{z}}\mathbf{Q}$ is the gradient vector.

III.Experimental results

The algorithm was validated in two steps. First, the algorithm was applied to synthetic mixture data. The data was generated randomly and the spectral observations generated using an endmember matrix that closely reflected the spectral reflectances found typically in AVHRR data. The errors involved in estimating the original fractions (which were, of course, known deterministically) are listed below.

class1	class2	class3	class4	class5	class6
0.02	0.009	0.009	0.012	0.009	0.013

On this test data set, we also find that the iterative algorithm yields much more accurate and robust results than "one-shot" (non-iterative) algorithms described in the literature.

The algorithm was then tested on the AVHRR data covering a large portion of the African continent. The size of the images is 433x487 and five spectral bands are used. Two of the bands are in the visual spectrum, two others are thermal bands and the last is NDVI. The NDVI band is actually derived from the red and infra-red bands as a ratio. It is the ratio of the difference of the observations to the sum of the observations in these two bands, and reflects the presence or absence of green vegetation. The endmember spectra for this image were derived from a thematic classification map that was obtained by the maximum likelihood method. Using the Classification map, regions that contained "pure" classes were selected. The spectral response for each class was averaged over its region, and the procedure was repeated for all the spectral bands. Although the classification map included several cover classes, many of these classes were themselves mixtures of the following three main cover types apart from water : unvegetated land,grassland and tropical forest. Accordingly these three cover types were chosen as the 'pure' classes.

The fractional maps generated by the algorithm were compared to known vegetation regions as well as the thematic classification map. The fraactional maps generated by the algorithm are shown below in figures 1-3. The results obtained with this algorithm are compared with those obtained using the conventional algorithm used commonly by Earth Scientists. It may be observed from Figure.1 that the conventional algorithm fails to produce estimates in a portion of the desert region in Africa. The algorithm illustrated in this paper is more robust. Also, the conventional algorithm does not reflect the presence of grassland in the rift valley. The iterative algorithm accurately estimates the presence of grass in this region (Figures 2 and 3). In order to facilitate comparison, the thematic classification map is also shown (figure.4).

IV.Parallel Implementation

We have also developed a parallel implementation of this algorithm on a 32-node Connection Machine. The algorithm has been implemented using the multiblock PARTI primitives[5] developed at the University of Maryland. Implementation of the algorithm using these primitives implies that it is machine independent. The algorithm may be used on any platform on which the multiblock PARTI primitives are established. This may either be a stand-alone massively parallel machine or a distributed computing environment.

In this case, the problem is trivially parallelizable since the computations are pixel-based and there is no correlation assumed between the pixels. This implementation provides a speedup of the order of 10 over a Sun Sparcstation 10. For example, the algorithm takes about 30 minutes to estimate fractions over an image of approximately 500 by 500 pixels. The parallel version of the algorithm takes about 3 minutes on the same image. The parallel implementation will result in a quick analysis turn around time on large global-sized imagery for the end users.

V.Future Directions

We are attempting to model the local correlations within the image through a quarter-plane causal model. The estimation of the fractions within each pixel could then be accomplished through a Kalman filter type of approach.

We are also attempting to improve the accuracy of the estimation of the end-member spectra. We propose to use principal component analysis to identify the "true" endmembers.

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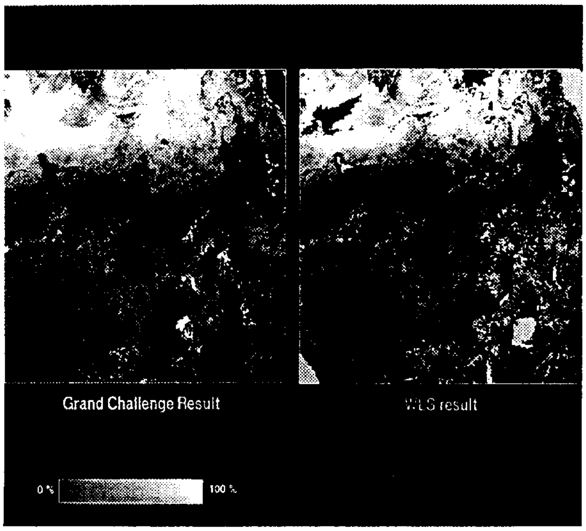

Figure 1: Desert Estimate

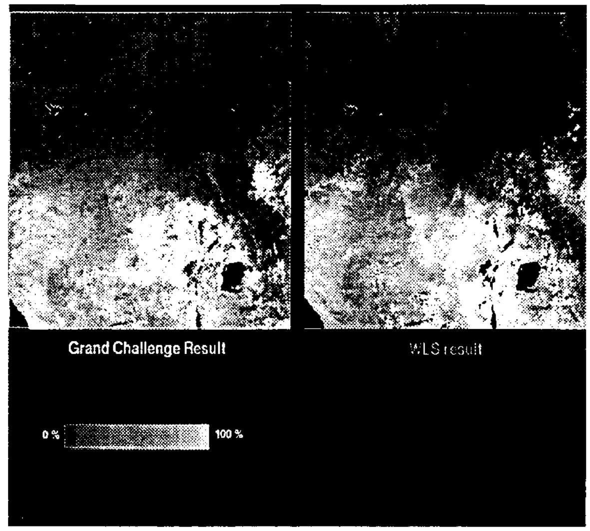

Figure 3: Forest Estimate

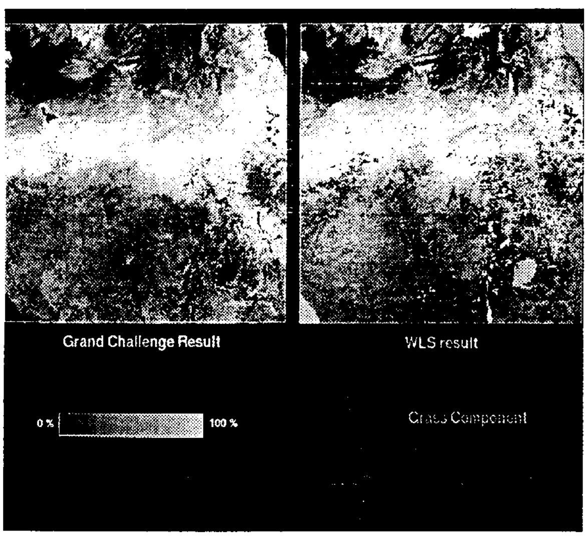

Figure 2: Grassland Estimate

Figure 4: Classification Result