POWER SYSTEM DISTURBANCE MONITORING USING SPECTRAL ANALYSIS TECHNIQUES

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ABSTRACT

This paper considers the problem of measuring and analysing the frequency and phase angle variations which occur in a power system after a fault or disturbance. This is an important problem because the results of the analysis are used in implementing control strategies to avert generator cutout. Commonly, the frequency is not measured directly, but is inferred from the instantaneous power at the substation. Since there is a complicated *non-linear* relationship between the power and the frequency, some bias is incurred in the measurement. This paper proposes instead that the angle and or frequency be obtained directly via an analytic signal readily constructed from the 3-phase output of the power system. The paper also proposes enhancements for some of the currently employed signal analysis techniques, and a new technique based on an extension of Thompson's harmonic line test.

1. INTRODUCTION

In a power system ideally all generators should run in synchronism so as to maintain system frequency within specified limits. When a disturbance occurs in a power system creating an imbalance between the mechanical power being supplied to a generator by its turbine and the electrical power being supplied to the power system, this imbalance is translated into a change in the kinetic energy of the rotor. In other words the generator begins to speed up or slow down. Normally various damping phenomena within the power system will act so that the system will attain a new steady state operating point.

However, should this change in speed continue unabated then the generator will lose synchronism with the power system and will be disconnected. Thus the remaining generators will be required to pick up extra load and should this load be too much for them to handle they in turn will drop out eventually leading to a system blackout.

The subject of this paper is the early detection of changes in generator speed which may enable power system controllers to take appropriate action. Although non-oscillatory instability can occur following very large disturbances, the usual course of events is for the power system generator angle to oscillate following a disturbance, their motions being described by a set of non-linear second order differential equations known as the swing equations:

$$\frac{2H}{\omega_0}\frac{d^2\delta_{in}}{dt^2} = P_{mi} - P_{ei} \tag{1}$$

where δ_{in} is the angle between generator, i, and the reference generator, n, H is an inertial constant, ω_0 is the system angular frequency, P_{mi} is the mechanical power supplied to the generator rotor, and P_{ei} is the electrical power supplied by the generator to the system. Note that the machine angle is, up to a multiplicative constant, the integral of the instantaneous frequency (IF). The frequency of the angle oscillations is usually in the range 0.1 Hz to 10Hz.

Equation (1) is non-linear due to the non-linear dependence of the electrical power on machine angles as expressed by equation (2):

$$P_{ei} = P_{ci} + \sum_{j=1, j\neq i} P_{Mi} sin(\theta_{ij} - \delta_{in} + \delta_{jn}) \tag{2}$$

where P_{ci}, P_{Mi} and θ_{ij} are constants related to network voltages and impedances. See [1], Chapter 2.

The behaviour of a set of generators following a disturbance is usually analysed by the numerical integration of the swing equation due to the non-linear nature of the problem. For small disturbances it is possible to obtain a linearized model around a given operating point, and to express the behaviour of the system in state space form. (See [5], Chapter 1). Once the linearisation has been performed, eigenanalysis may be used as an alternative to numerical integration. This technique yields a set of eigenvectors, corresponding to damped sinusoidal modes of oscillation. The modes that are poorly damped are a matter of particular concern, since these have the potential to cause generator cut-out.

Recently, to avoid the need to use the approximate linearisation inherent in eigenanalysis, much work has been done on the application of signal processing techniques such as Prony's method, [5] pp 105-115, [6] , and Fourier based techniques [3], [8] to the observed disturbance record directly.

The purpose of this paper is to propose a new way to obtain the post-disturbance phase/frequency measurement, and to

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propose some modifications to currently used spectral analysis techniques.

2. MEASUREMENT AND ESTIMATION OF ANGLE AND FREQUENCY

2.1. CURRENT APPROACHES

The current approaches used for the estimation of the damping and frequency of oscillation modes are 1) eigenanalysis and 2) spectral analysis techniques on observed data. With the latter approach spectral techniques have generally been applied to either average power or to RMS voltage magnitude estimates, both of which are non-linearly related to the actual quantity that is desired, namely the phase angle of the machines.

The use of average power for instance, which is seen in equation (2) to be a non-linear function of machine angle, will lead to the appearance of harmonic or ghost frequencies which in turn will cause Prony's method or any other method to give erroneous results.

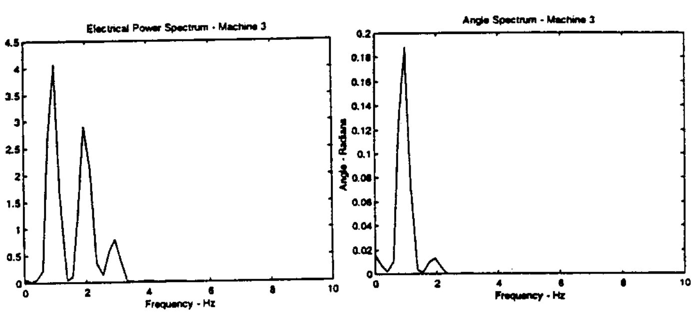
If the angle of machine i, namely δ_{in} is oscillating sinusoidally according to:

$$\delta_{in}(t) = \delta_{maxi} sin(\omega_m t) \tag{3}$$

and the angle at machine j oscillates at the same frequency, but with a phase delay of α, then from (2) the average power observed will be of the form:

$$P_{ei} = P_{ci} + P_{Mi} sin(\theta_{ij} - \delta_{maxi} sin(\omega_m t) + \delta_{maxj} sin(\omega_m t - \alpha)) \tag{4}$$

This means that the spectrum of the average power will consist of a component at ω_m plus components at integral multiples of ω_m since the second term may be expressed in terms of Bessel functions. The non-equivalence of the power and angle spectra is illustrated by figures 1 and 2 which depict respectively the electrical power and angle spectra for a machine in a model power system subjected to a disturbance. Details of the example may be found in [1], Ch.2.



(a) (b)

Figure 1. Spectrum of (a) Electrical Power (b) Generator Angle

Clearly, the presence of ghost harmonics will not only confuse identification of true modes, but it could also excaserbate resolution problems. The following section proposes an alternative method of obtaining frequency measurements which is less impaired by harmonic distortion problems.

3. DIRECT MEASUREMENT OF ANGLE AND FREQUENCY

The oscillation of generators in a power system has the effect of phase (and therefore also frequency) modulating the mains frequency. This fact can be used to obtain much faster and more direct estimates of the machine angles and frequencies, thereby enabling the use of Prony's method or other such spectral estimation techniques.

The definition for local or instantaneous frequency is:

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}, \tag{5}$$

where $\phi(t)$ is the phase of the analytic signal associated with the real signal. In the power system scenario, because of the presence of a three phase supply, the analytic signal can be readily generated. It is constructed from the A and B phase time domain voltages as follows:

$$z(t) = v_a(t) + j(1/\sqrt{3}v_a(t) + 2/\sqrt{3}v_b(t)). \tag{6}$$

Alternatively it can be obtained from all three phases, with possibly better performance, by the method of Clarke [4].

Once the analytic signal has been constructed, there is a need to estimate the phase, and possibly also its derivative, the frequency. There are many ways to do the estimation, depending on the signal to noise ratio (SNR) and the model. Because the model here is rather complicated (i.e. an unspecified number of damped sinusoids with unknown parameters), it is recommended that no model be assumed, and that the phase just be unwrapped, and if necessary, differentiated. At SNRs above about $7dB$, and assuming that the noise is additive white and Gaussian, it has been shown that this procedure produces a reliable phase estimate which is white and Gaussianly distributed [7].

Of course, it is also necessary to remove the 50Hz mains frequency from the frequency estimate to obtain the estimate of the disturbance alone.

The extraction of the frequency with this approach is also quite good under conditions of phase imbalance and harmonic distortion of the phase voltage waveforms. If the three phase voltages aren't perfectly balanced then the computation of the analytic signal in (6) will be imperfect and a negative mains frequency component will exist, mains frequency in this case being 50Hz. This causes a $100Hz$ component to appear in the estimated waveforms. This error component can be easily recognised and filtered out, as there will be no electro-mechanical mode of oscillation above +10 Hz. Alternatively it would be possible to compute a positive sequence set of three phase voltages from an unbalanced three phase set, see [2], Chapter 13, and apply the algorithm to that.

4. FOURIER BASED ANALYSIS

Two approaches have been used for the spectral analysis of the modes. Historically, the first was Fourier based. Because of the apparent problems with resolution for short data lengths, however, the application of parametric techniques such as Prony's method were introduced. This paper concentrates on Fourier based techniques, partly because the treatment of parametric techniques seems adequate. The Fourier techniques also have the advantage of greater speed (in general), and potentially better noise performance and robustness to signal model inaccuracies.

4.1. THE FOURIER BASED Z-TRANSFORM METHOD

Before analysing existing techniques it is instructive to recall the maximum likelihood (ML) technique for analysing a *single* mode. Consider the signal model:

$$z(n) = A_1 exp(j\omega_1 n + \sigma_1 n) + \epsilon(n), \; n = 0, \ldots, N-1. \quad (7)$$

where A_1 is the complex amplitude, ω_1 is the angular frequency, σ_1 is the damping factor of the mode, and $\epsilon(n)$ is complex white Gaussian noise. The sampling rate is assumed without loss of generality to be unity. The maximum likelihood estimates for the angular frequency and damping ratio can be shown to be:

$$\hat{\omega}_1, \hat{\sigma}_1 = \underset{\omega,\sigma}{\operatorname{argmax}} \frac{\sum_{n=0}^{N-1} z(n)\exp(-j\omega n)\exp(\sigma n)}{[\sum_{n=0}^{N-1}\exp(2\sigma n)]^{1/2}}. \quad (8)$$

These estimates essentially correspond to those parameter values which cause a unit energy estimate of the signal to give maximum correlation with the observation.

From (8) it is seen that the estimation procedure involves 1) appling a window which, ideally, is matched to the damping of the signal, 2) normalising the energy for this window, and 3) extracting the spectral peak. While this procedure can be expected to give good noise performance above threshold for a single mode, its extension to multiple modes is not straightforward. The reason for this is that the windowing creates a loss of resolution, and this loss is especially great for highly damped modes. The technique can therefore only be extended to multiple modes with substantial damping if they are quite well separated.

In [3] a technique is proposed for mode identification, based on z-transform maximisation. This technique, like the one in (8), involves applying a window to the signal and finding the spectral maximum. The difference is that the window is applied to match the *inverse* of the signal damping! The method in (8) and the one in [3] are the result of two different optimisation criteria. The one in (8) optimises noise performance (for the white Gaussian case), while the one in [3] optimises spectral energy concentration, and therefore resolution (assuming the SNR is high enough). These facts indicate that caution must be used in applying the method in [3]. While resolution is enhanced, this is achieved (for modes which are stable) by using a window which can dramatically exaggerate noise. The method can only be used reliably, therefore, if 1) the noise level is low, or 2) the damping factor is low, or 3) the signal can be truncated in time so that the windowing does not taper too close to zero. These restrictions do not preclude the use of the method in many practical situations because the noise *is* often low, and the low damping factor case is the one which is of most interest. It will also be possible to arrange for the window to be truncted, assuming this truncation still provides adequate resolution.

If the technique in [3] is used, it is necessary to normalise the energy of the window. For generalisation to *multiple* components, strictly speaking, it is necessary to separate the different components and normalise each individually. This is difficult here, because unlike the zero damping case, the components can overlap substantially in the frequency domain. A compromise is to choose that estimate of σ_1 which provides the narrowest energy peak in the spectrum. The ω_1 estimate is taken as the frequency corresponding to this peak. This approach has been found to work best.

4.1.1. THE SLIDING WINDOW METHOD

An alternative method for determining the parameters of a given mode is given in [8]. It relies on using two different windows applied at different starting points in the signal. A formula is given for determining the damping factor from the peak amplitudes of the two different windowed spectra. The formulae given in [8], however, assume a real signal is used, and consequently are unecessarily complicated. Even more importantly, some rather limiting conditions are placed on the window lengths and starting times which can be used. This section gives formulae which assume the analytic signal is used. The result is that there is no restriction on the window lengths or starting times for the two different windows, apart from those that one needs to ensure adequate resolution and good numerical conditioning. The formulae are presented below.

The Fourier transform of a single damped sinusoid conforming to the model in (7), windowed by a rectangular window starting at n_1 and ending at $n_1 + T_w$, is given by:

$$F_{n_1}(\omega) = \frac{e^{(j(\omega_1-\omega)+\sigma_1)n_1}\left[e^{(j(\omega_1-\omega)+\sigma)T_w} - 1\right]}{j(\omega_1-\omega)+\sigma}. \quad (9)$$

The formula in (9) enables the contribution of the mode to be calculated at all frequencies in the vicinity of the center frequency. It thus allows determination of the extent to which modes will interfere with one another spectrally. If the spectrum is evaluated at two different time positions along the signal, (9) also enables calculation of the damping factor. Assuming the first observation window is rectangular and is applied from $n = 0$ to $n = T_w$, and the second window is applied from $n = T_g$ to $T_g + T_w$, the damping factor estimate is:

$$\hat{\sigma}_1 = \frac{1}{T_g} log \left[\frac{F_{T_g}(\hat{\omega}_1)}{F_0(\hat{\omega}_1)} \right] \quad (10)$$

This result is the same as the one in [8], but there are no limiting restrictions on the choice of window length and window separation. The lifting of these restrictions, due to the use of the analytic signal, makes the method far more practically viable.

Because the method of damping factor estimation based on (10) does not suffer from the noise sensitivity of the method in [3], it is useful where resolution is not a problem.

4.2. THE ANALYSIS OF VARIANCE TEST FOR A DAMPED SINUSOID

In [9], an analysis of variance test is used in conjunction with Fourier analysis to effectively separate closely spaced *harmonic lines*. The achievable resolution is shown to be unlimited, provided the SNR is appropriately high. In this section, the analysis of variance test is extended to *damped sinusoids*. As in [9], the resolution is dependent on the SNR. If the latter is high, so is the resolution.

The test requires calculation of K different Fourier representations. The kth one, $y_k(f)$, is formed by windowing the data with the kth prolate spheroidal sequence, $v_k(n)$, and then Fourier transforming it (f is the cyclic frequency variable and typically $K = 5$). Assuming a model of the form in (7), an estimate of the frequency domain mean can be obtained, and so can an estimate of the variance. An F-statistic can then be calculated to determine the statistical significance of the existence of a damped sinusoid of a particular frequency and damping ratio. The formulae for determining the mean and the F-statistic:

$$\hat{\mu}(f,\sigma) = \frac{\sum_{k=0}^{K-1} D_k(\sigma) y_k(f)}{\sum_{k=0}^{K-1} D_k^2(\sigma)}, \quad (11)$$

$$F(f,\sigma) = \frac{(K-1)|\hat{\mu}(f,\sigma)|^2 \sum_{k=0}^{K-1} D_k^2(\sigma)}{\sum_{k=0}^{K-1} |y_k(f) - D_k(\sigma)\hat{\mu}(f,\sigma)|^2}, \quad (12)$$

where $D_k(\sigma)$ is defined by:

$$D_k(\sigma) = \sum_{n=0}^{N-1} v_k(n) \exp(\sigma n). \quad (13)$$

The F-statistic defined above provides exceedingly sharp peaks in the vicinity of the true parameters of a damped sinusoid. See for example, Figure 3 which shows the F-statistic for a damped sinusoid of frequency, $0.09Hz$, and damping ratio, $0.008sec^{-1}$. The F-statistic is plotted for a σ corresponding to $0.008sec^{-1}$. The very sharp peak is limited in magnitude only by roundoff errors used to enter the coefficients. Furthermore, the peak rolls off very quickly in both f and σ. The high resolution property of the F-statistic means that it can be used to separate closely spaced modes. For the latter, it is necessary to use a double mode statistic, as was done for the harmonic line case [9].

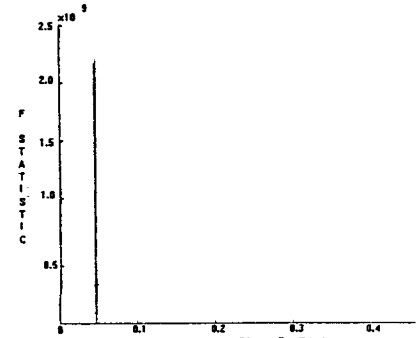


Figure 3. F-statistic for damped sinusoidal mode.

5. CONCLUSIONS

A new method for obtaining phase and frequency measurements at power substations has been proposed for the purposes of analysing post-disturbance oscillation. Some guidelines have also been given on how to enhance performance of Fourier based analysis of the post-disturbance record.

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