A REGULARIZATION APPROACH TO BLIND RESTORATION OF IMAGES DEGRADED BY SHIFT-VARIANT BLURS

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ABSTRACT

This paper presents shift-adaptive blind image restoration algorithms which can deal with realistic shift-variant blurs and which integrate the usually separate tasks of blur identification and image restoration. The key to success is the effective utilization of the piecewise smoothness of both the image and the PSF to compensate for the severe lack of information in this type of problems. This is achieved through regularization of the image and the PSF by anisotropic diffusion which has the property that smoothing is allowed only in the direction of edges.

1. INTRODUCTION

A fundamental issue in image restoration is blur removal in the presence of observation noise. This problem is usually addressed under the assumption that the underlying blur operation is shift-invariant. Since real-world blurs are often shift-variant, there is a need for adaptive restoration algorithms which can handle these types of blurs.

Several methods have been forwarded to handle the problem of restoration under the assumption that the underlying shift-variant blur is exactly *known*. They include coordinate transform [6], sectional processing [9], iterative methods [7], 2-D Kalman filtering [8], and POCS [5]. Since the blur operation is most likely unknown in practice, a more realistic and challenging problem is restoration under the condition that the shift-variant blur operation is unknown and is posed as a function (point spread function (PSF)) to be identified either initially or as an integral part of the restoration procedure.

The literature has recorded few works on the identification of shift-variant blurs. In [1], a sliding-block approach based on Kalman filtering is proposed to adaptively identify shift-variant blurs. It assumes that the point spread function (PSF) within a small block (subimage) can be considered as shift-invariant. So a Kalman filtering algorithm previously developed to identify shift-invariant blurs can be used to identify this PSF. By overlapping subsequent subimages, the results obtained at one subimage can be used to initialize the identification process in the next subimage. In [3], the sliding-block idea is followed but an expectation maximization (EM) algorithm previously developed to identify shift-invariant blurs is proposed to replace the Kalman filtering in each of such subimages. In [4], a multi-resolution approach based on local Fourier transform is proposed for blur identification. Since this method employs a Fourier analysis window which slides across the image, it may still be regarded as the sliding-block approach where an earlier technique which focuses on inherent regular zero patterns in the image spectrum is employed to identify the shift-invariant blur in each of the subimages. In order for these sliding-block based approaches to succeed, it is necessary that the underlying PSF be spatially stationary so that the PSF is approximately spatially invariant in each of the subimages. Unfortunately, this is usually not true for many photographically blurred images. Examples include motion blur when objects move against the background and the camera, and out-of-focus blur when the scene has depth. If the objects are solid with sharp edges, the PSF will undergo sharp transitions around the edges though it will be smooth in the rest of the image.

This paper presents shift-adaptive blind restoration algorithms which can account for this type of sharp PSF transitions. Specifically, the simple and efficient regularization approach to joint blur identification and image restoration forwarded in [10, 11] is successfully extended to dealing with shift-variant blurs. The key to success is the effective utilization of the piecewise smoothness of both image and PSF to compensate for the severe lack of information in this type of problems. This is achieved through regularization of the image and PSF by anisotropic diffusion which has the property that both the image and PSF are smoothed only in the direction of edges.

2. A GENERAL FORMALISM

A linear, shift-variantly degraded image may be modeled as

$$\begin{aligned} g(x,y) &= \int_{\mathbf{D}} d(x,y;s,t) f(x-s,y-t)\,ds\,dt \\ &\quad + n(x,y),\ (x,y) \in \Omega, \end{aligned} \tag{1}$$

where $f(x,y)$, $g(x,y)$, $d(x,y;s,t)$, and $n(x,y)$ represent the original image, observed image, PSF, and observation noise, respectively. The task of blind image restoration is to restore the original image $f(x,y)$ given only the observed image $g(x,y)$. This is equivalent to the decomposition of

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$g(x,y)$ in the presence of noise. Even when the blur operation is shift-invariant, this decomposition is not unique [10, 11]. With a shift-variant PSF, this problem may be more serious. In addition, this estimation problem also suffers from the usual ill-conditioning associated with deconvolution.

In order for the restoration effort to succeed, appropriate properties of both the image and the PSF have to be fully exploited. The most prominent of them is the well-known piecewise smoothness of image intensity $f(x,y)$. This property is known to be critical to the well-conditioning of regularization methods in image restoration. The PSF $d(x,y;s,t)$ may also be assumed to be piecewise smooth. As we noted in the introduction, the PSF undergoes sharp transitions with respect to (x,y) coordinates only where there is an edge (though an edge does not necessarily lead to a sharp PSF transition), and it is smooth otherwise. Of course, in the case of shift-invariant blurs, the PSF is constant with respect to (x,y): $d(x,y;s,t)=d(s,t)$. The PSF may be piecewise smooth with respect to the (s,t) coordinates. Such is the case for shift-variant motion blur and out-of-focus blur.

The piecewise smoothness property of the image is usually incorporated into the restoration operation in the form of weighted smoothing (shift-adaptive regularization) [2, 10, 11]. The basic idea behind this is that more smoothing is applied where the image is smooth and less where the image has sharp intensity transition. This, to some extent, preserves image edges while the restoration operation is properly regularized. But a little thought indicates that this approach is not enough to preserve image edges because the smoothing operation is indiscriminate with respect to the direction of edges in the sense that it smoothes both along and across the edges. Image edges would be better preserved if the smoothing operation is allowed only along the edges and inhibited across the edges. This objective, as well as the weighted smoothing, is very well achieved by anisotropic diffusion [12, 13]. Similarly, the piecewise smoothness property of the PSF can also be well incorporated into the identification process through anisotropic diffusion. Therefore, we propose to solve the blind restoration problem by minimizing the following functional

$$L(\hat{d},\hat{f}) = \int_{\Omega} A(e(x,y))dxdy + \lambda \int_{\Omega} B\left(|\nabla \hat{f}(x,y)|\right) dxdy + \gamma \int_{\Omega}\int_{\mathbf{D}} C\left(|\nabla \hat{d}(x,y;s,t)|\right) dsdtdxdy \quad (2)$$

subject to the following constraints:

$$\hat{d}(x,y;s,t) \geq 0,\ (x,y)\in\Omega, (s,t)\in\mathbf{D}. \quad (3)$$

$$\int_{\mathbf{D}} \hat{d}(x,y;s,t)dsdt = 1,\ (x,y)\in\Omega, \quad (4)$$

and

$$a \leq \hat{f}(x,y) \leq b,\ (x,y)\in\Omega. \quad (5)$$

The $e(x,y)$ in the first term of (2) is the restoration residual:

$$e(x,y) = g(x,y) - \int_{\mathbf{D}} \hat{d}(x,y;s,t)\hat{f}(x-s,y-t)dsdt, \quad (6)$$

which represents the fidelity of the estimate $\hat{d}$ and $\hat{f}$ to the observation $g(x)$. The $A(\cdot)$ in this term is usually a quadratic function: $A(e(x,y)) = e^2(x,y)$. However, since it has been observed that a quadratic function would lead to ringing artifacts [2], a different function may be used to reduce such artifacts. Since the residue $e(x,y)$ tends to be large around edges and small in smooth areas, more restoration is needed around edges than in smooth areas. This may be achieved by a function $A(\cdot)$ such as $A(e(x,y)) = e^4(x,y)$, which increases faster than the quadratic function. Direct minimization of the first term would lead to excessive noise magnification due to the ill-conditioning of the problem, so two piecewise smoothness constraints (second and third terms) are imposed. The $|\nabla\hat{f}(x,y)|$ in the second term of (2) represents the magnitude of the image gradient at (x,y) and the $B(\cdot)$ is an increasing function, so the the minimization of this term represents a decrease in image gradient, that is, it is a smoothing operation. It is proved in [12, 13] that the minimization of this term leads to anisotropic diffusion, and that if the function $B(\cdot)$ is properly designed, the diffusion operation can progress in such a way that the image is smoothed only in the direction of edges and the directional smoothing operation is encouraged at large intensity transitions and discouraged in smooth areas. The minimization of $C|\nabla\hat{d}(x,y;s,t)|$ in the third term of (2) plays a similar role for the PSF. The λ and γ are the regularization parameters which control the trade-off between fidelity to the observation and smoothness of the estimates $\hat{d}$ and $\hat{f}$. The first and second constraints (3) and (4) simply state the facts that the image intensities involved are non-negative and an imaging system normally neither absorbs nor generates energy. The third constraint (4) merely states that the image intensities involved are within a certain range [a,b].

Note that the cost functional (2) is obviously not convex, so our blind restoration effort suffers from the problems associated with local minima, in addition to the unresolved non-uniqueness problem of blind deconvolution. When $L(\hat{d},\hat{f})$ is practically minimized, a particular support for $\hat{d}(x,y;s,t)$ has to be specified, and its support $\mathbf{D}$ for the (s,t) coordinates is usually small compared with the image support Ω, reflecting the fact that blurring is usually a local operation. This is an important condition imposed on $\hat{d}(x,y;s,t)$ which might significantly reduce the number of possible decompositions as well as local minima. The constraints (3), (4), and (5) may also serve this purpose.

3. ALTERNATING MINIMIZATION

In order to minimize the functional (2), we need to obtain its gradients with respect to $\hat{d}$ and $\hat{f}$. The gradient of (2) with respect to $\hat{f}$ may be obtained as

$$\nabla_{\hat{f}} L(\hat{d},\hat{f}) = -\int_{\Omega} A'(e(u,v))\hat{d}(u,v;u-x,v-y)dudv - div\left(\frac{B'(|\nabla\hat{f}|)}{|\nabla\hat{f}|}\nabla\hat{f}\right). \quad (7)$$

The first term in (7) corresponds to the gradient with respect to $\hat{f}$ of the first term in (2), its derivation involves some work of Gâteaux variation and is not included here due to space limitation. The second term in (7) corresponds to the gradient with respect to $\hat{f}$ of the second term in (2), its derivation was presented in [12, 13]. Similarly, we give the gradient of (2) with respect to $\hat{d}$ as

$$\nabla_{\hat{d}} L(\hat{d}, \hat{f}) = -A'(e(x,y))\hat{f}(x-u, y-v) - div\left(\frac{C'(|\nabla \hat{d}|)}{|\nabla \hat{d}|}\nabla \hat{d}\right), \quad (8)$$

where the first and second terms correspond to the gradients with respect to $\hat{d}$ of the first and third terms in (2). Similar to steepest descent, the functional (2) may be minimized by moving $\hat{d}$ and $\hat{f}$ in their respective negative gradient directions with time:

$$\frac{\partial \hat{f}}{\partial t} = -\nabla_{\hat{f}} L(\hat{d}, \hat{f}), \quad (9)$$

$$\frac{\partial \hat{d}}{\partial t} = -\nabla_{\hat{d}} L(\hat{d}, \hat{f}). \quad (10)$$

Note that the discretization of (9) and (10) leads to the conventional steepest descent. Since $\hat{d}$ and $\hat{f}$ are in different scales and so are their gradients, it is impossible to find a common time step size which is optimal (in the sense of fast convergence and stability) for both of (9) and (10). Instead, we follow the idea of alternating minimization [10, 11] stated as follows. We initialize $\hat{d}$ to random numbers satisfying the constraints (3) and (4) and $\hat{f}$ to the observed image g. We first partially solve (9) to obtain an estimate of $\hat{f}$. Based on this estimate of $\hat{f}$ we then partially solve (10) to obtain an estimate of $\hat{d}$. We repeat this procedure until satisfactory estimates of $\hat{d}$ and $\hat{f}$ are obtained. In its simplest form, each step of the alternating minimization procedure may consist of merely one time step forward of (3) and (4), respectively, though more sophisticated methods may be used.

4. PARAMETERIZED BLURS

Since the number of unknowns in $\hat{d}$ and $\hat{f}$ is far greater than the observed data g, it is unlikely that, in general, a satisfactory solution to the blind restoration problem can be found. An alternative is to construct a simple PSF model and parameterize the PSF to reduce the number of unknowns. Due to the simplification of the parameterized PSF model, the restoration quality may not be satisfactory. In case of this, the general method proposed in last section may be used to fine-tune the restoration. Motion blur and out-of-focus blur may be easily parameterized, but we only present the identification of parameterized motion blur due to space limitation.

The PSF of a shift-variant motion blur may be parameterized by $u(x,y)$ and $v(x,y)$ which denote the horizontal and vertical motion components at a location (x,y):

$$d(x,y;s,t) = \begin{cases} \frac{1}{\sqrt{u^2+v^2}}, & s=\theta u, t=\theta v, 0 \le \theta \le 1; \\ 0, & otherwize. \end{cases} \quad (11)$$

Then the functional (2) takes the following form

$$\begin{aligned} L(\hat{u}, \hat{v}, \hat{f}) = & \int_\Omega A(e(x,y))dxdy \\ & + \lambda \int_\Omega B\left(|\nabla \hat{f}(x,y)|\right) dxdy \\ & + \gamma_u \int_\Omega C_u\left(|\nabla \hat{u}(x,y)|\right) dxdy \\ & + \gamma_v \int_\Omega C_v\left(|\nabla \hat{v}(x,y)|\right) dxdy. \end{aligned} \quad (12)$$

It is obvious that its gradient with respect to $\hat{f}$ is still given by (7) and the descent equation (9) still holds. The gradients of (12) with respect to $\hat{u}(x,y)$ and $\hat{v}(x,y)$ may be obtained as

$$\begin{aligned} \nabla_{\hat{u}} L(\hat{u}, \hat{v}, \hat{f}) = & -\frac{A'(e(x,y))}{\hat{u}} \cdot \\ & \cdot \left[f(x-\hat{u}, y-\hat{v}) - \int_0^1 f(x-\theta\hat{u}, y-\theta\hat{v})d\theta\right] \\ & - div\left(\frac{C_u'(|\nabla \hat{u}|)}{|\nabla \hat{u}|}\nabla \hat{u}\right) \end{aligned} \quad (13)$$

$$\begin{aligned} \nabla_{\hat{v}} L(\hat{u}, \hat{v}, \hat{f}) = & -\frac{A'(e(x,y))}{\hat{v}} \cdot \\ & \cdot \left[f(x-\hat{u}, y-\hat{v}) - \int_0^1 f(x-\theta\hat{u}, y-\theta\hat{v})d\theta\right] \\ & - div\left(\frac{C_v'(|\nabla \hat{v}|)}{|\nabla \hat{v}|}\nabla \hat{v}\right), \end{aligned} \quad (14)$$

and the PSF descent equation (10) takes the form of

$$\frac{\partial \hat{u}}{\partial t} = -\nabla_{\hat{u}} L(\hat{u}, \hat{v}, \hat{f}) \quad (15)$$

$$\frac{\partial \hat{v}}{\partial t} = -\nabla_{\hat{v}} L(\hat{u}, \hat{v}, \hat{f}). \quad (16)$$

5. NUMERICAL SIMULATION

The three discs in Fig. 1(a) are subjected to horizontal motion blur ($v \equiv 0$) with $u = 5$ for the largest disc, $u = 8$ for the middle disc, and $u = 10$ for the smallest disc. Gaussian noise is then added to the blurred image at $SNR = 30$ dB. The degraded image is shown in Fig. 1(b). Fig. 1(c) is the image restored by the parameterized algorithm developed in last section with $A(x) = x^2$ and $C_u(x) = x$. The regularization parameters used are 0.5 and 50 for the image and the PSF, respectively. The time step for the image is 1 and that for the PSF is 0.001.

6. CONCLUSION

This paper presented shift-adaptive blind image restoration algorithms which integrate the usually separate tasks of shift-variant blur identification and image restoration. This is achieved through the regularization of the image and the PSF by anisotropic diffusion which has the property that

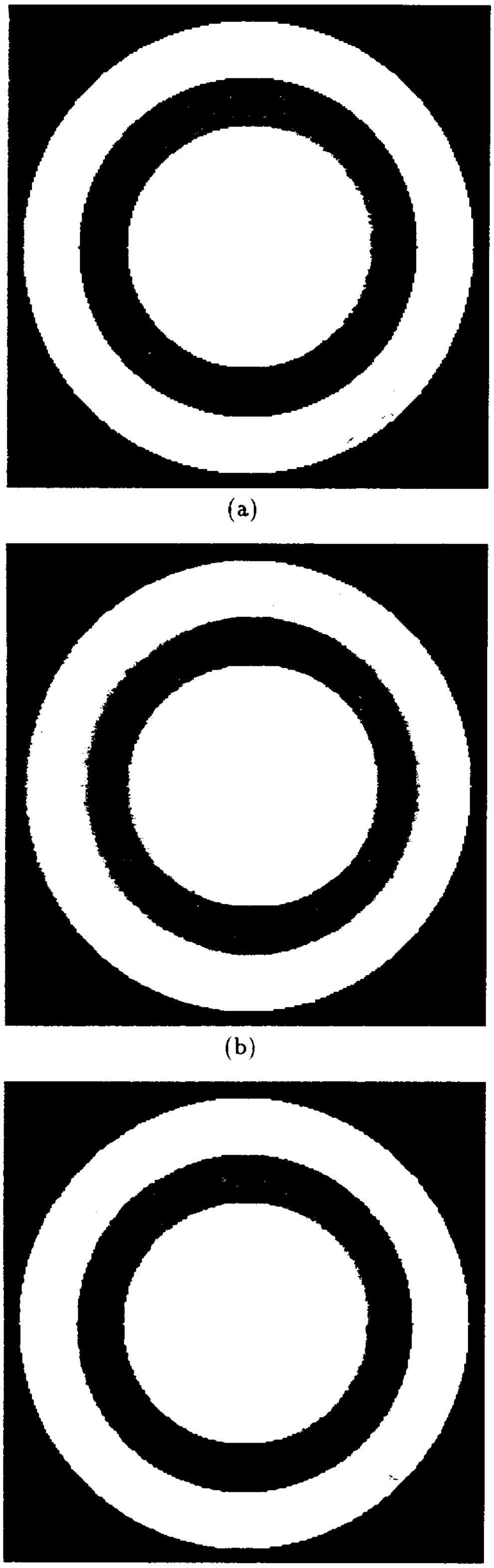

Figure 1: (a) Original image. (b) Image degraded by shift-variant horizontal motion blur and white Gaussian noise at 30 dB SNR. (c) Restored image.

smoothing is allowed only in the direction of edges. As with other regularization methods, the estimation of optimal regularization parameter is an open problem. The proposed method also suffers from the problems associated with local minima and the uniqueness issues of blind deconvolution.

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