DESIGN OF ZERO-PHASE, MULTIFORM, TILTABLE TWO-DIMENSIONAL FILTERS

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ABSTRACT

This paper advances a novel and relatively simple scheme for designing two-dimensional (2-D) filters with zero phase. The proposed filters are obtained by a nonlinear mapping of a one-dimensional (1-D) magnitude response into a 2-D filter. By appropriately constraining the filter parameters, a wide variety of passband iso-contour shapes can be generated in the frequency-frequency plane, e.g., tilted or untilted ellipses, circles, diamonds, parallel strips at arbitrary angles, crosses, and "snowflakes". Simple equations for designing the filter's parameters that meet or exceed user specifications are given for the special cases when the 1-D prototype is the magnitude response of a Gaussian, Butterworth, Chebyshev, or inverse Chebyshev filter. A scheme for designing fan filters with arbitrary angle is also provided.

1. MULTIFORM, TILTABLE LOWPASS FILTERS

We propose the following frequency responses for the two-dimensional multiform, tiltable (MT) Gaussian lowpass filter (MTGLF)

$$H_{\mathrm{MTGLF}}(f,\nu)=\exp\left\{-\pi\left[\mu^2\left(\frac{f}{f_0},\frac{\nu}{\nu_0};r,\beta,\gamma\right)\right]^{\lambda}\right\}, \qquad (1)$$

the MT Butterworth lowpass filter (MTBLF)

$$H_{\mathrm{MTBLF}}(f,\nu)=\left\{1+\left[\mu^2\left(\frac{f}{f_0},\frac{\nu}{\nu_0};r,\beta,\gamma\right)\right]^{\lambda}\right\}^{-1}, \qquad (2)$$

the MT Chebyshev lowpass filter (MTCLF)

$$H_{\mathrm{MTCLF}}(f,\nu)=\left\{1+K_pC_\lambda^2\left[\mu\left(\frac{f}{f_0},\frac{\nu}{\nu_0};r,\beta,\gamma\right)\right]\right\}^{-1}, \qquad (3)$$

and the MT inverse Chebyshev lowpass filter (MTICLF)

$$H_{\mathrm{MTICLF}}(f,\nu)=\left\{1+K_pC_\lambda^{-2}\left[\mu^{-1}\left(\frac{f}{f_0},\frac{\nu}{\nu_0};r,\beta,\gamma\right)\right]\right\}^{-1} \qquad (4)$$

where $\mu\left(\hat{f},\hat{\nu};r,\beta,\gamma\right)=\hat{f}^2+\hat{\nu}^2+2r\left[\left(\hat{f}\,\hat{\nu}\right)^{\beta}\right]^{\gamma}$. (5)

The "design" parameters of these filters are a positive power λ used to control the transition region width, two positive frequency scaling constants f_0 and ν_0 used to control the passband edge, a tilt or rotation parameter r usually in the range $r \in [-1,1]$, and β and γ which are powers coupled in such a way that either $(\beta,\gamma) = (1,1)$, for causing no change, or $(\beta,\gamma) = (2,½)$, for producing the magnitude of the product term $\hat{f}\hat{\nu}$. $C_\lambda(\cdot)$ denotes the Chebyshev polynomial of order λ, $K_p=\left(1-k_p\right)/k_p$ where k_p is the minimum amplitude of the filter allowed in the passband, and $K_s=\left(1-k_s\right)/k_s$ where k_s is the maximum amplitude of the filter allowed in the stopband. These 2-D filters satisfy many of the properties of the corresponding 1-D prototype counterparts. For example, the MTCLF has an equi-ripple passband that is bound between unity and the user specified k_p.

The first advantage of the proposed filters is their ability to generate arbitrarily narrow transition regions and at least six passband support regions by properly choosing combinations of the filter parameters: *parallel strips* at arbitrary angles ($r = \pm1$, $\beta=\gamma=1$), *crosses* ($r=-1$, $\beta = 2$, $\gamma = ½$), "*snowflakes*" ($r < -1$, $\beta=2$, $\gamma = ½$), *untilted ellipses* ($r = 0$), *tilted ellipses* ($-1 < r < 1$, $\beta=\gamma=1$), and *diamonds* ($r = 1$, $\beta = 2$, $\gamma = ½$). See Figs. 1 and 2. They extend the filters in [1,2]. The filters in [1] are restricted to circular or rectangular passband regions, and those in [2] are restricted to fan, diamond, and untilted elliptical filters.

The second advantage of the new filters is that we derive simple, easy to use, closed form design equations to select filter parameters that meet or exceed a given set of user specified passband and stopband design criteria in the frequency-frequency (f-ν) plane. They provide an "easy to design" alternative to the (Chebyshev) McClellan transformation filters [3,5].

Thirdly, by construction, these versatile filters have zero phase. Consequently, they should prove useful in image processing applications [4,5].

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2. MT LOWPASS FILTER PARAMETER DESIGN

The parameter design equations for the four lowpass filters advanced in (1)-(4) are summarized in Tables 1 and 2. It is assumed that the passband region must have filter values greater than or equal to k_p whereas the stopband region has filter values less than or equal to k_s. All the user needs to provide is k_p, k_s, the desired passband shape, and a set of $N_p \geq 2$ passband points, $\{(f_{p_i}, \nu_{p_i})\}_{i=1}^{N_p}$, which must lie in the passband, i.e., it is desired that $H_{\mathrm{MT}}(f_{p_i}, \nu_{p_i}) \geq k_p$, and a set of $N_s \geq 1$ stopband points, $\{(f_{s_i}, \nu_{s_i})\}_{i=1}^{N_s}$, which must lie in the stopband, i.e., it is desired that $H_{\mathrm{MT}}(f_{s_i}, \nu_{s_i}) \leq k_s$. From these user parameters, a relatively fast computer algorithm determines the filter parameters λ, ν_0, and f_0 which meet or exceed the user specifications. The algorithm works as follows. Since the iso-contours of the filters in (1)-(4), i.e., $H_{\mathrm{MT}}(f, \nu) = k_i$, occur along concentric ellipses or diamonds or parallel strips or crosses, the filter design algorithm searches for elliptical or linear boundaries to the given set of passband and stopband points and uses the parameters defining these boundaries to solve for the unknown filter parameters. For example, when $r = 0$, the algorithm fits an ellipse, $\nu^2 = mf^2 + b$, to each pair of given passband points (f_{p_i}, ν_{p_i}), $i = 1, \cdots, N_p$, and searches for the outermost or "critical" such passband ellipse that has all given passband points inside it or on it. Concentric, i.e., equal curvature m to the passband ellipse, ellipses are then fit through each stopband ellipse and a search is made for the critical stopband ellipse that has all given stopband points outside or on it. Table 1 lists the equations for the unique curvature and intercepts used to define these critical ellipses. The unknown filter parameters λ, ν_0, and f_0, are given in Table 2 which set $H_{\mathrm{MT}}(f, \nu) = k_p$ along the critical passband ellipse and $H_{\mathrm{MT}}(f, \nu) = k_s$ along the critical stopband ellipse. The derivations can be found in [6,7].

3. MT NON-LOWPASS FILTERS

The frequency responses of the Gaussian and Butterworth filters in (1)-(2) are always monotonically decreasing away from the origin of the frequency-frequency plane. Hence, using either of these 2-D lowpass filter responses, it is relatively straightforward to design other types of 2-D filters. See Fig. 3. Let $H_{\mathrm{LP}_i}(f, \nu)$ denote a lowpass filter. Then, a highpass filter can be generated as [4]

$$H_{\mathrm{HP}}(f, \nu) = 1 - H_{\mathrm{LP}_1}(f, \nu). \tag{6}$$

A bandpass filter can be generated as [4]

$$H_{\mathrm{BP}}(f, \nu) = H_{\mathrm{LP}_2}(f, \nu) - H_{\mathrm{LP}_1}(f, \nu) \tag{7}$$

where $H_{\mathrm{LP}_2}(f, \nu)$ has a higher "volume", i.e., cutoff boundary, than $H_{\mathrm{LP}_1}(f, \nu)$. A bandstop filter can be created from a highpass and a lowpass filter as [4]

$$H_{\mathrm{BS}}(f, \nu) = H_{\mathrm{HP}}(f, \nu) + H_{\mathrm{LP}_1}(f, \nu). \tag{8}$$

Also, a fan filter can easily be obtained through a phase shift transformation of a lowpass diamond filter. See Fig. 3(d). This transformation is basically the inverse of the mapping employed in [2] to obtain a full-plane diamond filter from a ±45° fan filter. Moreover, since our design scheme permits arbitrary angles for the linear boundaries of the diamond, the resulting fan filters also have arbitrary angle.

4. GENERALIZATION/EXTENSION

There are other interesting extensions to the formulation presented in this paper. For example, any of the multiform, tiltable filters mentioned can be easily extended by re-defining the nonlinear mapping in (5) as

$$\mu(\hat{f}, \hat{\nu}, \alpha_1, \alpha_2, \alpha_3, \alpha_4, r, \beta, \gamma) = \left(\hat{f}^2\right)^{\alpha_1}\left(\hat{\nu}^2\right)^{\alpha_2} + \left(\hat{f}^2\right)^{\alpha_3}\left(\hat{\nu}^2\right)^{\alpha_4} + 2r\left[\left(\hat{f}\hat{\nu}\right)^{\beta}\right]^{\gamma} \tag{9}$$

With this new formulation, it is possible to generate filters with iso-contours that look like hyperbolas and rectangles (see Fig. 4), etc., as well as the shapes mentioned above. Moreover, it is possible to get filters which are constant along one or both of the frequency axes. Such filters are useful in designing mixed time-frequency representations [6,7].

5. CONCLUSIONS

We have proposed versatile 2-D filters capable of attaining many possible passband support "shapes", including ellipses (tilted and untilted), parallel strips, crosses, diamonds, "snowflakes", rectangles, and hyperbolas. For given passband and stopband constraints, we developed closed form design equations for computing the filter parameters which meet or exceed user specified passband/stopband criteria. In contrast to the McClellan transformation filters [3,5], the new filters have stopband iso-contours identical in shape to those of the passband and have design equations that are simple to use for all shapes.

REFERENCES

[1] M. Ahmadi and A. Chottera, "An improved method for the design of 2-D FIR digital filters with circular and rectangular

cut-off boundary using Kaiser window," *Can. Electron. Eng. J.*, vol. 8, pp. 3-8, 1983.

[2] Q. Gu and M. N. S. Swamy, "On the design of a broad class of 2-D recursive digital filters with fan, diamond and elliptically-symmetric responses," *IEEE Trans. Circuits and Systems*, vol. 41, pp. 603-614, 1994.

[3] R. M. Mersereau, W. F. G. Mecklenbräuker, and T. F. Quatieri, Jr., "McClellan transformations for two-dimensional digital filtering: I - Design," *IEEE Trans. Circuits and Systems*, vol. 23, pp. 405-414, 1976.

[4] W-S Lu and A. Antoniou, *Two-Dimensional Digital Filters*. New York, NY: Marcel Dekker, Inc., 1992.

[5] J. S. Lim, *Two-Dimensional Signal and Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1990.

[6] A. H. Costa, *Multiform, Tiltable Time-Frequency Representations and Masked Auto Wigner Distribution Synthesis*. Ph.D. dissertation, University of Rhode Island, Kingston, RI, 1994.

[7] A. H. Costa and G. F. Boudreaux-Bartels, "Design of time-frequency representations using multiform, tiltable kernels," *Proc. IEEE International Symposium on Time-Frequency/Time-Scale Analysis*, pp. 205-208, 1994.

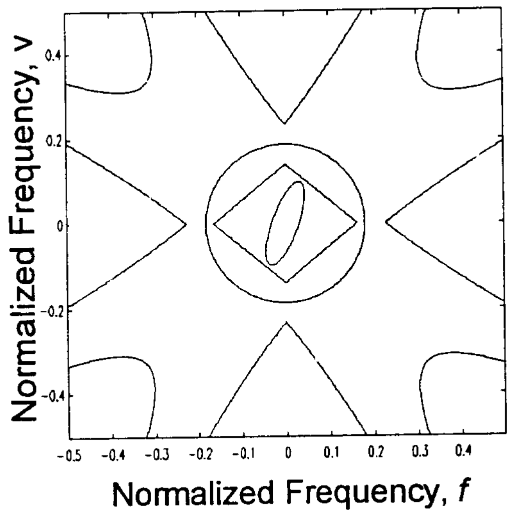


Figure 1: 50% iso-contours in the f-ν plane demonstrating some of the possible passband support regions (snowflake, circular, diamond, and elliptical (tilted)) for the MT lowpass filters.

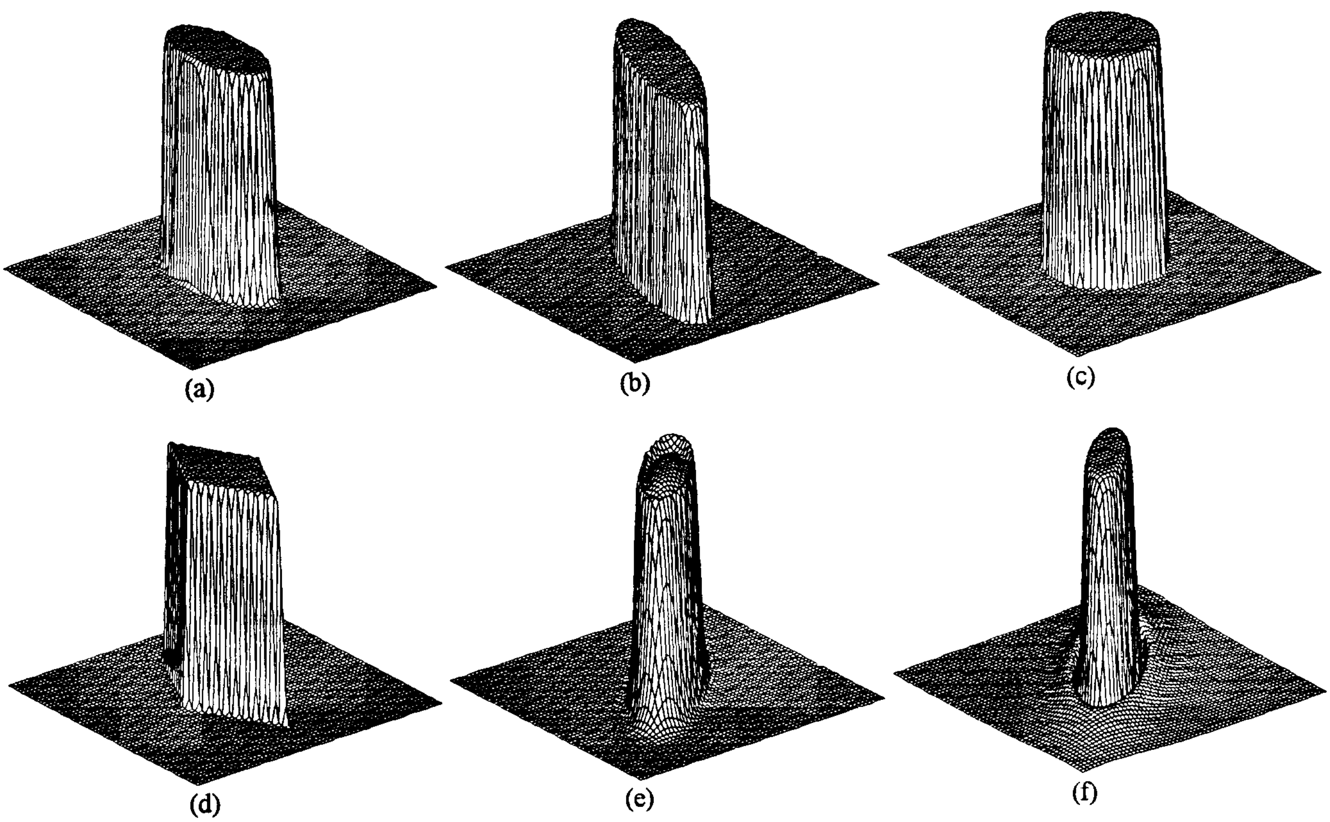


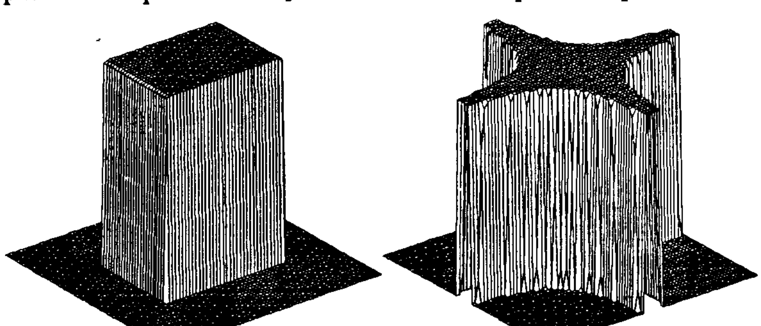
Figure 2: Multiform, tiltable lowpass filters. (a)-(d) MT Butterworth filter with elliptical (untilted or tilted), circular, and diamond shaped, respectively, passband/stopband regions; (e) MT Chebyshev filter; (f) MT inverse Chebyshev filter. In (e)-(f), the passband/stopband error has been chosen relatively large to emphasize where the ripple occurs; both filters have the same order. In all mesh plots in this paper, the origin of the f-ν plane is in the center. The amplitude is plotted using a linear scale between zero and one.

MT Iso-Contour Shape	Intermediate Design Parameters			Constraints		
	m	b_p	b_s	r	β	γ
Untilted Ellipse	$\frac{\nu_{cp_2}^2 - \nu_{cp_1}^2}{f_{cp_2}^2 - f_{cp_1}^2}$	$\nu_{cp_1}^2 - mf_{cp_1}^2$	$\nu_{cs}^2 - mf_{cs}^2$	0		
Parallel Strip Cross Diamond	$\frac{\nu_{cp_2} - \nu_{cp_1}}{f_{cp_2} - f_{cp_1}}$	$\nu_{cp_1} - mf_{cp_1}$	$\nu_{cs} - mf_{cs}$	±1 −1 +1	1 2 2	1 ½ ½

Table 1: Design equations for the intermediate parameters for the multiform, tiltable (MT) filters. (f_{cp_i}, ν_{cp_i}), i = 1, 2 are the two passband points used to form the outermost or critical passband curve while (f_{cs}, ν_{cs}) is the stopband point used to form the critical stopband curve. These equations hold for all the MT filters in (1)-(4).

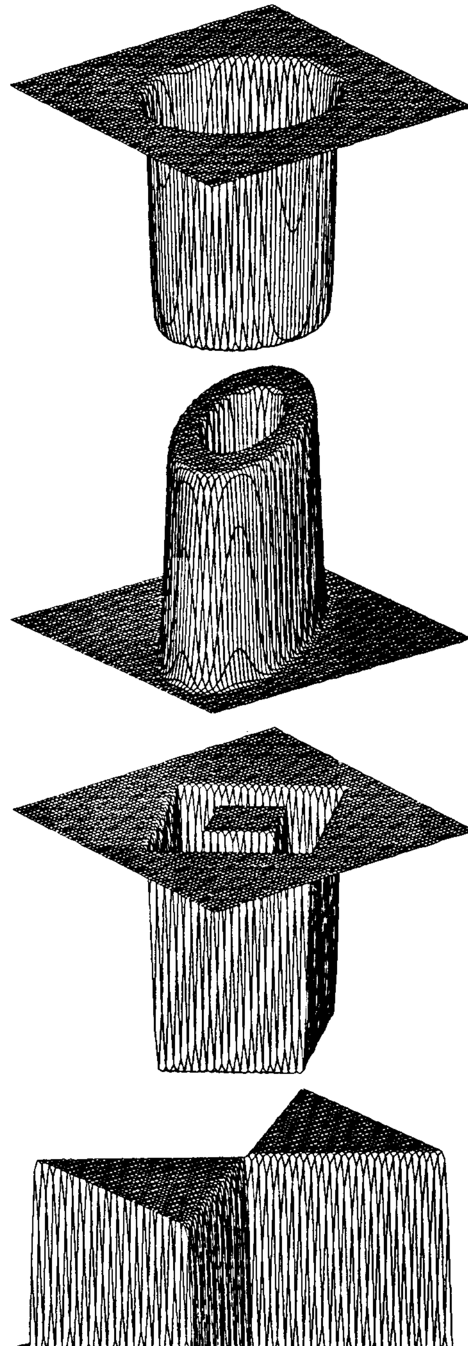
	Multiform, Tiltable (MT) Filter Design Parameters		
MT Filter Form	λ	ν_0	f_0
Untilted Ellipse: MTGLF	$\log\left(\frac{\ln k_p}{\ln k_s}\right) / \left[2\log\left(\frac{b_p}{b_s}\right)\right]$	$\sqrt{b_p}\left(\frac{\pi}{-\ln k_p}\right)^{\frac{1}{4\lambda}}$	$\frac{\nu_0}{\sqrt{-m}}$
MTBLF	$\log(K_p / K_s) / [2\log(b_p / b_s)]$	$\sqrt{b_p}(K_p)^{-\frac{1}{4\lambda}}$	
MTCLF	$\cosh^{-1}\left(\sqrt{K_s / K_p}\right) / \cosh^{-1}[(b_s / b_p)]$	$\sqrt{b_p}$	
MTICLF		$\sqrt{b_s}$	
Parallel Strip, Cross, Diamond: MTGLF	$\log\left(\frac{\ln k_p}{\ln k_s}\right) / \left[4\log\left(\frac{b_p}{b_s}\right)\right]$	$b_p\left(\frac{\pi}{-\ln k_p}\right)^{\frac{1}{4\lambda}}$	$\frac{-r\nu_0}{m}$
MTBLF	$\log(K_p / K_s) / [4\log(b_p / b_s)]$	$b_p(K_p)^{-\frac{1}{4\lambda}}$	
MTCLF	$\cosh^{-1}\left(\sqrt{K_s / K_p}\right) / \cosh^{-1}\left[(b_s / b_p)^2\right]$	b_p	
MTICLF		b_s	

Table 2: Design equations for the parameters of the MT lowpass filters in (1)-(4) using m, b_p, b_s, r, β, and γ listed in Table 1. $K_p = (1-k_p)/k_p$, $K_s = (1-k_s)/k_s$, where k_p is the minimum passband amplitude and k_s is the maximum stopband amplitude.



(a) (b)

Figure 4: (a) MT Gaussian lowpass rectangular filter and (b) MT Gaussian lowpass hyperbolic filter generated using the mapping in (9). See [6] for derivation of design equations when $\alpha_1 = \alpha_4 = 1$ and $\alpha_2 = \alpha_3$.



(3a)

(3b)

(3c)

(3d)

Figure 3: (a) MT Butterworth, circular highpass filter. (b) MT Butterworth, elliptical bandpass filter. (c) MT Butterworth, diamond bandstop filter. (d) MT Butterworth, fan filter.