POINT SPREAD FUNCTION ESTIMATION USING THE MEAN FIELD APPROXIMATION

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ABSTRACT

Any imaging system has to deal with the problem of degradation of images due to blurring and noise. In this paper, we consider the problem of blur identification, in the presence of noise. We use a stochastic model for the blur matrix. Then, we use the mean field approximation technique which enables us to obtain a closed-form expression for the mean values of the blur elements. This technique is proven to be versatile enough to handle wide varieties of blur matrices. Simulation results for estimation of the blur matrix using this formulation have also been presented.

1. INTRODUCTION

The degradation processes of blurring and noise acting on an image are modelled as follows

$$y_{(i,j)} = \sum_{(m,n)\in W} b_{(m,n)} x_{(i-m,j-n)} + n_{(i,j)} \qquad (1)$$

where $x_{(i,j)}$ and $y_{(i,j)}$ denote the elements of the original image X and the distorted image Y respectively, defined on the square $M \times M$ lattice I, $b_{(m,n)}$ represents the blurring process and W the blur window and $n_{(i,j)}$ represents the additive all-white Gaussian noise. The elements $b_{(m,n)}$ constitute the blur matrix or the point spread function (PSF) B. The PSF estimation problem [1] involves identification of the blur matrix, given knowledge about X and Y.

Techniques used for estimation of the PSF have generally concentrated on the use of suitable models for the original image X. These include the Maximum-Likelihood estimate, which used the Expectation Maximization algorithm [2]-[5]. The Generalized Cross Validation (GCV) approach has been used by Reeves and Mersereau [6] to obtain the PSF. Chalmond [7] has used a multiresolution approach combined with a Markov Random Field (MRF) model to obtain an estimate of the PSF.

In this paper, we outline a direct approach to the estimation of blur using an appropriate model for the blur itself. The blur elements are modeled as continuous valued random variables using an MRF model. This model helps us to develop an energy formulation that is versatile enough to handle many kinds of blur functions. Thus, we can dispense with regulations on "smoothness" of the blur matrix.

2. BLUR MODEL

We assume a continuous Gibbsian energy distribution for the blur matrix B i.e. it's *a priori* probability density function is given by

$$f_B(b) = \frac{1}{Z}\exp(-U(b)/T) \qquad (2)$$

where b is the lexicographically ordered column vector of blur elements, $U(b)$ is the Gibbs energy, T is the temperature and Z is the *a priori* partition function. The *a posteriori* partition function Z_p is given by

$$Z_p = \int \exp(-\beta[\frac{\|y - Bx\|^2}{2\sigma^2} + \mu U(b)])db \qquad (3)$$

where x and y are the lexicographically ordered column vectors corresponding to X and Y respectively. The integral above is of order N^2 for an $N \times N$ blur window. μ is the regularization parameter and $\beta = 1/T$.

The energy function $U(b)$ can be tailored to meet the demands of different varieties of blur matrices, as is shown below.

If we can evaluate the integral in (3), we can obtain closed form expressions for the mean values of the blur

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elements, since

$$y_{(i,j)} - \sum_{(m,n)\in W} \bar{b}_{(m,n)} x_{(i-m,j-n)} = -\frac{\sigma^2}{\beta Z_p} \frac{d}{dy_{(i,j)}}(Z_p) \quad (4)$$

where $\bar{b}_{(m,n)}$ is the mean value of the random variable $b_{(m,n)}$.

However, the integral in (3) is, in general, too complicated to evaluate, due to the dependence of the integrand on the neighbourhood of $b_{(i,j)}$. To simplify this dependence, we resort to mean field theory [8]-[10].

3. THE MEAN FIELD APPROXIMATION

The mean field approximation was originally used to solve the many-spin problem in thermodynamics. The complex effect of spins of neighbouring bodies on the spin of the body under consideration was simplified using this technique. In an analogous manner, we use the mean field approximation by assuming that the random variables associated with neighbourhood blur element values can be replaced by their means.

We present two examples. If

$$\begin{aligned} U_1(b) &= \sum_{(i,j)\in W} (b_{(i,j)} - b_{(i,j-1)})^2 + (b_{(i,j)} - b_{(i,j+1)})^2 \\ &\quad + (b_{(i,j)} - b_{(i-1,j)})^2 + (b_{(i,j)} - b_{(i+1,j)})^2 \end{aligned} \quad (5)$$

along with (3), this yields

$$\begin{aligned} Z_p &= \exp(-\frac{\beta}{2\sigma^2} \sum_{(i,j)\in I} y^2_{(i,j)}) \times \\ &\quad \prod_{(m,n)\in W} \int_{-\infty}^{+\infty} \exp\{-\frac{\beta}{2\sigma^2}[-2b_{(m,n)}P_{(m,n)} + \\ &\quad b^2_{(m,n)}U + b_{(m,n)} \sum_{(p,q)\in\overline{W}} \bar{b}_{(p,q)}U_{(m,n;p,q)}] \\ &\quad - 2\beta\mu[b_{(m,n)} - \bar{b}_{(m,n-1)}]^2 \\ &\quad - 2\beta\mu[b_{(m,n)} - \bar{b}_{(m-1,n)}]^2\} db_{(m,n)} \end{aligned} \quad (6)$$

where

$$P_{(m,n)} = \sum_{(i,j)\in I} y_{(i,j)} x_{(i-m,j-n)}$$

$$U_{(m,n;p,q)} = \sum_{(i,j)\in I} x_{(i-m,j-n)} x_{(i-p,j-q)}$$

$U = U_{(m,n;p,q)}$ and $\overline{W}$ is the blur window W excluding the pixel location (m,n).

Thus, using the mean field technique, we have converted an integral of order N^2 into N^2 integrals of order 1. Evaluating the integral in (6) and using (4), we have

$$\begin{aligned} \bar{b}_{(m,n)} &= \frac{1}{\frac{U}{2\sigma^2} + 4\mu}[\frac{1}{2\sigma^2}(P_{(m,n)} - \\ &\quad \sum_{(p,q)\in\overline{W}} \bar{b}_{(p,q)}U_{(m,n;p,q)}) \\ &\quad + 2\mu(\bar{b}_{(m,n)} + \bar{b}_{(m-1,n)})] \end{aligned} \quad (7)$$

Similary, if

$$\begin{aligned} U_2(b) &= \sum_{(m,n)\in W} [b_{(m,n)} - b_{(m,n-1)}][b_{(m,n)} - b_{(m-1,n)}] \\ &\quad + [b_{(m,n)} - b_{(m,n-1)}][b_{(m,n)} - b_{(m-1,n-1)}] \\ &\quad + [b_{(m,n)} - b_{(m-1,n)}][b_{(m,n)} - b_{(m-1,n-1)}] \end{aligned}$$

this yields

$$\begin{aligned} \bar{b}_{(m,n)} &= \frac{1}{3\mu + \frac{U}{2\sigma^2}}[\frac{1}{2\sigma^2}(P_{(m,n)} - \\ &\quad \sum_{(p,q)\in\overline{W}} \bar{b}_{(p,q)}U_{(m,n;p,q)}) \\ &\quad + 6\mu\bar{K}_{(m,n)}] \end{aligned} \quad (8)$$

where

$$\bar{K}_{(m,n)} = \bar{b}_{(m-1,n)} + \bar{b}_{(m.n-1)} + \bar{b}_{(m-1.n-1)} - \frac{1}{2}$$

We note that as $\mu \to \infty$ in equation (7), $U_1(b)$ is finite only for the uniform blur matrix. From equation (13), we see that the mean field approximation provides the uniform blur matrix solution as $\mu \to \infty$. Thus, if we consider annealing of the regularization parameter μ, the result minimizes the objective function, as is borne out by our simulation results (Table 1). Now, consider Equation (8). Here, in the $\mu \to \infty$ case, the solution is not the uniform blur case. This is useful when one is trying to estimate blur matrices which have sharp changes in blur values.

Further, by using linear combinations of these two energy functions in the form $\lambda_1 U_1(b) + \lambda_2 U_2(b)$, we can handle all kinds of blur matrices by using appropriate values for λ_1 and λ_2.

4. THEORETICAL INSIGHT

In the spirit of similar investigations by Geiger and Girosi [11], we have studied the relations in (7) and (8) and the part played by μ in them

If $\mu = 0$ in (7), we get

$$\bar{b}_{(m,n)} = \frac{P_{(m,n)} - \sum_{(p,q)\in\overline{W}} \bar{b}_{(p,q)}U_{(m,n;p,q)}}{U} \quad (9)$$

In order to prove that this formulation satisfies $y = Bx$, consider

$$\begin{aligned} P_{(m,n)} &= \sum_{(i,j)\in I} y_{(i,j)} x_{(i-m,j-n)} \\ &= \sum_{(i,j)\in I} \sum_{(p,q)\in W} x_{(i-p,j-q)} b_{(p,q)} \times \\ & \quad x_{(i-m,j-n)} \end{aligned} \tag{10}$$

Simplifying,

$$P_{(m,n)} = b_{(m,n)} \sum_{(i,j)\in I} x^2_{(i-m,j-n)} + \sum_{(p,q)\in \overline{W}} b_{(p,q)} U_{(m,n;p,q)} \tag{11}$$

Finally, we get

$$b_{(m,n)} = \frac{P_{(m,n)} - \sum_{(p,q)\in \overline{W}} b_{(p,q)} U_{(m,n;p,q)}}{U} \tag{12}$$

Thus, we have proved that the mean field approximation does indeed give a solution to the equation $y = Bx$ when $\mu = 0$.

Substituting $\mu \to \infty$ in (7), we get

$$\bar{b}_{(m,n)} = \frac{\bar{b}_{(m,n-1)} + \bar{b}_{(m-1,n)}}{2} \tag{13}$$

This implies that for large values of μ, we have "no trust" on the observed data and, instead, rely on smoothing to provide an estimate of the blur.

Thus, μ indicates the amount of "trust" placed on the data. If there is a lot of noise present, then we place very little trust on the data and μ is high i.e. smoothing is high. If noise is negligible, then we place a lot of trust on the data. Hence, in this case, μ should be small.

5. SIMULATION RESULTS

We have made a comparative study of algorithms based on (7) and (8). In both cases, we observe convergence in norm of the blur matrix as μ increases. The energy function used in (7) works better for uniform blurs (Table 1) (or, in general, "smooth" blurs), while the formulation in (8) works better for delta blur (Table 2) (or, in general, "non-smooth" blurs). Thus, by using appropriate weights for $U_1(b)$ and $U_2(b)$, we can handle any kind of blur matrix by developing the appropriate mean field equations.

μ	uniform blur case
0.0	0.000587
100.0	0.000577
500.0	0.000532
1000.0	0.000497
2000.0	0.000451
4000.0	0.000247
4700.0	0.000169

Table 1:Error norms for uniform blurs for $\sigma = 2.95$ corresponding to $U_1(b)$

μ	delta blur case
0.0	0.01640
100.0	0.02714
500.0	0.03585
1000.0	0.03559
2000.0	0.02619
4000.0	0.01275
4700.0	0.00997

Table 2:Error norms for delta blur for $\sigma = 2.95$ corresponding to $U_2(b)$

6. CONCLUSION

By using the technique of the mean field approximation, we have been able to analytically calculate the partition function and estimate the PSF for all kinds of blurs. Although the algorithm is essentially a supervised learning process, we believe that this would help in developing better algorithms for the blind deconvolution problem.

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