OPTIMUM PARAMETERS FOR HYBRID FRACTAL IMAGE CODING

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ABSTRACT

We evaluate the fidelity/compression performance of fractal transforms over a range of parameters, using an rms error metric. We consider order of approximation, different (fixed) block sizes and various degrees of image searching. We find that higher orders of the Bath Fractal Transform (BFT) are a better means of gaining accuracy at a given bit rate than searching of the image. The best rate/disortion performance is obtained with a lightly quantized higher order BFT, whose optimum block size increases with compression.

1. INTRODUCTION

The Bath Fractal Transform (BFT) [1] is a hybrid of earlier fractal image coding techniques. All fractal coding methods work by contraction mapping of portions of images onto smaller image fragments. The Iterated Transform Technique (ITT), published by Jacquin [2, 3] and patented by Barnsley [4] used a low order grey scale mapping and incorporated searching to improve fidelity. This corresponds to the zero order (flat) polynomial BFT. Beaumont [5, 6] investigated a number of complexity options for the zero order case. Monro and Dudbridge [7] encoded a block without searching, using a bilinear mapping of grey scales, which is a first order polynomial BFT. Monro's generalization of this process [1] demonstrated the trade-off between higher order mappings and searching, with useful gains in fidelity achieved up to the biquadratic case. Monro and Woolley [8, 9, 10] reported on the rate-distortion performance of the BFT without image searching, and with limited and full image searching based on different selection criteria for fixed 8x8 blocks. An important conclusion was that searching over block symmetries was counter-productive in rate-distortion terms. We now consider the effect of block size and image searching to derive the optimum combination of parameters.

2. METHODS

The BFT is defined in earlier work [1]. A fractal function is defined on a support which is an adjoint tiling of the plane. Some useful grey scale mappings of an image are of the form:

$$v_k(x, y, f) = a + b_x x + b_y y + c_x x^2 + c_y y^2 + d_x x^3 + d_y y^3 + e f(x, y) \quad (1$$

A zero order (flat) fractal transform has all coefficients zero except a and e, and corresponds to the conventional fractal transform used by other workers [3, 5, 12]. The first order transform (bilinear) adds b_x and b_y, and so on.

3. ACHIEVING USEFUL COMPRESSION

Fractal transforms are lossy approximations to tilings of an image. A quadratic BFT without searching, for example, can be used to approximate a block of pixels using six coefficients. Compression of an image block can be achieved by quantization of the coefficients, or by increasing the block size. Additional compression will come from entropy coding of the quantized coefficients.

We have earlier investigated the quantization and entropy coding of fractal coefficients [9]. This identifies four levels of quantization; namely High, Medium, Low and Poor fidelity. Improved compression is obtained by differential coding of one fractal coefficient in the horizontal direction.

In Fig. 1 we show the results of quantizing a quadratic BFT at these levels over a range of blocksizes, using the standard CCITT test image Gold Hill. An rms error metric is used, and the compression shown is based on the entropy of the block. The error obtained without quantization is shown on the zero compression axis for comparison. From the lower envelope of the curves in Fig. 1, we conclude that the best error/compression performance is obtained with a lightly quantized transform. Compressin is obtained by increasing the block size. Similar results are obtained for other orders of the BFT and on other images.

4. ORDER OF FRACTAL TRANSFORM

From many experiments such as those in Fig. 1, it is possible to evaluate the optimum combination of parameters for fractal image coding. These experiments are carried out using a fixed block size over an image. However they indicate the optimum combination of parameters for a particular block

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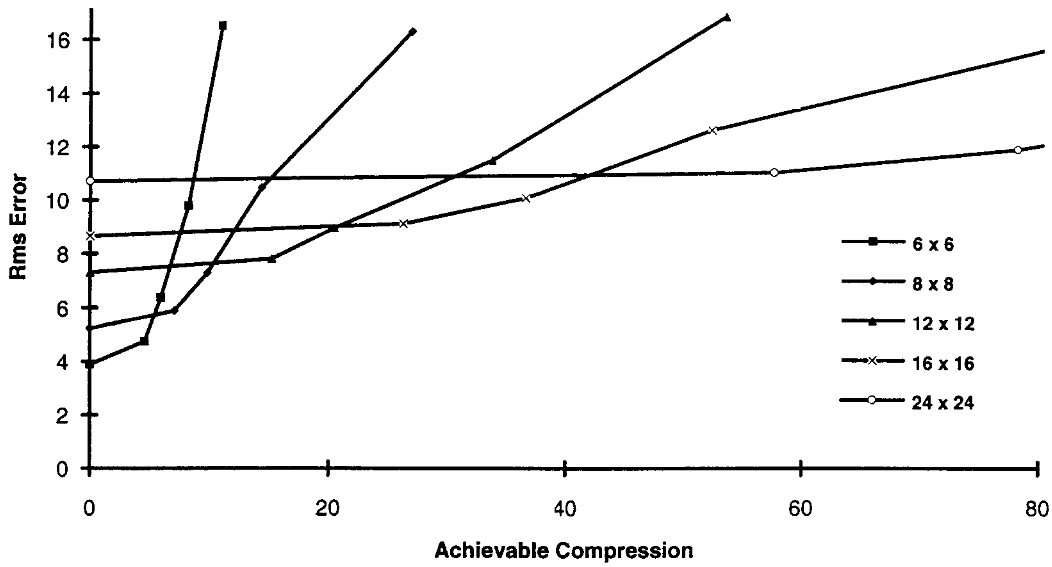

Fig. 1. Rate-distortion performance of the quadratic BFT. The quantization and block sizes are varied, and compression is predicted theoretically from the entropy of the coefficients.

size even in images which are decomposed into blocks of different size.

In Fig. 2 is shown the envelopes of best rate-distortion performance for zero searching BFT coding of order 0 (flat), 1 (bilinear) 2 (quadratic) and 3 (cubic). The performance improves as the order increases from flat to quadratic, and then decreases for the cubic case. At each higher order the accuracy is improved as might be expected. Up to order 2 the gain in accuracy more than compensates for the loss in compression caused by the additional coefficients. The cubic transform, while slightly more accurate, has sufficiently lower compression to give and inferior rate-distortion performance.

Oher results (not shown) confirm that for any compression ratio below 100:1, the quadratic BFT gives the lowest rms distortion if the correct block size is chosen.

5. SEARCHING ACROSS AN IMAGE

It has become conventional with fractal transforms to search the image to determine the optimum mapping of parent blocks onto child blocks. As the searching distance is increased better blocks may be found, but more bits are required to specify the translation across the image of these blocks. The value of this has not previously been investigated fully in terms of the tradeoff between accuracy and compression.

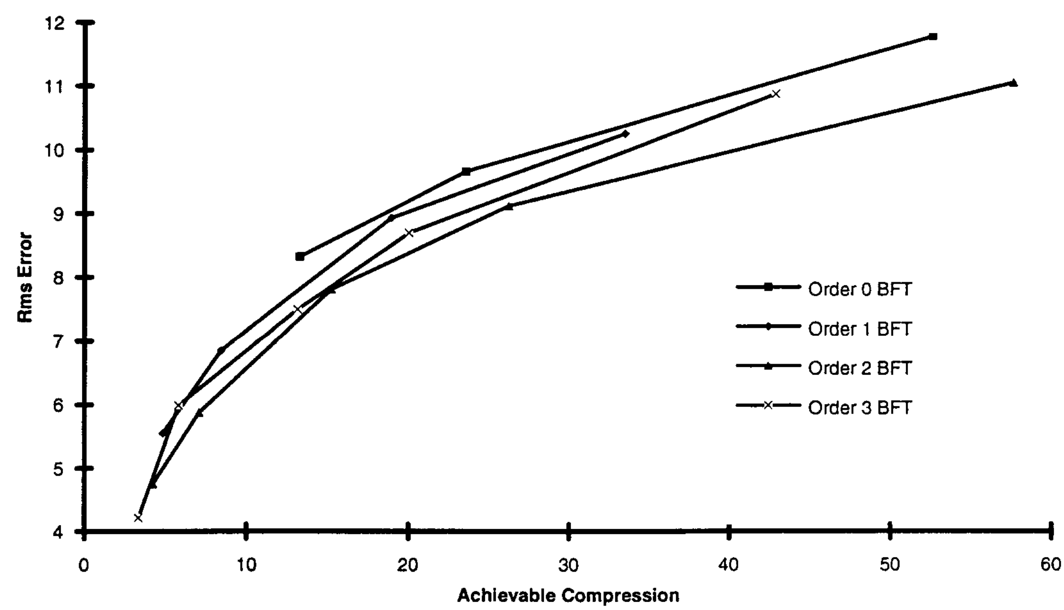

Fig. 2. Rate-distortion envelopes for various orders of the BFT, derived from many curves such as those in Fig. 1.

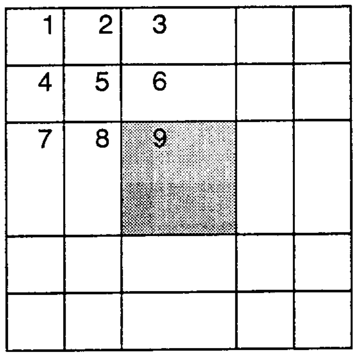

Fig. 3. Level 1 searching of a 3 x 3 array of parent blocks centred on the shaded child block.

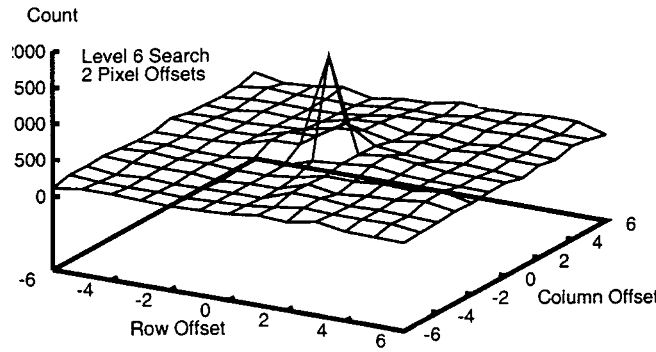

Fig. 4. Probability density function of block offset.

We have studied the case where the contraction of a parent block onto a child block is in the ratio 2:1. In Woolley and Monro [11] searching over an entire image, and limited searching based on the local edge value of an image were evaluated for the parent/child block combination of 8/4 pixels wide. Searching was found to degrade the rate-distortion performance due to the compression cost of storing the block offsets. We now consider the case of larger blocks.

Searching proceeds around the current child block, which may be of any size. The resolution of the search could be any integral number of pixels. Varying search 'levels' are used which dictate the number of possibilities investigated for the best match. The child is always in the centre of the searchnig range, and the 'level' indicates how many steps are taken on each side of the child block to locate the best parent. A level one search, such as is illustrated by Fig. 1, covers 9 parent blocks, while a level 6 search covers 169.

For each block size and search level a probability density function (pdf) was determined. Fig. 4 shows one such result of many, for a level 6 search with parent block size 8x8, and searching resolution of 2 pixels.

From each pdf an entropy can be calculated for the coding of the (x, y) offset. In combination with an rms error measure, it provides a point on a rate-distortion graph. In Fig. 5 we show as an example the optimum envelope from Fig. 2, together with the rate-distortion curves for different degrees of searching using the quadratic BFT over a range of compressions.

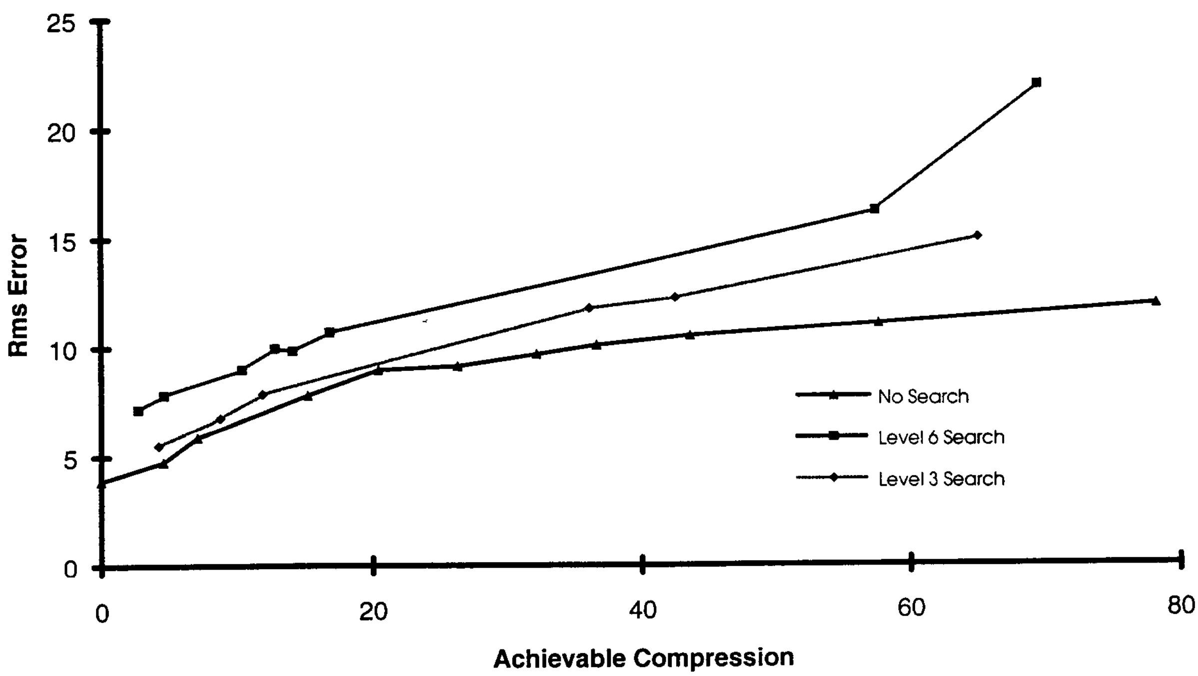

Fig. 5. The effect of searching on the Quadratic BFT. For the rms error metric we conclude that searching of the image, as well as being costly in time, does not help the rate-distortion performance.

6 DISCUSSION AND CONCLUSIONS

In this paper we have derived lower bound rate-distortion envelopes for the BFT with fixed size blocks from 8x8 to 24x24, both with and without the use of searching.

The quadratic transform performs consistently well across all block sizes. The results shown are for offsets which are a multiple of 2 pixels; similar results are obtained with 4 pixel offsets. We conclude that searching of the image is not useful when the consequences for the rate-distortion performance are considered. Since we have done this over a wide range of block sizes and compression ratios, we believe that a high order transform with increasing block size is the best way to achieve increasing compression of an image with minimum distortion.

We have also previously demonstrated that searching over block rotations and reflections is not useful [10]. Taken togetner, these are significant results which contradict widely held assumptions about fractal transforms.

Earlier workers have assumed that a block offest is an essential component of the fractal transform. Having devoted bits to describe these offsets, and being constrained to low order transforms, searching did indeed improve the rate-distortion performance.

However when the possibility of higher order transforms is introduced, a trade between searching and transform order is available. It is an unexpected but significant result of this investigation that zero searching transforms offer the best tradeoff between accuracy and compression, according to the rms metric used.

7. ACKNOWLEDGEMENTS

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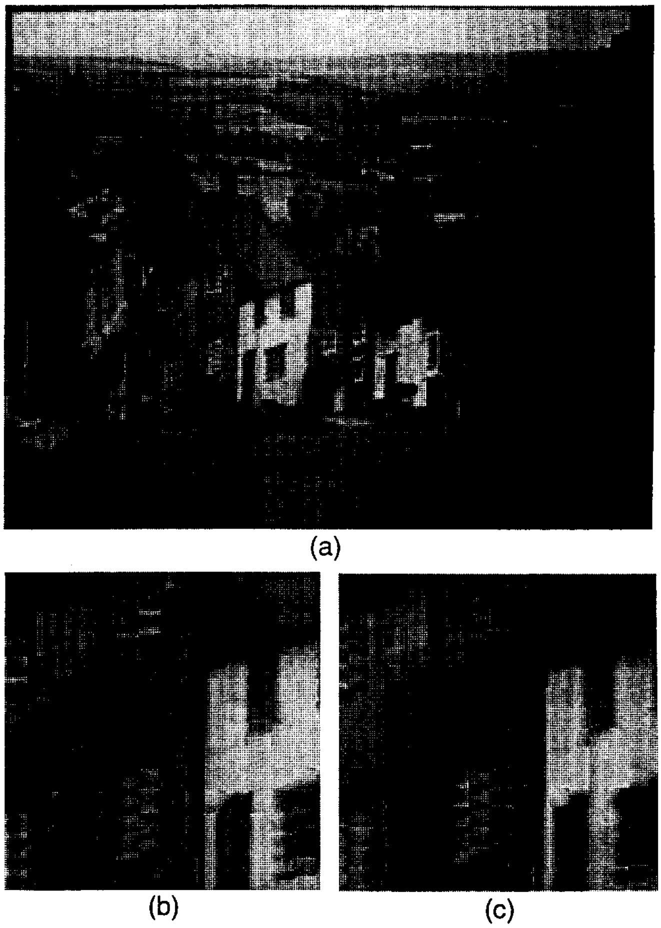

Fig. 6. Zero searching BFT with Gold Hill. (a) Cubic BFT, 18x18 parent blocks, 0.325 bpp, e_{rms}=9.2. (b) Detail from (a). (c) Quadratic BFT with similar result, 16x16 parent blocks, 0.305 bpp, erms=9.1.