A MODIFIED FRACTAL TRANSFORM

Hui Zhang Xiqi Gao and Zhenya He
Digital Signal Processing Division
Radio Engineering Department
Southeast University, Nanjing 210018, P.R.China

ABSTRACT

A Modified Fractal Transform (MFT) is presented in this paper. In the function part of MFT, the conventional greyscale function of an image block is replaced by the greyscal function of an error image block which mean is removed. This fractal transform is used to approximate an image which we want to encode. The simulation results show that with MFT the image decoding process is very fast, typically only 1 to 3 iterations are required to reconstruct the image while the quality of reconstructed image remains high.

1.INTRODUCTION

Recently fractal-based method for digital coding and compression of images has been received a high degree of interest since it can offer high compression ratios and simple decoding process[1-9]. Barnsley[7] first addressed the possibility of using fractal shapes for image compression, but the first practical coding method using fractal transform was developed by Jacquin[1]. Following them, other researchers over the world proposed several fractal-based methods for image coding purpose, such as BFT[5,6],etc. The main idea using fractal transform to encode images is that by deviding images into square blocks and designing fractal transformations for each block, one can exploit the self-transformability, as a form of redundancy in real-world images. An image we want to encode can be approximated by another image by using some simple fractal transforms acting blockwise with fewer parameters.

There are three main tasks in determining the set of fractal transforms to represent an image: (1) how to partion the image into blocks, (2) for each transform an appropriate domain must be chosen, and (3) we must determine a proper transform for each block and obtain the transform parameters through some error minimization technique. We mainly focus in the third problems in this paper because a proper fractal transform is the critical factor in fractal-based image coding methods.

In this paper, we propose a fractal transform which is similar to the BFT (Bath Fractal Transform)[6] but different from BFT in the function part of the fractal transform. This fractal transform takes advantages over other fractal transforms in two aspects: (1) iterations required for reconstructing an image from its fractal code are fewer, typically 0 to 3 are sufficient, and (2) fractal coefficients distribute over a more narrow range than ones obtained from other fractal transforms, thus it can be expected that higher compression ratio can be achieved. This paper is arranged as follows: a brief review of basic theory for fractal transform is described in section 2 and some investigating results are also given in this section. In section 3 a modification of well-known BFT is presented. Some simulation results are given in section 4 for comparing the performance of MFT used to encode images to that of BFT. Finely in the end of this paper we give some concluding remarks.

2.BASIC THEORY OF FRACTAL TRANSFORM

The fractal transform for image coding purpose is a strategy for approximation in three dimensions. An fractal transform is composed of two parts:

(1) *the domain part*:

$$T=\{t_k;k=1,\dots,K\} \tag{1}$$

T is an iterated function system, a finite set of contraction mappings whose attractor A defines a non-overlapping tiling of an image.

(2) *the function part:*

$$f(t_k(x,y))=F_k(x,y,f(x,y)) \tag{2}$$

where $f(x,y)$ is a fractal function defined on A to approximate the greyscal function of tile k (the range

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block defined by Jacquin[1]). The F_k, $k=1,\ldots,K$ are mappings to specify the fractal function $f(x,y)$ and take the form of

$$F_k = g_k(x,y) + s_k f(x,y) \quad (3)$$

where

$$g_k(x,y) = a_k + \sum_{p=1}^{P} b_{x,p} x^p + \sum_{p=1}^{P} c_{y,p} y^p \quad (4)$$

The $a_k, b_{x,p}, c_{y,p}$ and s_k are fractal coefficients of the fractal transform, e.g. the fractal code of tile k. P denotes the order of the fractal transform. Typically, P is taken up to 3 because that little is gained by a higher approximation[4]. Self-similarity is introduced by contracting an image fragment $f(x,y)$ (domain block) onto an image and when iterated [1], the ensemble function formed by the mappings $F_k, k=1,\ldots K$, construct an approximation to the image by the Collage Theorem[7]. To obtain the parameters (fractal code) of a fractal transform, the fractal transform solves for a least-squares F_k. This is the basic theory of Bath Fractal Transform (BFT) [5].

By using the BFT to encode an image, we found that fractal coefficients a_ks distribute over a very broad range (as shown in fig.1), thus affect the coding performance. In addition, when reconstructed the image from its fractal code by iterated technique, the iterative process will converge after many iterations, typically from 7 to 10. The reasons are that the term $g_k(x,y)$ in equ.3 only represents the translation of the slowly varying part or average part in greyscal function between the tile k (range block) and the image fragment $f(x,y)$ which gives the best approximation to tile k, while the $s_k f(x,y)$ introduces not only the detail information into tile k but also the slowly varying part or average part into tile k. If we want to improve the coding performance of fractal-based image coding methods, the distribution of fractal coefficients should be over a more narrow range. The iterations required to reconstruct an image will be very few if the fractal coefficients can fit the *main contents* included in tiles of the image to be approximated. This is the motivation that we introduce a modification into BFT.

3.MODIFIED FRACTAL TRANSFORM

In this section we present a fractal transform named MFT (Modified fractal transform) by introducing some modification into BFT. The domain part of MFT is the same as BFT, but in function part of MFT the greyscale function of an error image block named error domain block, which mean is removed, is used instead of the conventional greyscale function used in BFT

$$F_k = g_k(x,y) + s_k f_{me}(x,y) \quad (5)$$

where $f_{me}(x,y)=f(x,y)-m_f$ denotes the greyscal function of an error domain block and m_f is the mean of the original domain block located at (x,y). The $g_k(x,y)$ is the same as equ.4. For obtaining the parameters (fractal code) of MFT, MFT solves for a least-squares F_k as [5]. This modification has two features:

(1)*Because $f_{me}(x,y)$, rather than $f(x,y)$ is used in fractal transform, the term $s_k f_{me}(x,y)$ in equ.(5), which is dependent on particular image to be encoded, mainly introduces the "texture information" into the range block k.*

(2)*The other terms except $s_k f_{me}(x,y)$ in equ.(5), independent of particular image, can approximate the "smooth part" of the range block k closely.*

Therefore, it can be expected that by using MFT to encode an image, only few iterations, typically from 1 to 3, are required to reconstruct the image from its fractal codes because of the second feature mentioned above. Higher compression ratios can be achieved because the parameters for F_k ($k=1,\ldots,K$) distribute over a more narrow range. The following simulations will show this

In [8] and [9], a totally orthogonal process is introduced into fractal transform for reducing the redundant information involved in the fractal function $f(x,y)$ in order to speed up the decoding process and reduce the number of domain blocks. Though non-totally orthogonal process is involved in our methods, the simulation results are similar to the results described in [8] and [9], e.g., the iteration for reconstructing images are from 1 to 3, and our method is much simpler. Beaumont [3] presented a fractal-based image coding method in which the mean of each range block is removed before fractal transform is performed on that range block. The image formed by means (mean-image) is quantized, encoded and transmitted to decoder firstly and the iterative technique is used to reconstruct the image with the mean-image in decoder. In our method, removing the mean is involved in fractal transform and no extra step is needed to process the mean.

4.SIMULATION RESULTS

To evaluate the MFT, we applied it to the standard test image "LENA". The tiles (range blocks) with fixed size 4 by 4 pixel are used to encode the image. An image block (named "parent block" in [6]), which gives an approximation to a particular tile by MFT, is chosen from among all the 8 by 8 domain blocks contained in a proper chosen neiborhood of the tile. The searching scheme to select the domain block whose mapping gives lowest distortion measure is the same as the Moron's [6]. To test the performance of the MFT, we use the MFT with different complexity, from the simplest (zero-order) case to the biquadratic (second-order) case without cross-product of x and y, to encode the image. Also we consider the different area searching to find the best mapping to each tile, such as level-zero, level-one, level-two area searching [6] and full image searching. The BFT , with the same combination of area searching level and order of fractal function as MFT, is used to encode the same image for comparison with MFT.

We here give examples of "LENA" encoded by using MFT and BFT with the same combination of level-two searching and the first-order fractal function. We have not considered reflections or rotations in mappings and the quantization of the fractal coefficients in MFT and BFT both. First we investigate the value ranges of a_k. The maximum of $a_k$coefficients in MFT is 265.1 and the minimum is -44.1, while the maximum 420.8 and the minimum -124.9 in BFT. The varying range of a_k in MFT is much smaller than that in BFT (about 1:2), as illustrated by Fig.1. The value-ranges of other coefficients , either in MFT or in BFT, is the same. Therefore, if the same number of bits are used to quantize fractal coefficients, the quality of decoded image by MFT will be better than that by BFT because finer quantization step can be adopted to quantize a_k coefficients in MFT.

Also we investigate the error signals, denoted by S_d, between a_ks and means of range blocks with size 4 by 4 to be approximated by fractal transform. The distribution of the error signals S_d^M where M means Moron's method and the distribution of S_d^o (our methods) are shown in Fig.2. From Fig.2, it can be found that the S_d^o mainly distribute over a narrow range near to zero, from -44.9 to 42.7, while the varying range of S_d^M is from -288.7 to 250.2. The result shows a_ks in MFT can fit the means of tiles closer than that in BFT. So we can conclude that the iterations required to reconstruct the image with any initial image, when MFT used to encode the image, will be much less. Beginning with the black image, e.g. an image with its pixel values equal to zero, the iterative process has already converged after three iterations, as illustrated by Fig.3. The original image and the images resulting from the iterative reconstructing process after the first, and third iterations are shown in Fig.4. With the MFT, the errors of the reconstructed images remain almost constant after three iterations and the final PSNR of 31.93 dB is very close to the PSNR of 32.15 dB estimated in the encoding process.

5.CONCLUSIONS

A modified fractal transform (MFT) with an improved performance over the conventional fractal transform has been proposed. The basic difference is that in the function part of the MFT we use the greyscale function of an error image instead of the greyscale function of an image block . The simulation shows that with the MFT the reconstructing process of image is very fast while the image quality remains high. Another improvement is that at the same quality of decoded image, it is possible to achieve lower bit-rates with the MFT than with other fractal transforms. So in picture archiving systems where images are compressed once and decompressed often , MFT could become a preferred method.

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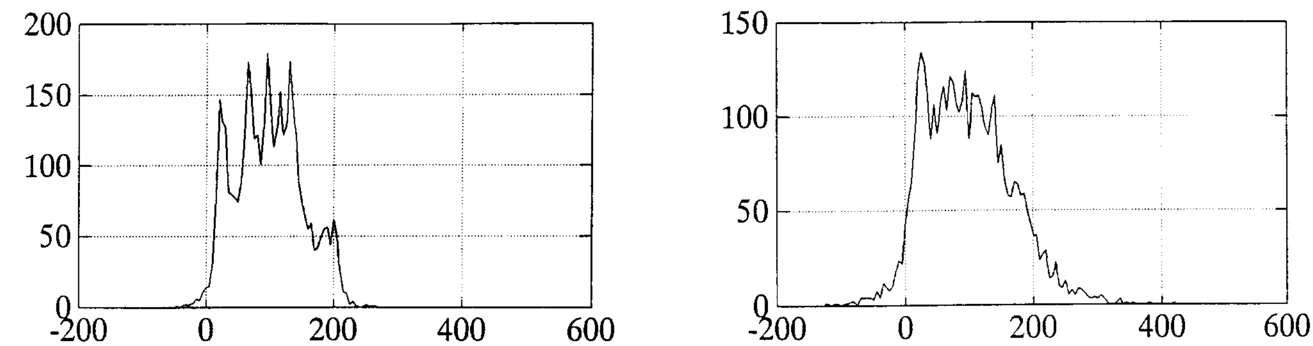


Fig.1. The distribution of fractal transform parameter a_k
(a) distribution of a_k of MFT. (b) distribution of a_k of BFT.

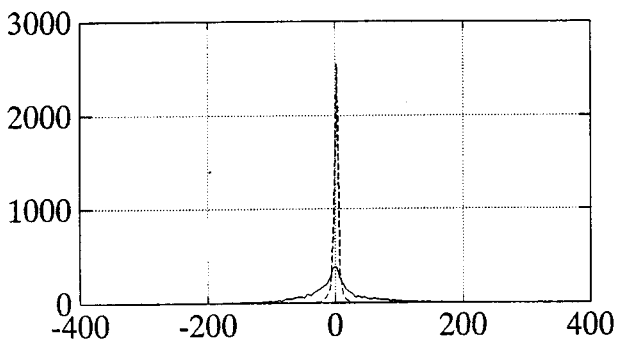


Fig.2. The distribution of error signals
dashed line - distribution of $S_d^{\,0}$.
solid line - distribution of $S_d^{\,M}$.

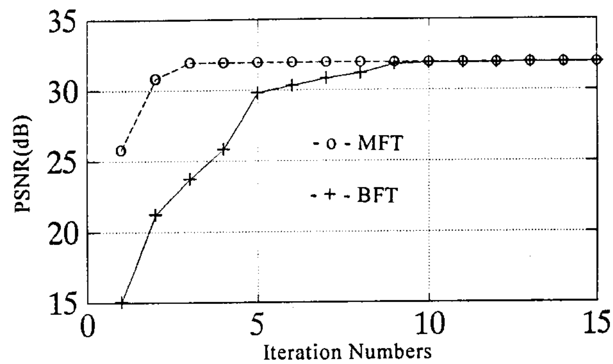
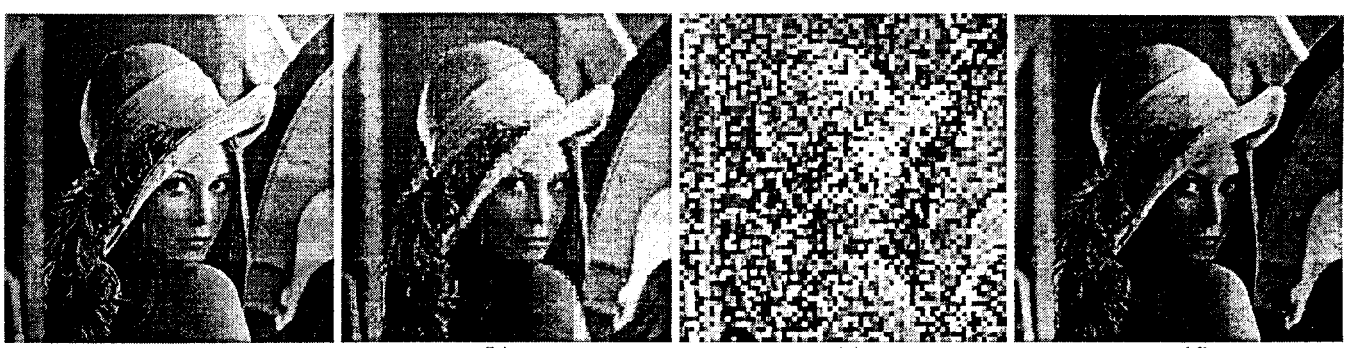


Fig.3. PSNR-iteration curves for fractal transform reconstruction of the test image "LENA" by MFT and BFT.



(a) (b) (c) (d)

(e)

Fig.4. (a) Original test image "LENA"
(b) Resconstructed image after one iterations, PSNR=25.41dB(MFT).
(c) Resconstructed image after one iterations, PSNR=14.73dB(BFT).
(d) Resconstructed image after three iterations, PSNR=31.93dB(MFT).
(e) Resconstructed image after three iterations, PSNR=25.63dB(MFT).