Model Selection and Texture Segmentation using Partially Ordered Markov Models[1]

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Abstract

Texture is a phenomenon in image data that continues to receive attention due to its wide-spread applications, ranging from remotely sensed data, to medical imaging, to military applications. In this paper we use a new class of spatial stochastic models called *partially ordered Markov models* (POMMs) for texture analysis and model selection. POMMs are a generalization of Markov mesh models that have the property that their joint probability density function is an exact, closed form expression in terms of conditional probabilities. Markov random fields (MRFs) do not, in general, have this property. This property of the POMMs has lead to exact and fast computations involving the joint probabilities. We show how these fast algorithms allow POMMs to be used for fitting models to textures, and for supervised texture segmentation. Applications to real data show that the model selection technique gives very good results. POMMs are a broad and general class of models, and have the potential to be applied to diverse areas beyond imaging, such as probabilistic expert systems and artificial intelligence.

1. Markov Random Fields and POMMs

Markov random fields are a versatile tool that have many uses in image processing [1]. A spatial stochastic model, it is well-known that a MRF can be expressed as a Gibbs distribution [1]. However, in computer implementations of algorithms which require calculations of the joint probability density function (pdf), such as the maximum likelihood estimator (MLE) or maximum a posteriori (MAP) estimator, only approximations of the Gibbs normalizing constant are typically available [1]. Thus, the efficacy of the MRF model can be severely limited in practical applications.

Partially ordered Markov models (POMMs) are a new class of models recently introduced [2] that generalize Markov mesh models (MMMs) [3]. It has been shown [4] that the particular relationship between random variables (r.v.s) in a POMM is a *partial ordering* of the set of pixel sites. As classes of models, we have MMMs $\subset$ POMMs $\subset$ MRFs. POMMs have the attractive property of closed joint pdf.

Let $\mathbf{A} = \{a_{ij}\}$ represent the set of r.v.s in an M x N array = Ω. Assume that the M x N array of r.v.s has an acyclic digraph imposed upon it. For this acyclic digraph denote the associated partial order by $\prec$. Define the set $adj_{\prec} a_{ij}$ to be those r.v.s which are "less than" and adjacent to a_{ij} under the digraph relation. Let L^0 be the minimal elements of the partially ordered set A. These are elements which have no other elements "less than" them. Then we have the following result:

$$P(\mathbf{A}) = P(L^0) \cdot \prod_{\{b : b \in \mathbf{A} \setminus L^0\}} P(b \mid adj_{\prec} b). \qquad (1)$$

That is, the joint pdf can be expressed as a product of conditional probabilities (modulo the "boundary" effects L^0). More results extending MMM properties to POMMs can be found in [4].

Note that because the joint pdf in Eq. (1) is exact, no approximations of the normalizing constant are necessary. Also, the specification of parameters for the POMMs is straightforward, through the use of Eq. (1) and the specific form of conditional pdf used. For all models here, a conditional binomial distribution was used. Any conditional pdf that satisfies all conditions for a valid probability distribution can be used to model the conditional probabilities of the POMM. In the next section, we introduce the models we developed for texture analysis.

2. POMMs for Model Selection and Texture Segmentation

While stochastic texture models have been used in many areas of image processing, not much emphasis has been given to the problem of model selection. In most literature on texture segmentation using MRFs, a fixed order MRF is assumed for all textures in the image. However, one simple model is usually not sufficient to model all texture classes. We believe that the choice of a model that "best" fits a class of textures is data dependent. Model selection is an important criterion that needs to be considered in texture segmentation, especially when the image is composed of a wide range of micro and macro textures.

In this research effort, we used POMMs in a model selection criterion for textures, and also for supervised segmentation of textured data. We incorporated the model selection criterion in a supervised texture segmentation algorithm using a MAP criterion and iterated conditional

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modes (ICM). Due to the exact closed form of the joint pdf, POMMs have a closed form expression for the likelihood function, and thus allow an exact calculation of the MLE, and a swift computation of the MAP using ICM. The ICM algorithm is computationally efficient and tends to avoid phase-transitions that are inherent in MRFs.

Model Selection. We employed a Bayesian approach for model selection. This has proved to be successful in other approaches [5], [6]. The Bayesian decision rule is consistent, that is, the probability of choosing the correct model approaches one as the number of observations goes to infinity. The algorithm assumes knowledge of the maximum likelihood estimator, which is available for the POMM. In the Bayesian approach to model selection, prior probability distributions for several possible models are assumed, and the model that maximizes the posterior probability conditioned on the data is chosen. Given a collection of K models, $C_1, \ldots, C_K$, the Bayesian approach chooses the model that maximizes the posterior probability $P(C_i \mid A)$, where A is the sample image of a texture. The posterior probability is written as $P(C_i \mid A) = \int P(A \mid \theta_i)P(\theta_i \mid C_i)P(C_i)d\theta_i$, where θ_i is the vector of parameters for model i. If we assume that $P(C_i)$ is constant for all models, the decision function used for model selection is given by

$$d_i = \int P(A \mid \theta_i)P(\theta_i \mid C_i)d\theta_i. \qquad (2.2)$$

The asymptotic expansion of Eq. (2.2) as obtained from the results in [6], is given by $d_i = (2\pi)^{-K/2}P\left(A \mid \hat{\theta}_i\right)P\left(\hat{\theta}_i \mid C_i\right)n^{-K/2}\Delta_i^{-1/2}$, where $\hat{\theta}_i$ is the maximum likelihood parameter estimate, K is the number of parameters in model i, n is the number of pixels in the array, and Δ_i is the Hessian of the likelihood at the maximum likelihood estimate. Since $\ln(2\pi)^{-K/2}$ does not vary much with models, and $\ln\Delta_i$ is small compared to the log likelihood $\ln P(A \mid \theta_i)$, the decision rule we used for model selection is $d_i = \ln P\left(A \mid \hat{\theta}_i\right) - K/2\ln(n)$, which is similar to Schwarz's criterion. Note that this model selection technique uses the maximum likelihood estimate, which is not easily computable for general MRFs.

Given a set of POMMs $C_1, \ldots, C_K$, we choose the model j as the "best" fitting model if $j = \arg\left(\max_i d_i\right)$. In Fig. 1, we show four of the nine POMMs we used for testing the model selection method.

We applied the model selection technique to several of the benchmark Brodatz textures. The images were of size 128 x 128, and had 16 gray levels. We used K=9 POMMs as representative models against which we matched. We found that all textures were modeled better by high order POMMs than by low order ones. This may be because the Brodatz textures are composed of macrotextures, and high order POMMs incorporate dependencies on pixel sites that are spatially distant. We display one of the original Brodatz textures of tree bark in Fig. 2. The selection process found that the 11–th order POMM gave the best fit to the image data in Fig. 2; in Fig. 3, we display the parameters found. In Fig. 4, we regenerated the texture from the model in Fig. 3, using a new, fast one-pass algorithm called the *level set algorithm* [4] that synthesizes an image in one pass of the pixel sites. For comparison in Fig. 5, we give the texture synthesized from the maximum likelihood estimates using the fourth order POMM shown in Fig. 1 (a). Obviously, the high order POMM provides a more accurate fit to the data.

In order to represent the conditional probabilities of the gray values at a pixel conditioned on its adjacent lower neighbors, we used a conditional binomial model that was introduced in [7].

$$P(a_{ij} = m | adj_{\prec} a_{ij}) = \binom{G-1}{m}\theta^m(1-\theta)^{G-1-m},$$
$$m = 0, 1, \ldots, G-1, \text{ where } \theta = \frac{e^T}{1+e^T}, \text{ and}$$
$$T = \alpha + \beta a_{i-1,j} + \gamma a_{i-1,j-1} + \delta a_{i,j-1} + \epsilon a_{i+1,j-1}. \qquad (2.3)$$

POMMs used for Textures and Region Distribution. Texture segmentation using a model based approach involves a method for estimating model parameters, and maximizing or minimizing a suitable cost function. A common measure used for texture segmentation with MRFs is maximum a posteriori estimation of the label image B, conditioned on the observation image A, $P(B \mid A)$ [7]. However, lack of a closed form expression for the joint distribution of MRFs makes the simultaneous job of parameter estimation *and* MAP segmentation mathematically intractable. We propose a segmentation technique using POMMs that simplifies parameter estimation and MAP segmentation, and incorporates model selection into the texture segmentation problem.

We used two different POMMs to model the two processes of texture data and region labels in the image. We assumed that for every observed pixel a_{ij} in the data image A, there exists a labeled pixel b_{ij} in the segmented image B that represents the texture class to which pixel a_{ij} belongs. We used a conditional binomial distribution as given in Eq. 2.3 to model the texture data process.

In order to model the region distribution of the labeled image, we assumed that each region is reasonably large and contains no small isolated regions within, and that the texture boundaries are reasonably smooth. Based on these assumptions, we developed a POMM that is similar to the multi-level logistic model to model the region label process. The general equation for the conditional distribution on any POMM is

$$P(a_{ij} \mid adj_{\prec} a_{ij}) = H(a_{ij}, a_{mn} : a_{mn} \in adj_{\prec} a_{ij}),$$

where H can be any arbitrary function that satisfies the conditions for a valid probability density function. Since our aim is to model a process that avoids small, isolated regions, we assign larger "penalties" to isolated pixels or small regions. The conditional probabilities are

$$P(b_{ij} \mid adj_{\prec} b_{ij}) = \frac{\exp\left(-U_{b_{ij}}\right)}{\sum_{b_{ij}} \exp\left(-U_{b_{ij}}\right)}, \text{ where}$$

$$U_{b_{ij}} = \sum_{(p,q)\in adj_{\prec} b_{ij}} F_{pq}(b_{pq}) + \sum_{(p,q),(r,s)\in adj_{\prec} b_{ij}} F_{pqrs}(b_{pq}, b_{rs}).$$

Here, the single pixel interactions obey $F_{pq}(b_{pq}) = 0$, and

$$F_{(pq),(rs)}(b_{pq}, b_{rs}) = \begin{cases} -\beta & \text{if } b_{rs} = b_{pq} \\ \beta & \text{if } b_{rs} \neq b_{pq} \end{cases}, \beta > 0.$$

The larger the magnitude of β, the greater the penalty for small regions.

Supervised Texture Segmentation. MAP segmentation involves computing a segmented image B* such that

$$B^* = \arg\left(\max_B P(B \mid A)\right) = \arg\left(\max_B P(A \mid B)P(B)\right). \quad (2.4)$$

The calculation of the conditional probabilities $P(A \mid B)$ and $P(B)$ is trivial for a given B, but the evaluation of Eq. 2.4 is difficult. Hence, we used the ICM algorithm to compute Eq. 2.4, but instead used the quantity

$$b^*_{ij} = \arg\left(\max_{b_{ij}} P(b_{ij} \mid A, B\backslash\{b_{ij}\})\right), \qquad (2.5)$$

for all *(i,j)*. The quantity in Eq. 2.5 is easier to compute than Eq. 2.4, but at the cost of accuracy in labeling. The reason we chose ICM to compute the MAP, as compared to using a global optimization technique, is that the MAP is faster to compute.

We performed supervised texture segmentation with model selection in the following manner. First, given instances of all representative textures that may be present in an image, we use the model selection rule to compute the "best" model for each texture. If there are K possible classes in A, we determine the set of optimum models for the classes, say, $M^* = \{M_1, M_2, \ldots, M_K\}$. We compute the joint pseudo-local likelihood at each pixel for all possible textures in A,

$$J_{mn}(i) = P(A_{mn} \mid M_i, (m,n) \in \Omega)$$
$$= \prod_{(m,n)\in\Omega} P(a_{mn} \mid adj_{\prec} a_{mn}, M_i), i = 1, \ldots, K.$$

The pixel is assigned the texture class number that has the maximum joint pseudo-local likelihood among K classes, i.e, $b_{mn} = \arg\max_i J_{mn}(i)$. The ICM algorithm is then used iteratively to compute a B^* until none of the pixel labels in B^* change.

We show the supervised segmentation algorithm applied to two images in Fig. 6 and Fig. 8, each a combined Brodatz texture pair. The resulting segmented images are shown in Fig. 7 and Fig. 9, respectively. We chose a randomly placed border between the textures to simulate real world images. The algorithm performed an accurate classification of the textures, even at the texture borders. The results show that even choosing a local optimization technique such as ICM results in good segmentation results, when the models chosen for the textures are "close." This stresses the importance of using a model selection rule to choose the best model, instead of simply choosing one model.

3. Conclusions

We have presented the use of a new stochastic spatial model called partially ordered Markov models for texture segmentation and model selection. Due to the property of a closed joint pdf, POMMs allow exact computation of many useful statistical algorithms that use the joint pdf, such as maximum likelihood estimation and maximum a posteriori estimation. We have presented a model selection rule that is especially suited to POMMs. The model selection technique is incorporated into texture segmentation using POMMs, which allow very fast computation. This technique can be extended to unsupervised texture segmentation using global optimization techniques, such as simulated annealing, to obtain better MAP estimation. An alternative criterion to MAP, such as minimum mean square estimation, may also be used for segmentation.

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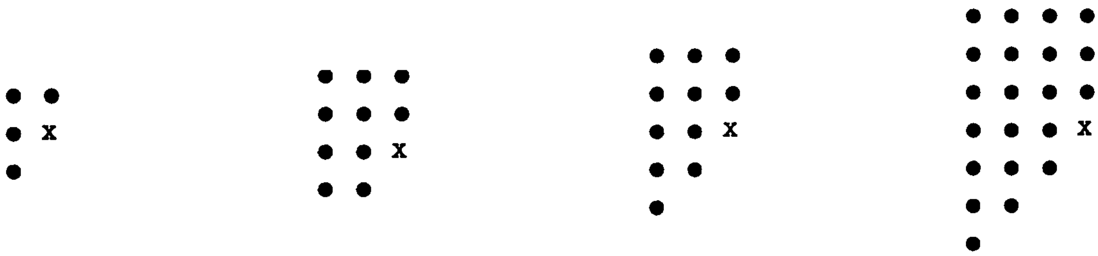

(a) (b) (c) (d)

Fig. 1. POMMs used for model selection. The symbol "X" marks the location (i,j). (a) Fourth order model. (b) 11–th order model. (c) 12–th order model. (d) 22–nd order model.

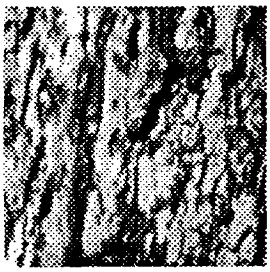

Fig. 2. Original texture used for model fitting.

-.02	-.03	0.0
-.03	.02	0.2
0.0	0.1	-2.6
-.03	0.1	

Fig. 3. Model and parameters giving best fit to image in Fig. 1.

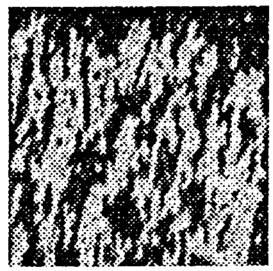

Fig. 4. Texture synthesized using best model as given in Fig. 3.

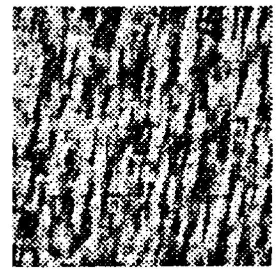

Fig. 5. Texture synthesized using fourth-order (lower order) POMM.

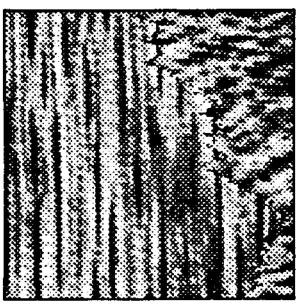

Fig. 6. Original image consisting of grain and water.

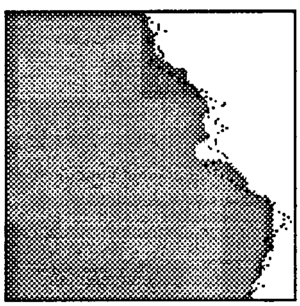

Fig.7. Supervised segmentation of image in Fig. 6.

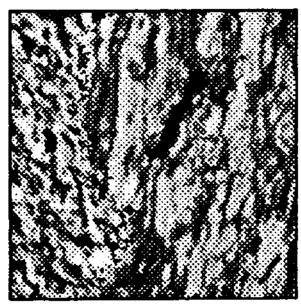

Fig. 8. Original image consisting of cork and tree.

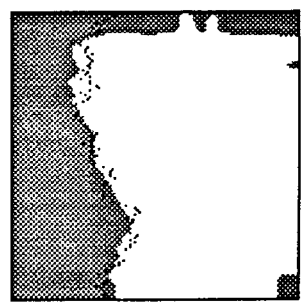

Fig. 9. Supervised segmetation of image in Fig. 8.