POLYNOMIAL IMAGE CODING
WITH VECTOR QUANTISED ERROR COMPENSATION

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ABSTRACT

We report a novel image coder which is a hybrid of fractal coding and vector quantisation. The approach to image compression is to form an approximate image by one method and clear up errors by another. In this realisation, image blocks are approximated by polynomial functions, and the residual image blocks (RIBs) are coded by vector quantisation into a code book which is small enough to transmit with an image. The method is evaluated on a number of parameters, and the results are found to be intermediate between fractal and JPEG coding in their rate/distortion performance. Possible further improvements are indicated.

1. INTRODUCTION

We report an image coding technique which combines principles from both fractal and vector quantisation methods.

Fractal image coding methods are based on contraction mapping of partitions of images (parent blocks) onto smaller image fragments (child blocks). Jacquin [1] suggested that an image could be coded using a grey scale approximation to each image child block, with a contraction mapping of a larger parent block in the image on to the child. To do this, an image is completely tiled by adjoint child blocks. Since the parent blocks can be anywhere in the image, the process of finding a parent block which best approximates a child block can be costly. The image code, which is lossy, consists of the mappings, and decoding can be accomplished by various rendering methods. Monro [2] extended this by using more general functions such as polynomials along with the contraction mapping to describe the grey scale. It was also shown [3] that the parent block enclosing the child was a good approximation to an ideal contraction mapping. Polynomial coefficients can be suitably quantised without much degradation of the image quality.

2. SUMMARY OF METHOD

It is proposed that an image could be approximated by polynomials which fit a blocking of the image. A few well chosen Residual Image Blocks (RIBs) can then be selected to improve the approximation by adding them to the polynomial fit. The general description of an image block might be:

$$v_k = a + b_x x + b_y y + c_x x^2 + c_y y^2 + e\,\{\, block(x, y) \,\} \quad (1)$$

This resembles a fractal transform, and the coefficient values can be calculated using a least squares fit as done by Monro [2] for fractals. The major difference from fractals is that the *block(x,y)* values are predetermined, and therefore to render the image there is no need for the iteration usually associated with fractals. The RIBs are chosen from the original image to approximate the difference between the polynomial coded image and the original. Since the RIBs are taken from the image the method is still fractal in that it depends on a degree of self-similarity in the residual error from the orginal coding method.

Vector quantisation provides a simple method for finding the RIBs which minimise the RMS error. A Lloyd Max method of quantisation as described by Gray [5] was used to determine the best RIBs to use for an image. The 'seed' for the Lloyd Max quantiser was formed from the linking of a large list of RIBs from the image and then pruning the linked list. The finished image code consists of the quantised coefficients from equation (1) together with the residual blocks. The code book is not normally transmitted in vector quantisation, so this is a major difference between the methods. It also means that the coding technique will work well for any set of images, rather than being affected by the class of image used to generate the code book.

3. GENERATION OF THE RIBS

For evaluation, an image was subdivided into 120x120 pixel sections in which separate sets of RIBs were calculated. This exploits the local symmetry of the image and allows a large number of RIBs to be used without the expense of transmitting an overly large, mainly redundant, vector for each block.

Each section of the image was approximated using a simple polynomial blocking and a list of residual error blocks was derived from the original image. The gray levels in each RIB were scaled to fit the range -1.0 to 1.0, so that only the shape of the RIB was preserved. The list of RIBs was linked

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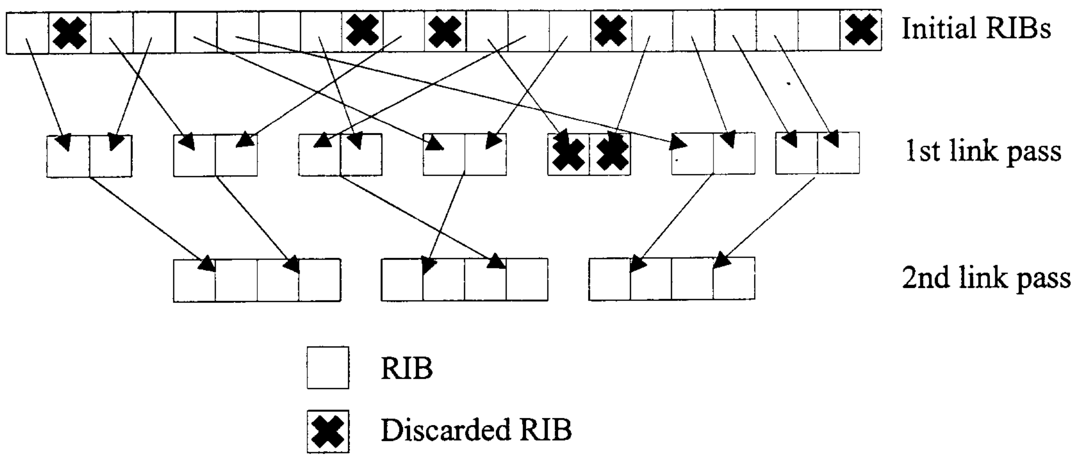


Fig.1. Illustrating the linking and pruning of similar Residual Image Blocks (RIBs) prior to vector quantisation.

into pairs of blocks and further pairs were linked into pairs of pairs as shown in Figure 1.

To do this, RIBs with similar shape were linked together, while blocks which were not sufficiently similar to others were discarded from the list. The shape similarity was found by scaling the blocks for minimum rms difference. Any RIBs not sufficiently similar to others in the list, based on an arbitrary rms threshold, were discarded, as shown in Fig. 1.

If the final of number of RIBs sought was n, then the linking was continued until approximately $2n$ RIBS remained. These RIB groups were averaged and normalised to preserve their shape, as before.

This linked list of RIBs was used as a 'seed' for a Lloyd Max quantiser which was then able to produce a near ideal group of RIBs for each section. The final group of RIBs used was found by pruning the list of RIBs from the Lloyd Max quantiser to the desired number.

The RIBs were scaled to the range -1.0 to 1.0 at each stage to prevent degradation of their shape information. They were also quantised, and it was found that a dynamic range of 5 bits for the RIB gray values showed negligible visible degradation. Since the RIBs are therefore fitted to equation (1) in quantised form, they produced very little additional quantisation error. No assumptions could be made about the probability distribution of the RIBs, so a linear quantiser was used.

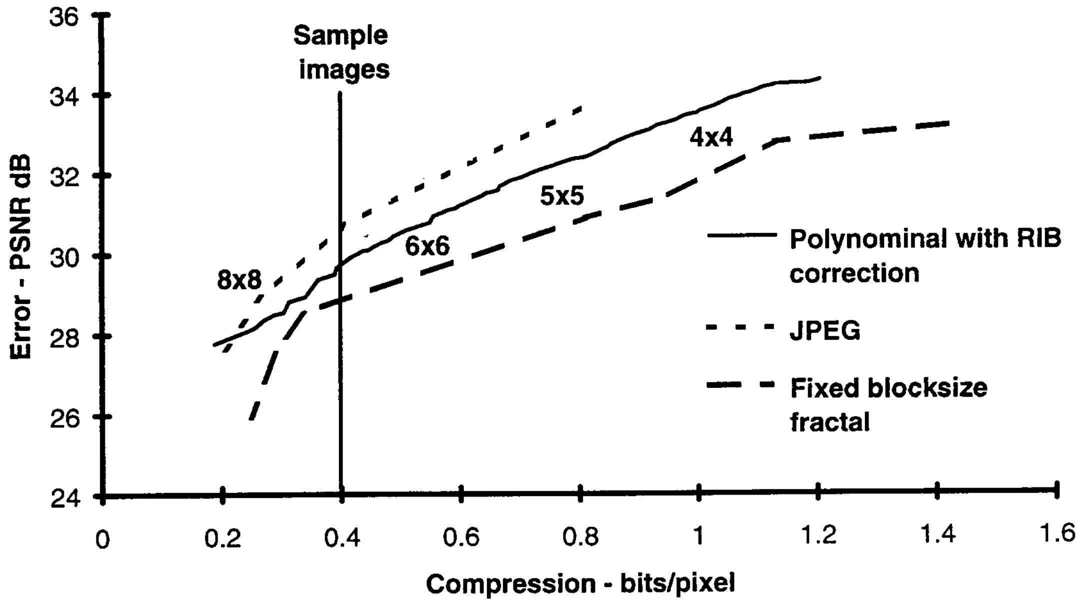
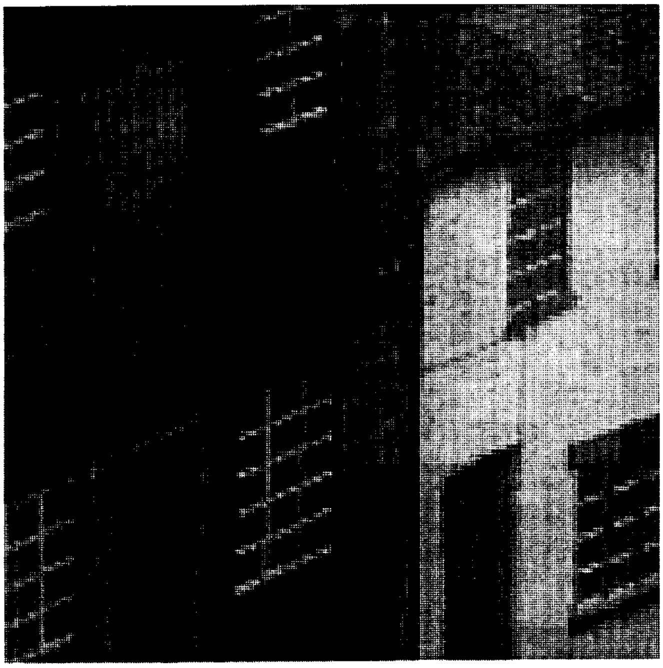
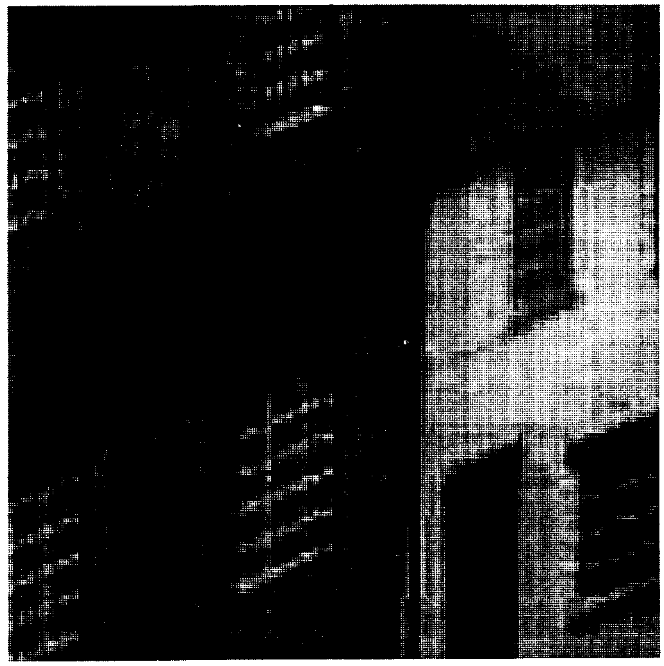


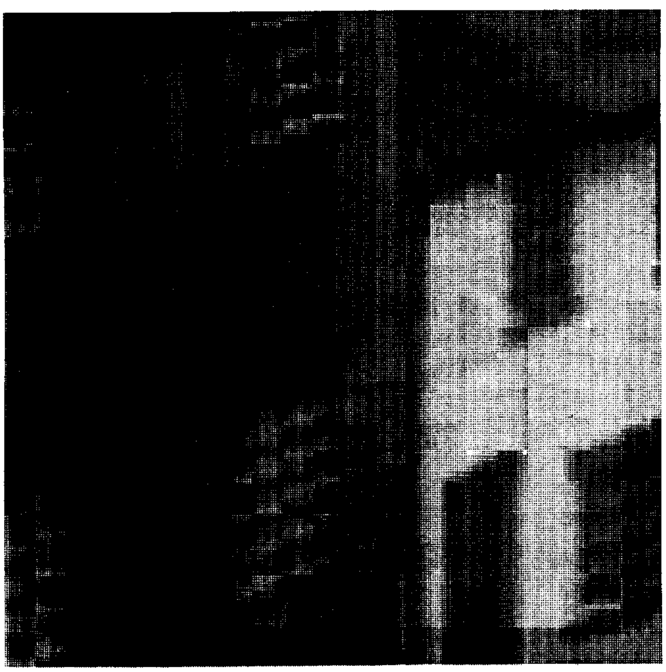
Fig 2. Rate/distortion graphs comparing three methods of compression for the CCITT test image Gold Hill.



(a) Original.



(b) JPEG 0.4 bpp, PSNR 27.9.



(c) Bilinear Fractal Transform 0.4 bpp, PSNR 24.9.

(d) Polynomial with RIB .4 bpp, PSNR 25.1.

Fig 3. Detail of the CCITT test image Gold Hill (a) original and (b) - (d) compressed by various methods. The compression and accuracy figures given are for the fragments.

4. CODING OF DATA FOR TRANSMISSION

After coding of an image block, the polynomial coefficients are still to be quantised. The grey level coefficient, a, was highly correlated in the horizontal direction and it was therefore difference coded to reduce the number of bits needed to store it, as δa. For a bilinear polynomial we quantised δa, b_x, b_y and e to 6, 5, 3 and 3 bits respectively, using a Lloyd Max quantiser. The entropy of all the quantised coefficients and the list containing the pointers to the RIBs was calculated, to determine the theoretical compression of the method. Entropy coding produced only slight improvement to the code. This is because a Loyd-Max quantiser was used rather than a linear one. Further work has shown that enhanced results could be achieved with a linear quantiser and efficient entropy coding.

This was done for each 120x120 segment giving the final coefficients and set of RIBs for the entire image.

5. TRANSFORM ORDER, BLOCK SIZE AND NUMBER OF BLOCKS

There are a number of parameters which determine the rate/distortion performance of this method including polynomial order, block size and RIB codebook size. The compression decreased rapidly when higher order transforms were used on small blocks. Similarly as the block size increased, so did the quantity of information transmitted to describe the RIBs. Reducing the number of RIBs used per section reduces the fidelity gradually until eventually the fidelity collapses, when only a few RIBs are transmitted per section.

The practical number of RIBs to transmit decreases from about 16 for 4x4 blocks to about 4 for 8x8 blocks. Although additional RIBs will always provide more accuracy, they also decrease the compression.

In this work, only zeroth and first order polynomials have been investigated. Results from fractal methods suggest that further improvement may occur at larger block size with higher order transforms [5]. Figure 2 shows the rate/distortion graph with varying numbers of RIBs, and block size using mainly (but not exclisively) the first order transform. For the standard CCITT test image Gold Hill, Figure 3 shows a fragment which is compared to other image coding techniques.

From the data it was clear that it was better to increase the block size than to reduce the transform order since there are very few order zero transforms on the best envelope line. This is because the first order transform is a very compact way of expressing shape. It was also evident that as block size increases, the performance begins to drop and the method starts to break down at 8x8 blocks. The overhead of transmitting the RIBs eventually outweighs the increase in compression gained by larger blocks. Also as the block size increased the accuracy of the polynomial approximation is reduced, but this is offset by the larger ratio of RIBs to original image blocks which were

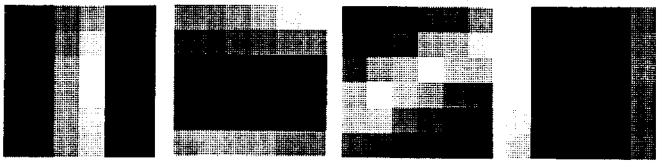


Fig 4 For the example of Fig 3, four 6x6 Residual Image Blocks are used, shown in normalised form.

transmitted.

6. CONCLUSIONS

As Figure 1 shows, this method gives a better rate/distortion performance than fractal transforms with fixed blocking over the image. This is because the RIBs can be selected from anywhere in the image without the overhead of describing their offsets.

Fisher [4] has shown that by using a variable blocking strategy, fractal transforms can outperform JPEG coding, which is based on the DCT. It seems probable that this method would similarly give superior results to JPEG when used with intelligent image partitioning strategies.

A number of extensions could be made:

1. Form fixed code books for block sizes of 6 and above, where the amount of data transmitted becomes significant.

2. Reduce the redundancy inside the lists themselves.

3. Subdivide the image into different sized blocks - such as by a quad tree.

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8. REFERENCES

1. Jacquin, A. E. : 'A novel fractal block-coding technique for digital images', Proc. ICASSP 1990, pp. 2225-2228.

2. Monro, D. M. : 'A hybrid fractal transform', Proc. ICASSP 1993, pp. V:1 69-192.

3. Monro, D. M. and Woolley, S. J. : 'Fractal image compression without searching', Proc. ICASSP 1994, pp. I:168-161.

4. Linde, Y. *et al* : 'An Algorithm for Vector Quantizer Design', IEEE Trans., Vol. 28, pp. 84-95 ,1980.

5. Fisher, Y. : 'Fractal image compression', Fractals, Vol. 2 No. 3 , pp. 347-363, 1994.

6. Wooley, S. J., and Monro D. M. : 'Optimum parameters for hybrid fractal image coding', Proc. ICASSP 1995 (in press).