FEATURE MEASUREMENT AND ANALYSIS USING GABOR FILTERS

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ABSTRACT

An innovative and powerful method is proposed for measuring physical parameters of lines using the responses from a bank of Gabor filters. These measurements are made without resorting to an image ruler. First the system is calibrated by establishing a relationship between the frequency of the Gabor filter and line length, then the length and angle of of isolated lines can be measured. A constraint on this method is that the lines in the scene need to be separated and isolated by a minimum distance. Results indicate that Gabor filters can be successfully applied to the measurement of geometric properties of objects, especially where Gabor filters are already being used for processing tasks. The best accuracies in terms of measurement error for the line length and angle measurements were 0.81% and 0.0% respectively.

1. INTRODUCTION

The Gabor filter was developed by D. Gabor [4] and used to define signals in both the frequency and time domains with minimum uncertainty. By applying methods from quantum mechanics, Gabor found that functions obtained by multiplying a complex exponential by a Gaussian having real variance achieves the smallest space or time bandwidth product permitted by the uncertainty principle of Fourier analysis.

Marcelja [5] was the first to introduce the Gabor filter to model the mammalian visual system. He realised that odd or even symmetric receptive field profiles of simple cortical cells approximated the elementary Gabor functions in the spatial domain. He also observed that the visual scene was analysed in terms of independent spatial frequency channels and that the cortical cells are tuned to specific spatial frequencies. Daugman [3] extended the original Gabor filter to a two-dimensional (2D) representation. Daugman also showed that the 2D Gabor filter meets the lower limit of the uncertainty principle of Fourier analysis.

Mehrotra *et al* [6] developed a step edge detector based on odd Gabor filters. The locations of the step edges were found from the local maximum in the absolute Gabor filter response. They showed that optimum performance was achieved with a Gabor filter oriented perpendicular to the step edge with $\sigma = 1/\omega$, and the width of the filter needs to be restricted to $2\pi/\omega$ to suppress all maxima and minima except for the one occurring at the step edge, where σ is the width of the Gaussian and ω is the radial frequency of the sinusoid. They also observed that the local maxima are located at the edge points and are independent of the scale of the Gaussian.

The Gabor filter has been used in applications ranging from texture analysis to image compression.

In this paper, we propose a new and accurate method using the Gabor filters to directly measure such metric properties of image features as length and angle information. Using filters to directly measure image information is inherently parallel and shift invariant. A benefit of using Gabor filters over more specific line-spread functions is that our aim here is to measure lines of arbitrary orientation, length and position in an image and, again, without any reference to an image ruler.

The Fourier spectrum has previously been used to measure the dominant direction of textured patterns as given in Bajcsy and Lieberman [1]. However, these Fourier domain paradigms are not suitable for our purposes as they often result in unreliable and inaccurate geometrical measurements due mainly to the low resolution of the fundamental frequency. These Fourier domain approaches are also unable to provide indices for individual lines.

The Gabor filter is a narrowband filter which can be oriented in any desired direction, where the width, frequency, and orientation of the filter can be adjusted. It is defined in the spatial domain as

$$g(x,y) = \exp\left\{-\pi(\frac{2}{3}f)^2(x^2+y^2) + \jmath 2\pi(ux+vy)\right\}$$

and the real part, used in this paper, is given by

$$g_r(x,y) = \exp\left\{-\pi(\frac{2}{3}f)^2(x^2+y^2)\right\}\cos 2\pi(ux+vy) \quad (1)$$

where $u = f\cos\theta$, $v = f\sin\theta$, f is the frequency and θ is the angle of the Gabor filter. The response of the real and imaginary components of the Gabor filter are identical except for a phase difference of 90°. In this form of Gabor filter, the spread of the Gaussian is proportional to $1/f$. This results in only a fixed number of oscillations appearing in the window, thus the number of oscillations within the window is independent of the frequency. The size (bandwidth), shape (symmetric or elongated) and location of the filter (in the spatial frequency domain) can be adjusted by the choice of σ, f, and θ parameters. An example of the Gabor filter, oriented at 45°, in the spatial domain is shown in Figure 1.

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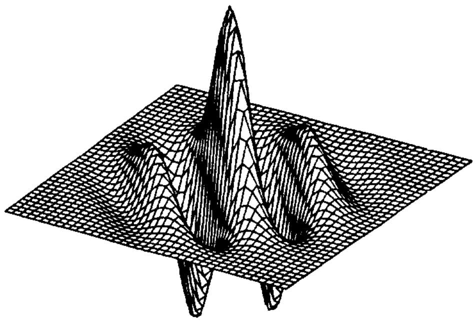

Figure 1: Spatial domain response of a 2D Gabor filter oriented at 45°.

2. MEASUREMENT METHOD

In this section we describe the method used to measure line lengths and angles of single, multiple, and connected lines in an image. We first describe a method of measuring the length of a single isolated line, then extend the method to measuring the angle of the line.

2.1. Length-Frequency Relationship

The Gabor filter is well suited to detecting lines in images as it will respond to changes, but not to areas of constant amplitude so long as $\int_{\tilde{u}} g(\tilde{u})\,\mathrm{d}\tilde{u} = 0$. For the case of a single line, say oriented at 0°, a Gabor filter oriented at 0° will produce responses at the two ends of the line, and a Gabor filter oriented at 90° will produce responses along the top and bottom edges of the line. It is the responses produced by the step edges at the ends of the line that we utilise in the measurement of the line length and angle.

A line can be represented by the interaction of two step functions. In the 2D case, the line (having thickness) and can be described by $\mathcal{L}(x,y) = u(x-\tau_{x1}, y-\tau_{y1}) - u(x-\tau_{x2}, y-\tau_{y2})$, where $u(\cdot)$ is the step function and τ is the delay of the step. The length is given by $\tau_{x2}-\tau_{x1}$ and the width given by $\tau_{y2}-\tau_{y1}$, for the case when the angle of the line is 0° with respect to the x axis. Applying the Gabor filter to this line image is described by $r(x,y) = g_r(x,y) * \mathcal{L}(x,y)$, where $\mathcal{L}(x,y)$ is the line image, $g_r(x,y)$ is a Gabor filter given by (1), and $*$ represents the convolution operation. This results in

$$
\begin{aligned}
r(x,y) \;=\; & \int_{-\infty}^{x-\tau_{x1}} \int_{-\infty}^{y-\tau_{y1}} g(\alpha,\beta)\,\mathrm{d}\beta\,\mathrm{d}\alpha \;- \\
& \int_{-\infty}^{x-\tau_{x2}} \int_{-\infty}^{y-\tau_{y2}} g(\alpha,\beta)\,\mathrm{d}\beta\,\mathrm{d}\alpha \qquad (2)
\end{aligned}
$$

where α and β are dummy variables.

Depending on the length of the line and the frequency and angle of the Gabor filter, there will be an interaction between the two Gabor step responses, as can be seen in

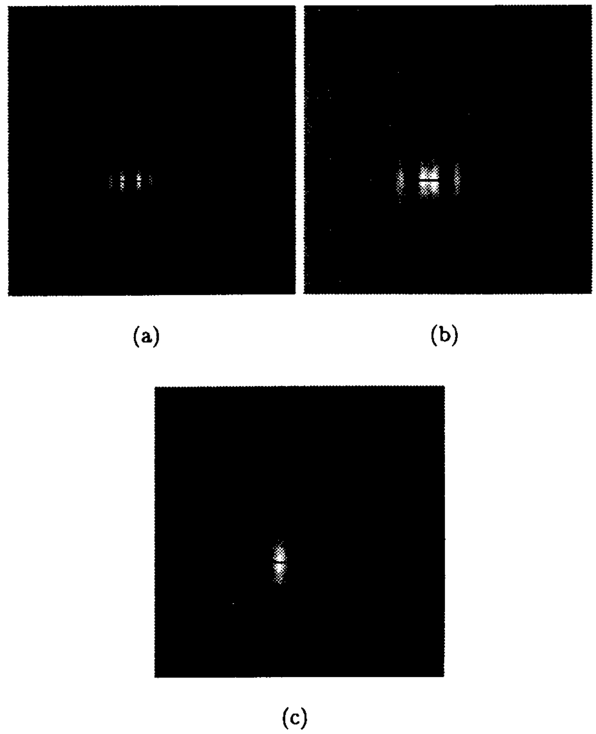

Figure 2: Response of the line image being filtered by the Gabor filter. (a) No interaction between the two step responses is taking place. (b) Gabor frequency corresponding to the minimum response point, f_d in Figure 3. (c) Gabor frequency corresponding to the maximum response point, f_c in Figure 3.

Figure 2. Due to the frequency dependent aperture size of the Gabor filter used, the interaction between the two step responses will be predominantly between the maximum positive and maximum negative and the first pair of oscillations. The other oscillations will have virtually no effect as they will be severely attenuated. When the frequency is sufficiently high (Figures 2a), there will be no interaction between the two step responses. As the frequency is reduced, destructive interaction between the two step responses occurs and a minimum, f_d (Figure 2b), results when the positive peak coincides with the first negative peak. When the frequency is at a point where the interaction between the two step responses causes the two positive peaks to constructively interact, there will be a maximum in the response, f_c (Figure 2c).

The effect of this interaction can be seen in Figure 3 which shows the peak response as the frequency is varied. A maximum and minimum correspond to the maximum constructive (at f_c) and destructive (at f_d) interferences respectively. This infers that the frequency corresponding to either of these two response points can be related to the length of the line, as these are unique points.

To find a relationship between the frequencies f_c or f_d and the length of the line, l, the following equations need

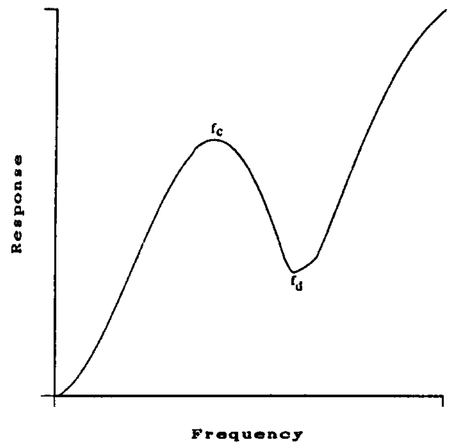

Figure 3: Maximum response in the line image convolved with Gabor filters. (Note f_d does not correspond to the fundamental frequency of the line).

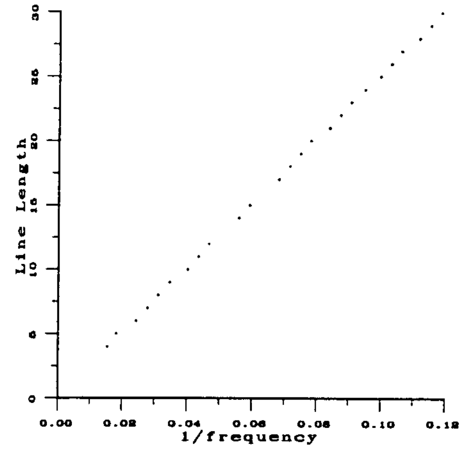

Figure 4: Relationship between the reciprocal of the frequency which generated the local minimum shown in Figure 3, $(1/f_d)$ and line length.

to be solved:

$$\frac{\partial r_i(x,y)}{\partial x} = 0, \quad \frac{\partial r_i(x,y)}{\partial y} = 0$$

which will determine the maximum response in the filtered image, giving $r_i(x_m, y_m)$ where x_m, y_m correspond to the maximum or minimum in Figure 3. Then

$$\frac{\partial r_i(x_m, y_m)}{\partial f_i} = 0$$

which determines the maximum and minimum points in the curve, and thus the relationship between the frequency and the line length. These operations are performed when the angle of the Gabor filter equals the angle of the line. There is no closed form solution resulting from these equations.

This relationship needs to be obtained through a numerical method. The operation is part of the calibration process and only needs to be performed once. The procedure used to find the minimum, f_d (the minimum is used as opposed to the maximum as will be explained in Section 2.2) was to iteratively search for the minimum in the response obtained by finding the maximum value from each filtered image. This is performed for a range of line lengths, and the actual line length is plotted against $1/f_d$ (as the required relationship is a distance) as shown in Figure 4. A least squares fit through the data points yields the relationship between the frequency and the line length. As indicated in Figure 4, the calculated minima fits the line extremely well, indicating the direct and linear relationship between l and $1/f$, and is given by

$$l = \frac{0.9866N}{f} \tag{3}$$

where N is the size of the image. The closeness of the least squares line to the data points is 99.9926%, thus indicating the high accuracy of the relationship.

2.2. Measurement of Line Length and Angle

The line length and angle can be determined by analysing the bank of Gabor filter responses. Figure 5 shows plots of the maximum responses from a full bank of Gabor filters applied to a line.

As can be seen from in Figure 5, there is a ridge, corresponding to f_c in Figure 3, followed by a distinctive local minimum, corresponding to f_d in Figure 3. Thus the reason for selecting f_d as the measurement point is that it is far easier to detect and calculate the minimum point of the local minimum than the corresponding point, f_c, on the ridge.

The absolute minimum point is calculated by first locating the minimum point of the local minimum in the matrix of response values, then fitting a paraboid of the form $(x+a)^2 + (y+b)^2 + c$ using a least squares fit to a small area around this point. The location (frequency and angle) of the absolute minimum can then be estimated from the values of a and b. Thus the angle of the line will be immediately extracted and the length of the line calculated from the frequency/length relationship. The relationship determined between l and $1/f$ is only valid when the angle is aligned with the Gabor angle.

This method produces excellent results as the surface around the local minimum is very smooth and reasonably symmetrical. Table 1 gives some examples of measurements made for various line lengths and angles. The majority of the measurement error is due to the discretisation of the lines in the rectangular grid, where they consist of 0°, 45°, 90° and 135° segments, thus resulting in jagged lines consisting of segments of the above angles. The most accurate results, for any length of line, are for the straight lines.

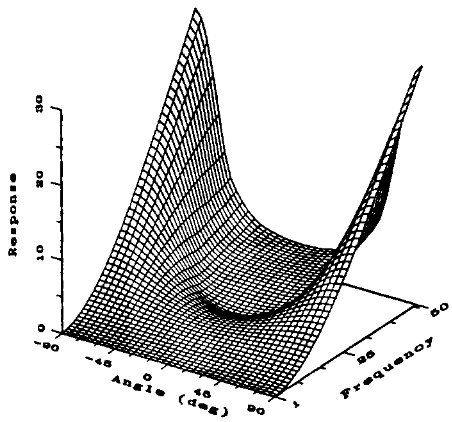

Figure 5: Maximum response from a bank of Gabor filters convolved with a line 10 pixels long and an angle of 0°.

Discretised		Measured			
ang	len	ang		len	
		value	%err	value	%err
0.00	10.00	0.00	0.00	9.85	1.50
0.00	20.00	0.00	0.00	19.75	1.25
19.18	24.35	18.09	5.68	24.48	4.84
30.26	13.89	27.45	9.29	13.00	6.41
29.74	24.19	28.52	4.10	24.49	1.24
45.00	9.90	45.00	0.00	9.82	0.81
45.00	24.04	45.00	0.00	23.78	1.08
59.53	19.72	63.78	7.14	18.90	4.16
60.26	24.19	61.40	1.89	24.49	1.24
81.03	19.24	84.53	4.32	18.98	1.35
80.54	24.33	83.96	4.25	24.11	0.90

Table 1: Results of measurements for various single line lengths and angles, minima found through a surface fitting method (angles specified in degrees, line lengths specified in pixels)

2.3. Separation between Lines

When more than one line appears in the image, it is necessary to maintain a minimum separation between these lines to prevent interactions between the responses from causing false maxima and minima. The two cases where maximum interference occurs is when two lines are parallel and their centres are aligned vertically, and when the two lines are aligned horizontally. In both of these cases, the two lines need to be separated by a minimum distance of the spread of the response.

The spread of the response is defined as the distance that is twice the distance from the centre of the step edge response to the second zero crossing. This distance is sufficient to prevent any interference between responses from different lines. The spread can be approximated by (see [2] for derivations)

$$S_t = \frac{2N}{f}. \quad (4)$$

3. CONCLUSIONS

We have proposed a measurement procedure to determine the line parameters (length and angle) by using only Gabor filters, and without the application of any Euclidean geometry. The processing time to determine the parameters is minimal as the bulk of the computational effort is associated with applying the bank of Gabor filters to the image.

The measurement process first needs to be calibrated to determine the relationship between the Gabor frequency and the line length. This would require recalibration if the constants associated with the Gaussian, the form of the Gabor filter, or the image size was changed.

The accuracy of the measurement is determined by the accuracy of the calibration constant, the approximation used in finding the minimum by fitting a paraboid to an area around where the discrete minimum was located, and the fact that the lines are discretised onto a rectangular grid. The limitation of this method is that the lines need to be isolated and separated by a small distance to prevent interaction between the different responses. When this limitation is considered, the method produces accurate measurements of line lengths and angles. It is also important to consider that the width of the line does not affect the measurement of its length.

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