EXPERIMENTAL EVALUATION OF CUMULANT-BASED CLASSIFIERS ON NOISY IMAGES

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ABSTRACT

The paper describes a cumulant-based classifier whose discrimination criterion exploits statistical signal characteristics of higher order than the second one. The performances of the classifier were tested on texture images. In texture classification, the discrimination criterion is usually based on structural characteristics (edge density, co-occurrence matrix) or on statistical parameters (Gaussian-Markov random fields, fractal dimension) of sample textures. As an alternative to statistical approaches, in this paper a third-order cumulant-based criterion is applied and the classifier's performances on images affected by different types of noise are assessed.

1. INTRODUCTION

In recent years, the problem of classifying or detecting signals or objects corrupted by noise has been one of the most extensively investigated in the area of signal processing. The general problem formulation can be reduced to a test of K hypotheses, and, in the case of additive noise, can be expressed as:

$$H_k:\ x(i)=n(i)+s_k(i) \qquad i=1...N;\ k=1...K \tag{1a}$$

where n(i) denotes noise samples, $s_k(i)$ is the signal template belonging to the class k, and x(i) are observation samples. For K=2, the formulation reduces to the following detection problem:

$$\begin{aligned} H_0&:\ x(i)=n(i) \\ H_1&:\ x(i)=n(i)+s(i) \qquad i=1...N \end{aligned} \tag{1b}$$

Under the assumptions of Gaussian noise and Gaussian signals, energy detectors, that perform a Maximum Likelihood (ML) test, turn out to be optimal classification/detection schemes. They become suboptimal if signals are non-Gaussian. To overcome this drawback, schemes based on higher-order statistics (HOS) have been proposed [1], [2] and [3]. Insensitivity to additive Gaussian noise and the feature of preserving phase information (i.e., sensitivity to signal shape) make HOS classification schemes attractive alternatives to energy detectors in all cases where signal spectrum characteristics do not provide sufficient information to discriminate between the hypotheses H_k. Within this context, the asymptotic theory for HOS estimators, as devised by Brillinger and Rosenblatt [4], is of notable importance, as it allows one to consider an HOS classifier/detector within the framework of an optimal decision rule that can be formulated as a hypothesis test and solved by means of the ML criterion. On the basis of the general aspects of this theory, the paper describes a cumulant-based classifier whose discrimination criterion exploits statistical signal characteristics of higher order than the second one. We discuss the performances of the classifier in two test cases.
In the first case, a detection problem is faced; the second test case concerns texture images. In texture classification, the discrimination criterion is usually based on the structural characteristics (edge density, co-occurrence matrix) or on the statistical parameters (Gaussian-Markov random fields, fractal dimension) of sample textures. As an alternative, we used a third-order cumulant-based criterion and evaluated its performances when we applied it to images affected by different types of noise.

2. FROM ASYMPTOTIC PROPERTIES OF HOS ESTIMATORS TO THE ML CLASSIFICATION RULE

When a non-Gaussian signal is corrupted by additive Gaussian noise, the observation vector $x(i)$ in (1a) can be replaced with the Mx1 vector $\hat{f}_M$, containing all estimated

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cumulant lags. Under the K-hypothesis, this vector behaves as [4]:

$$H_k:\ \hat{f}_M \xrightarrow{distr} N(f_{k,M}, \Sigma(H_k)) \quad (2)$$

In detection problems, (1b) reduces to:

$$\begin{aligned} H_0:\ \hat{f}_M &\xrightarrow{distr} N(0, \Sigma(H_0)) \\ H_1:\ \hat{f}_M &\xrightarrow{distr} N(f_M, \Sigma(H_1)) \end{aligned} \quad (3)$$

Thus, if we replace the non-Gaussian vector $x(i)$ with $\hat{f}_M$, we can still apply an optimal detection scheme, provided that a sufficient number of samples are available. From (2) or (3), a ML classifier can be derived by performing the following test:

$$\ln p(\hat{f}_{k,M} / H_k) \underset{<_{H_l}}{\overset{>^{H_k}}{}} \ln p(\hat{f}_{l,M} / H_l) \quad (4)$$

Replacing $\hat{f}_M$ with third-order cumulant samples, estimated for zero-mean processes as follows:

$$\hat{c}_3^{\,x}(\tau_1,\tau_2) = \frac{1}{N} \sum_{i=1}^{N-1-\tau_1} x(i)\cdot x(i+\tau_1)\cdot x(i+\tau_2) \qquad 0 \le \tau_1 \le \tau_2 \le N-1$$

we can implement a log-likelihood test. Alternatively, we can avoid ill-conditioning and increase computational efficiency, by reducing the above classifier/detector to a minimum distance or minimum HOS-energy classifier/detector by setting $\hat{\Sigma}(H_k)$=I. We choose H_k iff

$$\left|\hat{c}_3^{\,x} - \bar{c}_3^{\,s_k}\right|^2 < \left|\hat{c}_3^{\,x} - \bar{c}_3^{\,s_l}\right|^2 \quad (5)$$

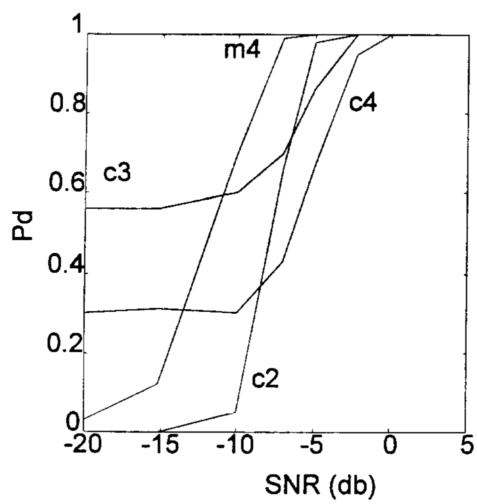

Fig.1: Detection probability vs. SNR for the four statistics considered.

3. EXPERIMENTAL RESULTS

Test Case #1:

We use the detection scheme defined by relation (4) and choose, as the vector $\hat{f}_M$, the statistics: $\hat{c}_2, \hat{c}_3, \hat{m}_4, \hat{c}_4$ evaluated at zero-lag sample a:

$$c_2^{\,x}(0) = R_x(0) = E\{x(k)^2\}$$

$$c_3^{\,x}(0,0) = E\{x(k)^3\}$$

$$m_4^{\,x}(0,0,0) = E\{x(k)^4\}$$

$$c_4^{\,x}(0,0,0) = E\{x(k)^4\} - 3 \cdot c_2^{\,x}$$

Test (4) was performed for different SNR values, different Gaussian noise, and exponentially distributed signal realizations (100). The detection probability vs. SNR is shown in Fig.1

Test Case #2:

The first five images shown in Fig.2 were used for this classification test. Each 256x256 image was subdivided into four 128x128 subimages. We presented each subimage corrupted by noise to the classifier, after training it (i.e., after estimating the variances and expected values in relation (4)) by using the remaining 19 images (leave-one-out-strategy). In the various tests, we superimposed different types of noise, with 100 independent realizations for each SNR value selected (Fig. 3). The classification results obtained by third-order cumulant samples of the set $\{\tau_1=(\tau_{11},\tau_{12})$ and $\tau_2=(\tau_{21},\tau_{22}),\ 0\le \tau_{21},\tau_{22} \le 3;\ \tau_{11}=0;\ \tau_{12}=0,1\}$ were compared with the autocorrelation results obtained using 36 lags of the set $\{0 \le \tau_1, \tau_2 \le 5\}$ (Tables 1 and 2).

4. CONCLUSIONS

Cumulant-based classifiers exploit HOS features, rather than spectral information, in discriminating between classes. For 1-D signals, they are particularly suited for detection of non-Gaussian signalscorrupted by Gaussian noise. For 2-D signals, they can be successfully applied to solve problems for which the statistical content is an important discriminating feature. However, HOS estimators generally present higher variances than 2nd-order estimators; therefore, larger sets are required to train HOS classifiers. Furthermore, computational efforts allow only some lags of cumulants to be computed in practice, thus the HOS potentiality is exploited only partially. These drawbacks make HOS classifiers' performances comparable to those of energy-based classifiers, in some practical cases where few samples are available. A discrimination criterion able to assign an efficiency weight to each cumulant sample can be adopted to select a limited number of significant features, without affecting correct classification results. However, a considerable improvement is obtained by HOS when classification is performed on noisy signals, thanks to HOS insensitivity to Gaussian noise.

The points that remain to be investigated are: a criterion to estabilish what and how many lags should be chosen; the invariance of cumulants to displacements, scalings and rotations; the possibility of building an a-priori signal model for an analytical computation of all cumulants.

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Table 1: Correct Classification results for $\hat{c}_3^x$

SNR (dB)	Gaussian Noise	Coloured Gaussian Noise	Exponential Noise	Coloured Exponential Noise
-15	61.40	50.36	39.50	20.00
-10	88.90	77.86	77.50	20.00
-5	97.14	96.07	97.25	55.25
0	100.00	99.23	99.50	97.50
5	100.00	100.00	100.00	100.00

Table 2: Correct Classification results for $\hat{c}_2^x$

SNR (dB)	Gaussian Noise	Coloured Gaussian Noise	Exponential Noise	Coloured Exponential Noise
-15	20.00	20.00	20.00	20.00
-10	38.75	20.00	32.76	20.00
-5	98.75	96.25	99.33	95.00
0	100.00	99.23	100.00	100.00
5	100.00	100.00	100.00	100.00

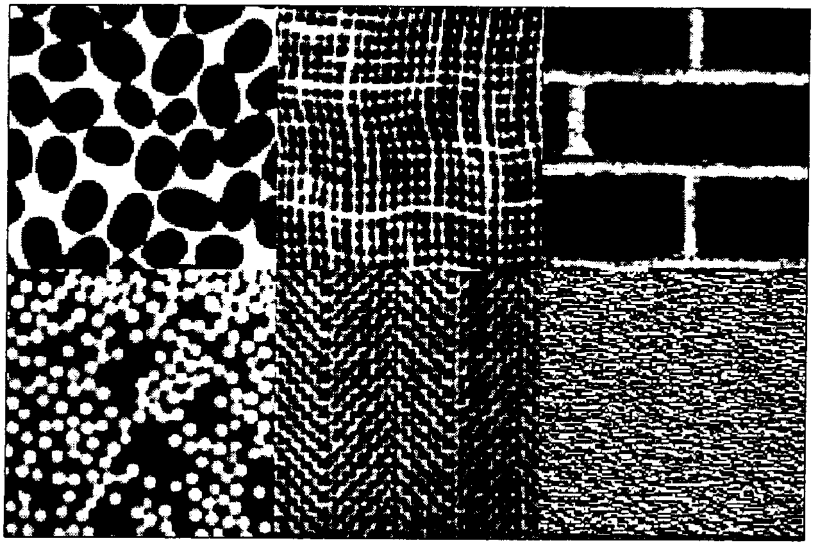

Fig. 2: Test images. From left to right: coffee, cloth, wall, naphtha, tweed and synthetic noise.

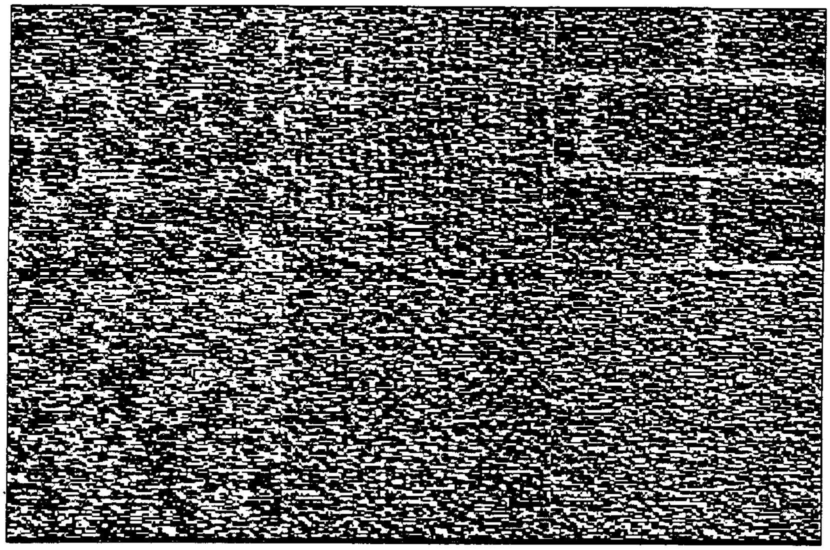

Fig. 3: Test images corrupted by coloured exponential noise. SNR=-10 dB