ENTROPY CONSTRAINED HALFTONING BY TREE CODING

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ABSTRACT

An entropy constraint is introduced into a tree coding halftoner, so that one can control the degree of compressibility of the bi-level output images. The algorithm essentially trades image quality with compressibility as indicated by rate distortion theory. We demonstrate that this algorithm can generate halftone images that are of higher quality than error diffusion, and yet are also more amenable to compression than error diffused images.

1. INTRODUCTION

The goal of image halftoning is to produce a bi-level image $b_{m,n}$ from a continuous tone image $x_{m,n}$ so that both images appear similar when viewed from a distance. Well known halftoning methods can be classified into three categories: ordered dithering [1], error diffusion [2,3], and optimization based techniques [4–8]. In [8], an optimization based halftoning algorithm using the concept of tree coding is proposed, that minimizes a mixture distortion criterion and produces an optimized halftone output. The tree coding based halftoning algorithm interprets the halftone output as a binary tree, and is similar to the Viterbi decoding algorithm [9] in that it looks a predetermined number of steps into the future before making a decision for each pixel location. As a result, the usual disadvantages of greedy optimization can be alleviated, and hence better halftone images can be produced.

Popular halftoning algorithms in the literature [1–8] often focus on the tradeoff between complexity and image quality. It is well known that high quality halftones produced by either error diffusion or optimization techniques are typically not amenable to lossless compression. To solve this problem, we introduce in this paper an entropy constraint into a recently proposed tree coding based halftoning algorithm [8], and consider explicitly the tradeoff between halftone compression and image quality. This problem is particularly important for rendering or printing halftones in high resolution ($\geq$ 600 dots per inch) devices, where the communication time for transmitting halftone images from a computer to the device is often substantial. In Section 2, we briefly review the tree coding halftoner. The entropy constrained tree coding halftoner is described in Section 3. Experimental results are also presented. Section 4 summarizes the results of this paper.

2. TREE CODING HALFTONER

A mixture distortion that uses a combination of frequency weighted mean square error and distances between minority pixels in a halftone image is proposed in [8]. Specifically, the distortion has the form

$$e_{m,n} = (x_{m,n} - (v * b)_{m,n})^2 + \gamma u_{m,n}$$

where $v_{m,n}$ is a low pass filter that approximates the characteristics of the human visual system, $*$ denotes convolution, γ is a weighting parameter, and $u_{m,n}$ is a distortion measure based on minority dot distances [8].

$$u_{m,n} = \begin{cases} 0 & \text{if } d_{m,n} \geq d_p(x_{m,n}) \\ & \quad \text{and } b_{m,n} = \rho_{m,n} \\ 0 & \text{if } d_{m,n} < d_p(x_{m,n}) \\ & \quad \text{and } b_{m,n} \neq \rho_{m,n} \\ \left(\dfrac{d_p(x_{m,n}) - d_{m,n}}{d_p(x_{m,n})}\right)^2 & \text{otherwise.} \end{cases} \tag{1}$$

In (1), $\rho_{m,n}$ is the value of minority pixel at location (m,n), $d_p(g)$ is the principal distance [10] between minority dots at the gray level g given by

$$d_p(g) = \begin{cases} \sqrt{1/g} & \text{if } 0 \leq g < 0.5 \\ \sqrt{1/(1-g)} & \text{if } 0.5 \leq g < 1, \end{cases}$$

and $d_{m,n}$ is the actual distance from location (m,n) to the nearest minority pixel.

The tree coding based halftoning algorithm is then used in conjunction with this distortion measure to generate halftones such as the image in Fig. 1. Using a standard JBIG (lossless) coder [11], one can encode

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Figure 1: Halftone generated using the tree coding halftoner with no entropy constraint. The compression ratio achieved using a JBIG coder is 1.52. The printing resolution is 150 dpi.

Figure 2: Halftone generated using Floyd-Steinberg error diffusion. The compression ratio achieved using a JBIG coder is 1.89. The printing resolution is 150 dpi.

this image to a compression ratio of 1.52. Experimental results using other images give similar compression performance. As a reference, Fig. 2 shows a halftone generated using standard Floyd-Steinberg error diffusion [2], where one can encode to a compression ratio of 1.89 using a JBIG coder. The image of Fig. 1 is apparently of higher quality than that of Fig. 2, particularly in the rendering of details. The error diffusion version, however, is more amenable to compression. If we interpret the halftoning procedure as a constrained output alphabet compression problem, one can explain this observation using rate distortion theory [12], in that one can trade distortion with rate.

3. A TREE CODING HALFTONER WITH ENTROPY CONSTRAINT

Although the tree coding algorithm is capable of generating high quality halftones, it is desirable to improve on the compression performance for the resulting bi-level images. This is not surprising because the optimization procedure in [8] has been performed to solely minimize distortion. Note that the tree coding based halftoner can be viewed as a lossy encoder for the continuous image, where the output alphabet is constrained to be binary. As indicated by Shannon's rate-

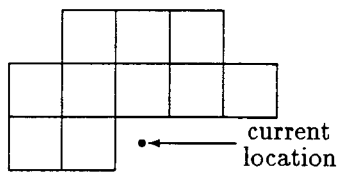

Figure 3: Template of previous output pixels used as a context for computing conditional entropy.

distortion theory [12], one can trade distortion for compression performance in any lossy coder. To this end, one formulates the optimization problem by adding an entropy constraint to the cost function as

$$J_{m,n} = e_{m,n} + \lambda h_{m,n}(c_{m,n}) \qquad (2)$$

where $h_{m,n}(\cdot)$ is an entropy measure at pixel location (m, n), and $c_{m,n}$ is a context [13] defined by a window of neighboring pixels. The parameter λ determines the location of the resulting halftone on the *operational* rate-distortion function, and has an interpretation of the gradient of the convex hull supporting the operational rate distortion function.

To ensure good performance at a reasonable complexity, we compose the context using a template of 10 previous output pixels as shown in Fig. 3. As a result, there are 1024 different possible values for the context. Since each output pixel is necessary binary, the conditional entropy is completely determined by

$$p_{m,n}(\beta, c_{m,n}) = \Pr\{b_{m,n} = \beta | C_{m,n} = c_{m,n}\} \qquad \beta = 0, 1.$$

Similar to what is done in adaptive arithmetic coding [13], these probabilities are estimated using the statistics of the past output pixels, and are continuously updated as the halftone image is being generated. More specifically, we use the estimate

$$\widehat{p}_{m,n}(\beta, \gamma) = \frac{N_{m,n}(\beta, \gamma) + 1}{N_{m,n}(\gamma) + 2} \qquad \beta = 0, 1, \qquad (3)$$

where $N_{m,n}(\gamma)$ is the frequency count of the context taking on the value γ upto the location (m, n), *i.e.*, the number of times that the context γ has occurred; and $N_{m,n}(\beta, \gamma)$ is the frequency count of the context γ and output β, *i.e.*, the number of times that the context γ has occurred together with the output value β. It is evident that

$$N_{m,n}(0, \gamma) + N_{m,n}(1, \gamma) = N_{m,n}(\gamma).$$

The bias values of "1" and "2" in (3) have been inserted to avoid the situation of division by zero when initially there is no data. It also has the property that when there is no data, we have $N_{m,n}(0, \gamma) = N_{m,n}(1, \gamma) = 0.5$, *i.e.*, neither 0 nor 1 is favored in such case.

During the halftoning procedure, we update the statistics at each pixel location after the decision on the value of $b_{m,n}$ has been made. Instead of (2), we use at each pixel location the cost function

$$\widetilde{J}_{m,n} = e_{m,n} + \lambda \log(p_{m,n}(b_{m,n}, c_{m,n})), \qquad (4)$$

where it is well known that in the long run, the term $\log(p_{m,n}(b_{m,n}, c_{m,n}))$ on the average approximates the average length of a codeword required to describe $b_{m,n}$. Using (4) and the tree coding algorithm, we can generate halftones with varying degrees of compressibility by changing λ.

The two images of Figures 4 and 5 are halftones generated at two different values of λ. Using a standard JBIG encoder, we can achieve a compression ratio of 2.09 for the image of Fig. 4, and 2.35 for that of Fig. 5. As a reference, halftones generated by error diffusion can be compressed using the same coder at compression ratios near 1.9. The image quality of error diffusion, however, is inferior to the halftone image of Fig. 4, which is more amenable to compression than error diffused images. It is evident that as the compression ratio improves, the distortion also increases as predicted by rate distortion theory [12]. Fig. 6 shows the plot of mixture distortion versus the average bit rate required for a JBIG encoder to encode the output generated by the entropy constrained tree coding halftoner. The curve exhibits the usual form as predicted by rate distortion theory.

Figure 4: Halftone generated using the entropy constrained tree coding halftoner. The compression ratio using a JBIG coder is 2.09. The printing resolution is 150 dpi.

Figure 5: Halftone generated using entropy constrained tree coding halftoner. The compression ratio using a JBIG coder is 2.35. The printing resolution is 150 dpi.

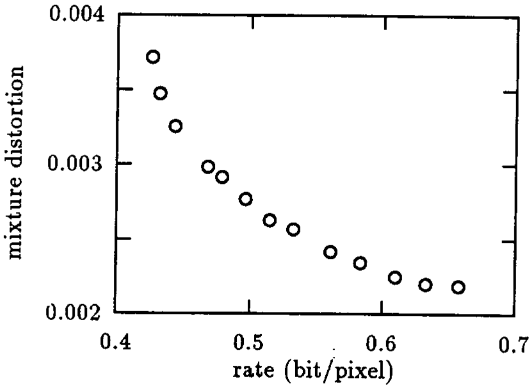

Figure 6: Distortion-rate performance of the entropy constrained tree coding halftoner.

4. DISCUSSION

We have proposed an entropy constrained halftoning algorithm by introducing an entropy constraint in a tree coding halftoner. This algorithm enables one to optimally trade image quality with compression performance, allowing halftones to be transmitted through rate constrained communication channels.

5. ACKNOWLEDGMENT

The author gratefully acknowledges the help of Dr. Gregory Yovanof, who supplied an experimental JBIG coder for testing the compression performance of the halftones.

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