SAR MOVING TARGET DETECTION AND IDENTIFICATION USING STOCHASTIC GRADIENT TECHNIQUES

S. Young, N. Nasrabadi and M. Soumekh
Department of Electrical & Computer Engineering
State University of New York at Buffalo
Amherst, New York 14260

ABSTRACT

This paper presents methods for detecting and identifying moving targets in a Synthetic Aperture Radar (SAR) scene. An analytical expression is derived for the coherent SAR signature of a target. SAR system model of a moving target is developed. These principles are then used to construct a SAR signal statistic (energy function) in a parameter space which is defined by the target's coordinates, speed, and coherent SAR signature. Stochastic gradient techniques are used to search for the maximum point of this energy function which is located at the desired target's parameters.

I. INTRODUCTION

This paper presents a Fourier-based multidimensional signal processing framework for high resolution UHF SAR moving target detection-identification. The dangers encountered by Scud missiles in the Gulf War created the need for developing FOliage PENetrating (FOPEN) SAR systems that utilize wideband UHF-VHF radars for detection and identification of concealed stationary and moving targets.

Our formulation brings out the special role of the target's radiation pattern as well as the radar's radiation pattern in constructing the SAR signal's signal subspace, and its Fourier properties (Section II). This will help us to develop the Fourier properties of a target's coherent SAR signature. This is the key discriminant for detecting and identifying man-made targets with FOPEN UHF SAR systems.

Next, we examine the effects of target motion in the SAR system model (Section III). The classical radar problem of detecting and estimating the range and speed of a moving target in the *one-dimensional* spatial domain is based on a two-dimensional reconstructed image or ambiguity function in the (x, v_x) domain. The problem that we encounter in reconnaissance with FOPEN SAR is to detect and estimate the location and velocity of a moving target in the two-dimensional (slant-range and cross-range) domain. One may intuitively rationalize that this SAR problem requires a *four-dimensional* reconstructed image or ambiguity function in the (x, y, v_x, v_y) domain. However, whether a target in a SAR scene is stationary or moving, its *relative* motion trajectory with respect to the radar can be modeled by at most *three* parameters [7] that we refer to as the (X, Y, α) parameters in our future discussion.

Based on this model, we show that a three-dimensional image of the moving target scene in the (X, Y, α) domain can be formed via the accurate and fast Fourier-based SAR reconstruction algorithm (Section IV). The SAR system model for moving targets may also be used to develop a statistic (energy function) from the measured SAR signal (Section V). Maximizing this energy function in the target parameter space yields the motion parameters and coherent SAR signature of a suspected target. Stochastic gradient techniques is used to search for the maximum point of the energy function.

II. COHERENT SAR SIGNATURE OF TARGETS

The type as well as shape of a target determines its interaction with the radiated radar signal. For instance, the characteristics of the SAR signature of a man-made metallic cylinder is quite different from those of a tree with the same size at low frequencies of the UHF band [3]. *This difference is a key feature which can be used to discriminate a man-made structure from foliage.*

Why not use shape information to distinguish targets in UHF SAR images? At the UHF frequencies, the resolution of a reconstructed SAR image is poor. In this case, the SAR *magnitude reconstruction* of both man-made targets and foliage (tree trunks) appear as *blobs*. Thus, there is not much *shape* information that may be used as discriminant. A target's *coherent* SAR radiation pattern is the signature that can be used for target detection and identification in reconnaissance with FOPEN SAR. We now present a study to quantify the coherent SAR signature of targets. In our present discussion, the target is assumed to be stationary.

A target exhibits an *amplitude pattern*, which contains *phase* as well as *magnitude*, when illuminated by a SAR system [5,6,8]. *A target's amplitude pattern varies with the radar's fast-time frequency and aspect angle.* For a radar located at $(0, u)$ and a target located at (x_n, y_n), we denote the target's amplitude pattern with $a_n(x_n, y_n - u, \omega)$, where $\arctan(\frac{y_n - u}{x_n})$ is the target's aspect angle with respect to the radar's broadside.

Coherent SAR signature of the n-th target is its contribution in the total echoed signal from the target scene. When the radar is located at $(0, u)$ and fast-time frequency ω, the coherent SAR signature of the n-th target can be expressed via the following [6,7]:

$$h_n(x_n, y_n - u, \omega) = \underbrace{a_n(x_n, y_n - u, \omega)\ a(x_n, y_n - u, \omega)}_{Amplitude\ Function\ (AM)}$$

$$\times \underbrace{\exp\left[-j2k\sqrt{x_n^2 + (y_n - u)^2}\right]}_{Spherical\ Phase\ Function\ (PM)},$$

where $a(x, y, \omega)$ is the transmit-receive mode amplitude pattern of the radar antenna.

Based on the Fourier properties of AM-PM signals, we have the following slow-time Fourier transform for the transmit-receive mode radar-target radiation pattern:

$$\mathcal{F}_{(u)}\left[h_n(x_n, y_n - u, \omega)\right]$$

$$= A(k_u, \omega)\ A_n(k_u, \omega)\ \exp(-j\sqrt{4k^2 - k_u^2}\ x_n - jk_u y_n),$$

where the target and radar amplitude patterns in the u and k_u domains are related via the following:

$$a_n(x_n, y_n - u, \omega) = A_n\left[2k \sin\theta_n(u), \omega\right]$$

$$a(x_n, y_n - u, \omega) = A\left[2k \sin\theta_n(u), \omega\right],$$

where

$$\theta_n(u) \equiv \arctan\left(\frac{y_n - u}{x_n}\right),$$

is the n-th target's *aspect angle* when the radar is located at $(0, u)$. Note that the amplitude functions $a(\cdot)$ and $A(\cdot)$ are *scaled* transformations of each other, and not Fourier transform pairs.

III. SAR SIGNAL MODEL FOR A MOVING TARGET

Next, we examine the effect of target motion in the SAR's spherical phase function (i.e., phase history). We denote the speed of the airborne aircraft which carries the radar with v_R. Suppose the velocity vector for the n-th target is

$$(v_{xn}, v_{yn}) = (a_n\ v_R\ ,\ b_n\ v_R),$$

in the spatial (x, y) domain; (a_n, b_n), the target's relative velocity with respect to the radar, is unknown. Thus, the target's distance from the radar at the slow-time u is

$$\sqrt{(x_n + a_n u)^2 + [y_n + (b_n - 1)u]^2}.$$

In this case, the target's SAR signature in the (u, ω) domain is [for notational simplicity, the amplitude patterns $a(\cdot)$ $a_n(\cdot)$ and $P(\omega)$ are suppressed, and $(b_n - 1)$ is replaced with b_n, that is, the motion parameters of a stationary target is $(a_n, b_n) = (0, -1)$]

$$s_n(u, \omega) = \exp\left[\ -j2k\sqrt{(x_n + a_n u)^2 + (y_n + b_n u)^2}\ \right]$$

$$= \exp\left[\ -j2k\sqrt{r_n^2 + 2(a_n x_n + b_n y_n)u + (a_n^2 + b_n^2)u^2}\ \right] \quad (1)$$

where $r_n \equiv \sqrt{x_n^2 + y_n^2}$.

Note that the target's motion trajectory can be uniquely identified via three parameters: r_n, $a_n x_n + b_n y_n$, and $a_n^2 + b_n^2$. In fact, the generalized SAR/ISAR target motion trajectory with respect to the radar can be expressed via the following:

$$\sqrt{X_n^2 + (Y_n + \alpha_n u)^2} = \sqrt{X_n^2 + Y_n^2 + 2\alpha_n Y_n u + \alpha_n^2 u^2}.$$

Equating this model to the SAR signal in (1) for the n-th target yields

$$\text{Radial range}: \qquad \sqrt{X_n^2 + Y_n^2} \equiv r_n$$

$$\text{Squint cross-range}: \qquad \alpha_n Y_n \equiv a_n x_n + b_n y_n$$

$$\text{Relative speed}: \qquad \alpha_n \equiv \sqrt{a_n^2 + b_n^2}$$

It can be shown that the (X_n, Y_n) is a linear (rotational and scale) transformation of the (x_n, y_n); the parameters of the linear transformation depend on the target's motion parameters [7].

IV. THREE-DIMENSIONAL IMAGING IN (X, Y, α)

The SAR reconstruction of a moving target appears smeared and shifted if it is treated as a stationary target [4,9]. However, the moving target's reconstruction is focused if the value of its α is incorporated in the reconstruction algorithm. The SAR detection of a moving target with an *unknown* velocity can be formulated as a *three-dimensional* imaging in the (X, Y, α) domain. The three-dimensional reconstruction in the (X, Y, α) domain is based on the following inversion.

We denote the two-dimensional Fourier transform of the measured stripmap data, that is, $s(u, t)$, with $S(k_u, \omega)$. Then, the inverse equation in the spatial frequency domain of (X, Y) for targets with relative speed α is [7]

$$F(k_X, k_Y, \alpha) = P^*(\omega)\ A^*(k_u, \omega)\ S(k_u, \omega)$$

where

$$k_X \equiv \sqrt{4k^2 - \left(\frac{k_u}{\alpha}\right)^2}$$

$$k_Y \equiv \frac{k_u}{\alpha}$$

The inverse two-dimensional Fourier transform of this signal with respect to (k_X, k_Y), that is,

$$f(X, Y, \alpha) \equiv \mathcal{F}^{-1}_{(k_X, k_Y)}\left[F(k_X, k_Y, \alpha)\right],$$

is the desired statistic for Moving Target Detection (MTD) of targets with relative speed α. For this purpose, the computer selects the α value that maximizes the reconstruction energy at the vicinity of a spatial point

(X, Y) domain, call that $\alpha_{max}(X, Y)$. If the maximum energy is beyond a prescribed threshold, a target is suspected to be at that pixel point. Clearly, it is not feasible to form the same three-dimensional statistic with Time Domain Correlator (TDC) [1,2] within a reasonable time period.

Considering the practical values for the speed of a moving vehicle on the ground, the values of α that one should use for the three-dimensional imaging is approximately within [.75, 1.25]. Moreover, for the parameters that we encounter in UHF SAR, the sample spacing of $\Delta_\alpha = .025$ can be shown to give a good initial estimate of the relative speed value of a moving target (i.e., α_{max}). The resultant digitally-spotlighted signature can then be used to obtain a more accurate estimate of the motion parameters.

We injected a realistic UHF SAR database with the signature of a moving target with $\alpha = .8$ and $(X, Y) = (525, 75)$ meters. Figure 1 is the SAR reconstruction of the target scene with $\alpha = 1$. The moving target appears as a smeared and shifted structure at the lower left corner of the image. The stationary targets and foliage appear relatively focused in this image. Figure 2 shows the SAR reconstruction of the target scene with $\alpha = .8$. The moving target appears focused at its coordinates that is $(X, Y) = (525, 75)$ meters. The stationary targets and foliage are smeared in this reconstruction.

V. ENERGY FUNCTION

After incorporating the radar's and target's amplitude patterns in the SAR system model of a moving target, one obtains

$$s_n(u, \omega) = \mathrm{a}_n(X_n, Y_n + \alpha u, \omega) \sqrt{X_n^2 + (Y_n + \alpha_n u)^2}$$

where $\mathrm{a}_n(\cdot)$ is the product of the two amplitude patterns with the appropriate transformations from (x_n, y_n) to (X_n, Y_n). The signature of this moving target appears smeared in the reconstruction $f(X, Y, 1)$. In spite of this fact, this smeared signature appears fairly localized in the reconstruction (though not focused). This is due to the fact that the practical values for the speed of a moving vehicle on the ground, that is, α, is approximately within [.75, 1.25]. This is the key for developing the MTD/MTI processor which is described next.

For a given pixel point (X, Y) in the spatial domain, the processor extracts (digitally spotlight) the coherent reconstruction $f(X, Y, 1)$ within a neighborhood of that pixel; the size of the window is related to the radar parameters and the range of practical α values. The coherent data are then transformed into the SAR signal in the (u, ω) domain via the inverse mapping from the (X, Y) domain to the (u, ω) domain. We denote the resultant database with $s_{XY}(u, \omega)$.

The energy function is defined via the following matched filtering:

$$E_{XY}(X_n, Y_n, \alpha_n, \mathrm{a}_n) \equiv \int_u |s_{XY}(u, \omega)\ s_n^*(u, \omega)|^2\ du$$

The optimization problem is to maximize the energy function over the variable (X_n, Y_n, α_n), and the amplitude function a_n. In practice, the amplitude function can be identified via a simple *differentiable* parametric model. For instance, the amplitude pattern can be modeled as a Gaussian function in the u domain; the standard deviation (support band) and location of the Gaussian function, call them (σ_n, U_n), as well as (X_n, Y_n, α_n) identify the target's parameter space.

For certain classes of targets, for example, trucks, the standard deviation σ_n may be known a priori. Moreover, the location of the Gaussian function U_n can be approximated with Y_n. This is due to the broadside nature of the radar's amplitude pattern $a(\cdot)$, and not the target's amplitude pattern $a_n(\cdot)$. A moving target appears as a *squint* structure. In this case, $a_n(\cdot)$ is likely to be a *constant* when the target falls under the radar's radiation pattern. The model may also be modified to contain multiple moving targets and a stationary background (foliage).

ACKNOWLEDGMENTS

The author acknowledges his collaboration with Michael Pollock and Robert Dinger (Radar Branch), and Chris Yerkes and Skip Hoff (Communication Systems Branch) of NRaD. The author is also grateful to Johnattan Schoenfeld of ARPA for providing FOPEN SAR data.

References

[1] M. Allen and L. Hoff, "Wide-angle wideband SAR matched filter image formation for enhanced detection performance," *SPIE Conference on SAR Imaging*, Orlando, April 1994.

[2] J. Curlander and R. McDonough, *Synthetic Aperture Radar*, Wiley, 1991.

[3] M. Pollock, "VHF/UHF ultra-wideband radar for concealed target detection," NCCOSC Technical Document, Radar Branch, 1994.

[4] R. Raney, "Synthetic aperture imaging radar and moving targets," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 7, pp. 499-505, 1971.

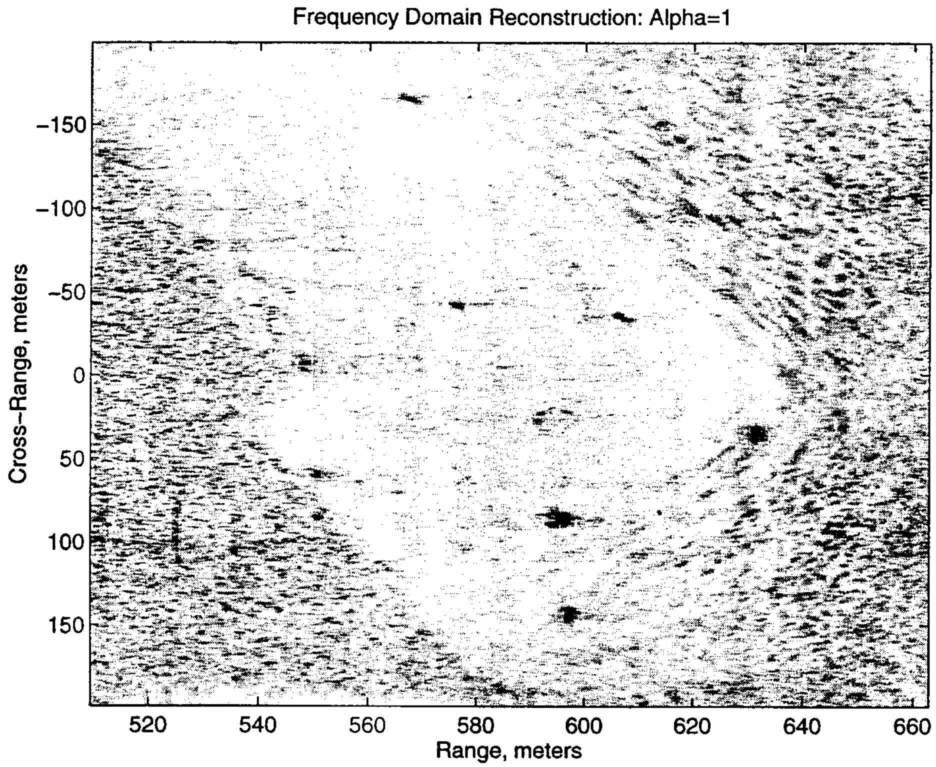
[5] A. Rihaczek, *Principles of High-Resolution Radar*, New York: McGraw Hill, 1969.

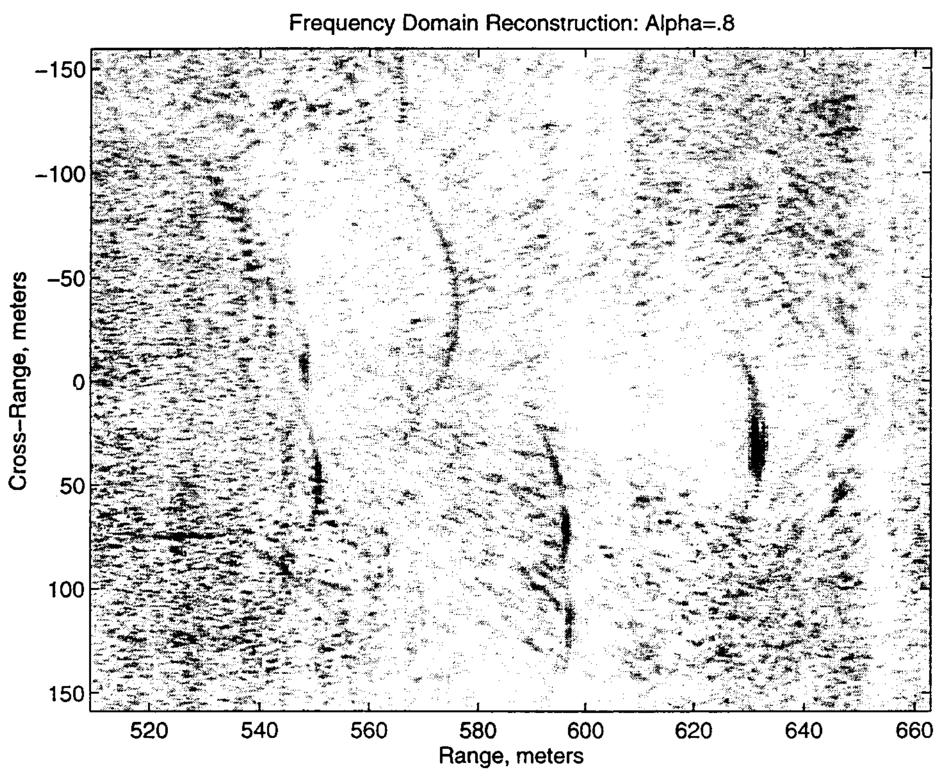
[6] M.I. Skolnik, *Introduction to Radar Systems*, New York: McGraw Hill, 1980.

[7] M. Soumekh, *Fourier Array Imaging*, Englewood Cliffs, NJ: Prentice Hall, 1994.

[8] D. R. Wehner, *High Resolution Radar*, Artech House, 1987.

[9] H. Yang and M. Soumekh, "Blind-velocity SAR / ISAR imaging of a moving target in a stationary background," *IEEE Transactions on Image Processing*, January 1993.


Frequency Domain Reconstruction: Alpha=1
Cross-Range, meters
-150
-100
-50
0
50
100
150
520
540
560
580
600
620
640
660
Range, meters


Frequency Domain Reconstruction: Alpha=.8
Cross-Range, meters
-150
-100
-50
0
50
100
150
520
540
560
580
600
620
640
660
Range, meters