# Spaceborne SAR Imaging in Wavenumber Domain*

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## ABSTRACT

An algorithm to process the Synthetic Aperture Radar (SAR) data in wavenumber domain, has been discussed in this paper. Generated from the seismic migration technique in the area of geophysics, this imaging method has been modified and developed in the area of SAR imaging. The basic idea of original seismic migration technique is analysed and modified to suit the feature of spaceborne SAR imaging. The result shows that the new algorithm will improve the image quality without additional computational time because the correction can be realized by phase shifting and fused to pulse compression in azimuth direction.

## 1. INTRODUCTION

A new algorithm, which is known as seismic migration technique in the area of geophysics, has been introduced in recent years, to process the Synthetic Aperture Radar (SAR) data in wavenumber domain. As a start, it is necessary for us to to review the basic idea of seismic migration technique, so as to get a better understand of the whole algorithm.

### 1. 1. The Basic Idea of seismic migration

Assume a point source at coordinate$(x_0, z_0)$, where z is the depth from the surface of the earth, and x is a straight line extended on the surface. The source generats a short pulse at $t = -z_0/c$, which transmits to the surface with the wave speed of c, and this is the seismic wave in the view of geophysics. A set of sensors are evenly located along the x axis to receive the upward wave, and the received signal of this array can be expressed as $d(x, t, z=0)$, which are wave field samples received at positions on x axis where $z=0$. To the whole wavefield $d(x, t, z=0)$ we have wave equation:

$$\frac{\partial^2 d}{\partial x^2} + \frac{\partial^2 d}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 d}{\partial t^2} \tag{1}$$

The basic idea of seismic migration technique is to reconstract the wavefield $d(x, t, z)$ with the wave equation and known data $d(x, t, z=0)$. Do 2-D fourier

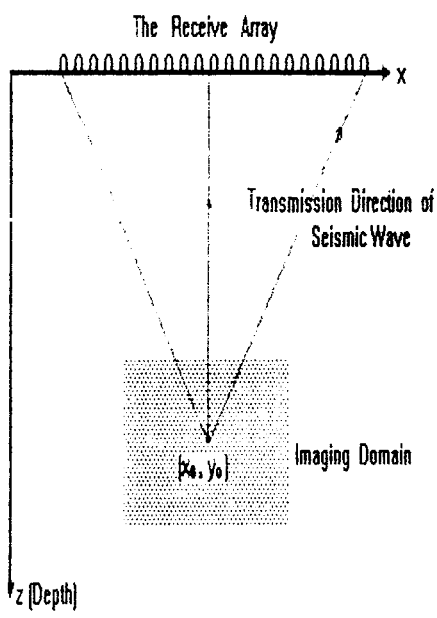


FIG. 1. Seismic Migration

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transform on both side of the wave equation, we have:

$$k_x^2 D(k_x, z, \Omega) + \frac{d^2 D(k_x, z, \Omega)}{dz^2} = \frac{\Omega^2}{c^2} \cdot D(k_x, z, \Omega) \tag{2}$$

Solve the equation, with the data $D(k_x, \Omega, z=0)$, we have:

$$D(k_x, \Omega, z_0) = D(k_x, z=0, \Omega) exp(jz_0 \sqrt{\Omega^2/c - k_x^2}) \tag{3}$$

Assume an imaging area from $z=z_1$ to $z=z_2$, which can be seen as a set of point sources cover the area, repeat the operation of (3) from $z_0=z_1$ to $z_0=z_2$, we can get the wave field and therefore the image of the desired area.

### 1. 2. The Fast Algorithm of Migration

The result of the operation in (3) is the wave field in particular depth $z=z_0$ and particular time segment from $t=0$ to $t=T$; while in fast imaging, the desired result is the wavefield in particular time $t=-z_0/c$ and particular depth range from $z=z_1$ to $z=z_2$. we can get the needed result with following operation:

$$\begin{aligned} & d(x, z, t=-z_0/c) \\ &= \frac{1}{2\pi} \cdot \iint D(k_x, z, \Omega) e^{-j\Omega z_0/c} e^{jk_x x} d\Omega dx \\ &= \frac{1}{2\pi} \cdot \iint D(k_x, z=0, \Omega) e^{jz\sqrt{\Omega^2/c - k_x^2}} e^{-j\Omega z_0/c} e^{jk_x x} d\Omega dx \end{aligned} \tag{4}$$

To get a fast imaging, we should transfer the variable of raw data from $D(k_x, \Omega)$ to $D(k_x, k_z)$ (which are known as wavenumber domain) by using the relation:

$$\Omega = c\sqrt{k_x^2 + k_z^2} \tag{5}$$

After then, the whole processing can be expressed as:

$$d(x, z, t=-z_0/c) = \frac{1}{2\pi} \iint D(k_x, c\sqrt{k_x^2 + k_z^2}, z=0) \cdot e^{-j\Omega z_0/c} e^{j(k_x x + k_z z)} dk_x dk_z \tag{4'}$$

In practical digital processing, the imaging procedures can be summerized as following sequence:

1) 2-D FFT of raw SAR data (have been compressed in range direction).
2) Change the variable from $\omega$ to $k_z$ by interpolation.
3) Multiplication by azimuth compression factor.
4) Inverse 2-D FFT.

This fast algorithm will be applied to SAR imaging in the following description, from the analyse we can know that the result is satisfied.

## 2. IMAGING IN WAVENUMBER DOMAIN

### 2. 1. The SAR Echo in Wavenumber Domain

The SAR system is a conventional pulse radar which takes advange of the relative motion between sensor and target to synthesize a very long "antenna" so as to obtain a high spatical reslution. From FIG. 2. we can see that the sensor moves along the x axis, and a point target is located at $(x_0, z_0)$. The range between the sensor and the target is:

$$R(x, x_0) = \sqrt{z_0^2 + (x - x_0)^2} \tag{6}$$

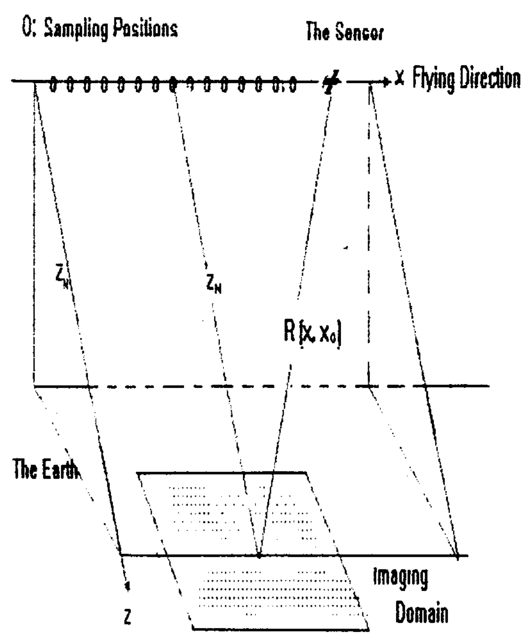


FIG. 2. SAR Imaging

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The sensor transmitt pulse to the target:

$$s(t) = A(t)cos(\omega_0 t)$$

(where A(t) is the pulse envelope, and $\omega_0 = 2\pi f_0$ is the carrier frequency).

Receive the returned wave from the target:

$$s_r(t) = \delta(x_0, z_0) \cdot A(t - 2R(x, x_0)/c) \cdot cos(\omega_0(t - 2R(x, x_0)/c) \quad (7)$$

(Where $\delta(x_0, z_0)$ is the reflective factor, $2R(x; x_0)/c$ is the double way time delay)

Think the target to be one of many point sources "explode" short pulse simultaneously at $t = -2z_0/c$, with a wave speed of c/2, we can establish a mathematical model of the returned wave in the view of seismic migration. After baseband complex demodulation and pulse compression, the returned wave, or we should say the samples of the wavefield at z=0, will be:

$(x, t, z = 0)$

$$\delta(x_0, z_0) \cdot \delta(t - \frac{2}{c}(R(x, x_0) - z_0)) \cdot e^{-j\frac{2\omega_0}{c}(R(x, x_0) - z_0)} \quad (8)$$

With the application of stationary phase principle, we can transfer d(x,t,k) into Ω-k domain ($\Omega = \omega + \omega_0$):

$$D(k_x, \omega, z = 0) = C_0 \delta(x_0, z_0) \cdot e^{-jz0\sqrt{4(\omega+\omega_0)^2/c - k_x^2}} \cdot e^{j(\frac{2\omega_0}{c}z_0 - k_x x_0)} \quad (9)$$

Where $C_0$ is a complex constant, and can be ignored. Think about the imaging domain, $z_0$ from $(z_N - z_A)$ to $(z_N + z_A)$ and $x_0$ from $-x_A$ to $x_A$, we have the sumerized echo:

$$D(k_x, \omega, z = 0) = \iint \delta(x_0, z_0 + z_N) \cdot e^{-j(z_0 + z_N)\sqrt{4(\omega+\omega_0)^2/c - k_x^2}} \cdot e^{j\frac{2\omega_0}{c}z_N - k_x x_0} dx_0 dz_0 \quad (10)$$

Where $z_N$ is the center of imaging domain, point sources in this domain "explode" pulse at $t = -2z_N/c$.

### 2.2. Theoretical Description of Imaging

With formular (4′) and (10), we have:

$$d(x, z, t = -2z_N/c) = \iint D(k_x, \omega, z) \cdot e^{-j\omega \cdot 2z_N/c} \cdot e^{jxk_x} dk_x d\omega$$
$$= \iint D(k_x, \omega, 0) \cdot e^{jz\sqrt{4(\omega+\omega_0)^2/c - k_x^2}} \cdot e^{-j\omega \cdot 2z_N/c} \cdot e^{jxk_x} dk_x d\omega \quad (11)$$

Let $k_0 = 2w_0/c$, transfer $D(k_x, \omega, z=0)$ into $k_x$-$k_z$ domain,

$$d(x, z, t = -2z_N/c) = \iint D(k_x, \frac{c}{2}(\sqrt{k_z^2 + k_x^2} - k_0), z = 0) \cdot e^{jz(k_z + k_0)} \cdot e^{-j\omega \cdot 2z_N/c} \cdot e^{jxk_x} dk_z dk_x \quad (12)$$

Let $z_a = z - z_N$, we have:

$$d(x, z_a + z_N, t = -2z_N/c) = \iint D(k_x, \frac{c}{2}(\sqrt{k_z^2 + k_x^2} - k_0), z = 0) \cdot e^{j(z_a + z_N)k_z} \cdot e^{-j\omega \cdot 2z_N/c} \cdot e^{jxk_x} dk_x dk_z \quad (13)$$

Therefore, the final result of imaging will be:

$$d(x, z_a + z_N, t = -2z_N/c) = \iint [\iint \delta(x_0, z_0 + z_N) \cdot e^{-j(z_0 + z_N)\sqrt{4(\omega+\omega_0)^2/c - k_x^2}} \cdot e^{j\frac{2\omega_0}{c}z_N - k_x x_0} dx_0 dz_0] \cdot e^{j(z_a + z_N)k_z} \cdot e^{-j\omega \cdot 2z_N/c} \cdot e^{jxk_x} dk_x dk_z$$
$$= \iint sinc(K_X(x - x_0)) \cdot sinc(K_Z(z_a - z_0)) \, dx_0 dz_0 \quad (14)$$

Where $K_X$ is determined by the bandwidth of transmitted signal, and $K_Z$ is determined by the length of synthetic aperture.

From (14) we can see that the phase error of returned wave can be completelycompensated.

## 3. SAR IMAGING PROCEDURE

The most diffculty in synthetic aperture imaging processing is the varible transform of data $D(kx, \omega, 0)$ from $\omega$ to $k_z$, which needs precious interpolation. Though interpolation is necessary in variable transform from $\omega$ to $k_z$, it is a troublesome operation, which will bring about additional computational cost. In this paper, consider the small relative bandwidth of the spaceborne SAR signals, we can use phase shifting on ω axis to take the place of interpolation.

In frequency domain, the SAR signals are limited in the range from $\omega_0$-Δω to $\omega_0+\Delta\omega$, where $\frac{\Delta\omega}{\omega 0}$ is less than .001. on the other hand, $k_x$ is much smaller than $k_z$, so we have:

$$\omega = c/2\ (k_z + k_x^2/k_0) \qquad (15)$$

this is an aproximation of (5), where $k_0=2/c\omega_0$, $kz \in (k_0-\Delta k_z, k_0+\Delta k_z)$, $k_x << k_0$.

*b2M With the application of this approximation, the practical imaging processing will be:

1. Do Fourier transform in azimuth direction to echo signal d(x, z=0, t).

2. Mutiple frequency shifting factor $e^{-j\frac{k_x^2}{2k_0}\cdot t}$, replace the interpolation by a frequency shifting, then do Fourier transform in range direction.

3. Multiple with azimuth compression factor in wavenumber domain (kx—kz).

4. Do 2-D inverse Fourier transform , we can get the final result.

## 4. SPACEBORNE SAR IMAGING

Because the sensor of spaceborne SAR moves along a circle orbit, the range relation between the sensor and the target should be modified to:

$$R(x, x_0) = \sqrt{z_0^2 + \frac{R_e\cos\theta_0}{R_s}(x - x_0)^2} \qquad (16)$$

Where $R_e$ and $R_s$ are radius of the earth and the satellite orbit; $\theta_0$ is the elevation angle , with respect to the orbit plane, and $x=R_s\Phi$, $x_0=R_s\Phi_0$.

Let $h = \frac{R_e\cos\theta_0}{R_s}$, we have:

$$R(x, x_0) = \sqrt{z_0^2 + h\cdot(x - x_0)^2} \qquad (17)$$

The rigorous relation between $k_x$, $k_z$ and ω become to:

$$\omega + \omega_0 = \frac{c}{2}\cdot\sqrt{(k_0 + K_z)^2 + k_x^2/h} \qquad (18)$$

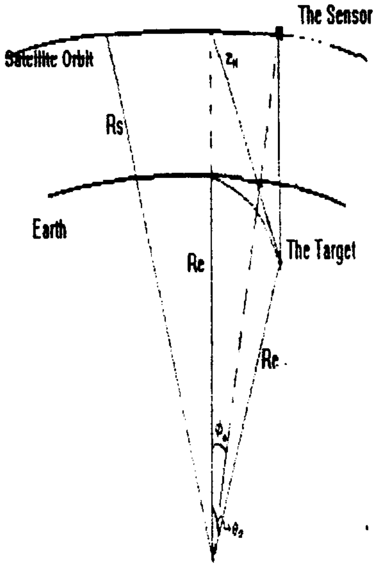


FIG. 3. Spaceborne SAR Imaging

So we can modify (15) to

$$\omega = c/2\cdot(k_z + \frac{k_x^2}{h\cdot k_0}) \qquad (19)$$

and repeat the imaging procedures of session 3 so as to get the result of spaceborne SAR imaging.


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