# GENERAL STRUCTURE FOR THE DESIGN OF MULTIDIMENSIONAL FILTERS WITH LOW SENSITIVITY PROPERTY

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## ABSTRACT

In this paper, a structure consisting of only all-pass subsystems for the design of n-dimensional IIR digital filters with arbitrary cutoff boundaries is presented. It is shown that allpass filter is a universal building block for multidimensional digital filter structures. Utilization of the proposed structure for n=2, and 3 is given in more details.

## 1. GENERAL STRUCTURE

In 1-D digital filter design, it is well known that IIR filters can be implemented using sum or difference of two all-pass blocks. The very important property of this structure is the "built-in" insensitivity in the pass-band [1-3]. In this paper the structure is extended to n-D filters. Suppose two stable all-pass systems, $A_{ki}(z_1, z_2, \ldots, z_n)$ for $i = 1,2$, in n-dimensional space are given. A connection as shown in Fig. 1(a) yields a universal building block. Every cutoff boundary may be obtained by connecting these blocks in sequential and/or parallel structure.

Fig.2 (a), (b), and (c) show all *critical frequency points (CFPs)* in one, two, and three dimensions, respectively. Each point is equivalent to $w_i = 0 / \pi$ for $i = 1,2,3$. Also $z_i = e^{jw_i}$, therefore each of them is equivalent to $z_i = 1$ or $-1$. Each building block, in the form of Fig.1(a), eliminates $2^{n-1}$ CFPs. Therefore, one, two, and four CFPs are eliminated in 1-D, 2-D, and 3-D cases respectively by each block element. The element of delay is the decision part of each block, that filters a CFP out or allows it to pass through. There exist $2^{n+1}$ types of delay elements, including no delay, in n-dimensional space. These elements are combinations of delays like "$z_i^{-1}$", or its complement "$-z_i^{-1}$" for $i = 1,2,..,n$. For instance in 2-D, types of delay elements are as follows

$$-1, z_1^{-1}, z_2^{-1}, z_1^{-1}z_2^{-1}, -z_1^{-1}, -z_2^{-1}, -z_1^{-1}z_2^{-1}, +1 \quad (1)$$

As it was pointed out before, each of the above elements eliminates $2^{2-1} = 2$ CFPs, except for no delay cases "-1", and "+1", which the former one filters all regions out and the latter one lets all regions to pass through it. In 3-D case each element of delay eliminates $2^{3-1} = 4$ CFPs. To have other shape of cutoff boundaries, like one CFP in 1-D, one may cascade building blocks with different delay elements in series. Cascading two blocks may create four different boundaries consisting of none, all, $2^{n-1}$ or $2^{n-2}$ CFPs for $n \geq 2$.

Some combinations of blocks with different delay elements may give the same boundary shape. To clarify this point, consider the following two different combinations of blocks

$$(A_1 + z_1^{-1}A_2)(A_3 + z_1^{-1}z_2^{-1}A_4) \quad (2)$$

and

$$(A_1 + z_1^{-1}A_2)(A_3 + z_2^{-1}A_4) \quad (3)$$

both (2), and (3) initially give the same cutoff boundary. To investigate these similarities, first we introduce the following notation

$$\alpha.\beta \equiv (A_1 + \alpha A_2)(A_3 + \beta A_4) \quad (4)$$

where $\alpha$, and $\beta$ are delay elements. Therefore, (3) simply is shown as "$z_1^{-1}.z_2^{-1}$". The following rules are true for cascading the blocks in series

$$\alpha.(-\alpha) \equiv -1, \quad \alpha.\alpha \equiv \alpha, \quad \alpha.\beta \equiv \beta.\alpha,$$
$$\alpha.\alpha\beta \equiv \alpha.\beta,$$
$$(-\alpha).\alpha\beta \equiv (-\alpha).(-\beta) \quad \alpha\beta.\alpha\gamma \equiv \alpha\beta.\beta\gamma \equiv \alpha\gamma.\beta\gamma$$
$$(5)$$



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From the above, one may conclude that not all shapes of boundaries for $n \geq 3$ are possible to be created by just cascading basic building blocks in series. To overcome this problem and complete the structure, we allow blocks to be combined in parallel connections as well. The following notation, similar to (4), is introduced for this kind of connections

$$\alpha + \beta \equiv (A_1 + \alpha A_2) + (A_3 + \beta A_4) \qquad (6)$$

Some of the rules for this kind of connection is as follows

$$\alpha + (-\alpha) \equiv +1, \quad \alpha + \alpha \equiv \alpha, \quad \alpha + \beta \equiv \beta + \alpha \qquad (7)$$

The general structure is depicted in Fig.3. Each "$B_{...}$" represents a universal building block. This structure is represented mathematically in equation (8)

$$\eta + (-1)^{\eta} \frac{1}{K} \sum_{k=1}^{K} \frac{1}{2^{L_k}} \prod_{l=1}^{L_k} B_{kl} \qquad (8)$$

where "$K$", and "$L_k$" are to be selected based on the shape of the cutoff boundary. The parameter "$\eta$" is a binary parameter. It is sometimes simpler to design the complement of the boundary of interest instead of itself and then assign $\eta=1$ [1].

To conclude this discussion, we give two examples. In the first example, we try to design a low-pass filter in 2-D. Considering Fig. 2(b), this filter consists of CFP number 1 only. Thus, the following combination might be appropriate for the case

$$(A_1 + z_1^{-1} A_2)(A_3 + z_2^{-1} A_4) \qquad (9)$$

A circular symmetric filter based on the structure of shown in (9) is designed by using non-linear optimization technique. 3-D plot of this filter in the first quaderant is shown in Fig. 4.

In the second example, we try to design a filter which has a cutoff boundary consisting of CFPs 1, 5, 6, and 7 shown in Fig. 2(c). For this filter the following structure is proposed

$$A_1 + z_1^{-1} z_2^{-1} z_3^{-1} A_2 \qquad (10)$$

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[1]For instance every boundary consisting of 5 CFPs is a complement of another boundary consisting of 3 CFPs in the case of n=3.

## REFERENCES


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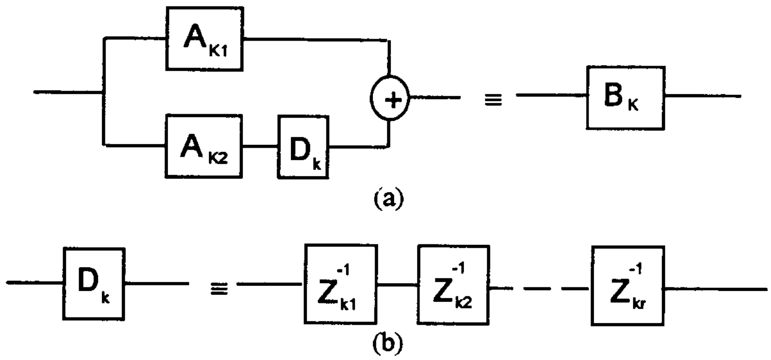


Fig. 1 (a) connection of the "kth" building block (b) delay element of the "kth" block

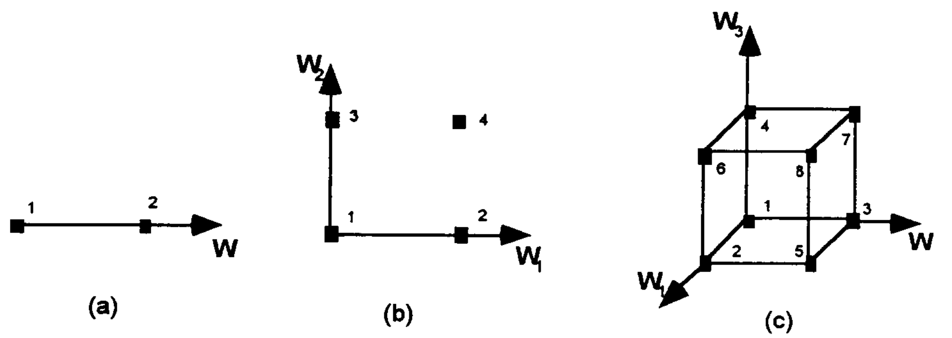


Fig. 2 (a) 1-D CFPs in the first half $\{(z^{-1}=e^{-jw})\}=\{(1),(-1)\}$

(b) 2-D CFPs in the first quaderant $\{(z_1^{-1},z_2^{-1})=(e^{-jw_1},e^{-jw_2})\}=\{(1,1),(1,-1),(-1,1),(-1,-1)\}$

(c) 3-D CFPs in the first octant $\{(z_1^{-1},z_2^{-1},z_3^{-1})=(e^{-jw_1},e^{-jw_2},e^{-jw_3})\}=\{(1,1,1),(1,1,-1),\dots,(-1,-1,-1)\}$

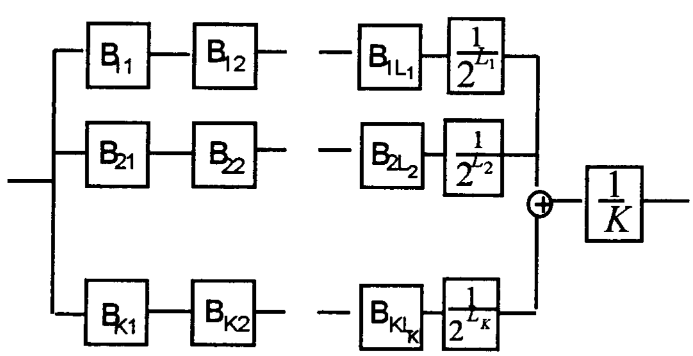


Fig. 3 General structure for IIR digital filters based on all-pass building blocks

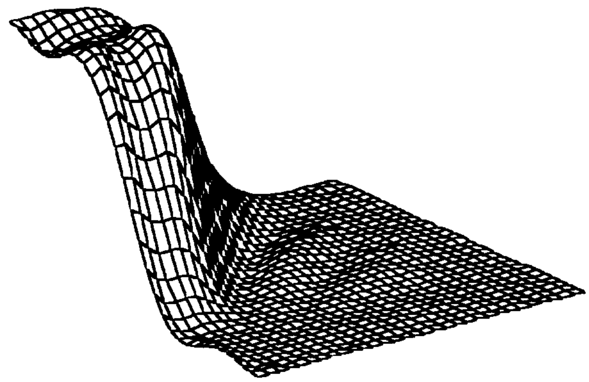

Fig. 4-Circular symmetric Low pass filter designed by using the structure shown in (9)