BLIND SEPARATION OF WIDE-BAND SOURCES IN THE FREQUENCY DOMAIN

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ABSTRACT

Conventional antenna array processing techniques are based on the use of second order statistics but rest on restrictive assumptions. Thus, when a priori information about the propagation or the geometry of the array are hardly available, the model can be generalized to a blind sources separation model. It supposes the statistical independence of the sources and their non-gaussianity. We focus in this paper on the generalization of the sources separation problem to convolutive mixtures of wide-band sources in the frequency domain.

1. INTRODUCTION

Usual array processing techniques are based on the second-order statistics of the received signals and use a priori information about signal propagation from sources to sensors (hypothesis of plane waves, linear array, ...).Several methods have been recently proposed [1] [2] [3] [4] when no a priori information is available. The problem, generally called "blind source separation", consists in identifying p independent and non-gaussian sources from M observed linear mixtures of these sources. Existing methods [1] [2] [3] [4] have been developed in time domain in the case of linear instantaneous mixtures, using higher-order statistics (usually fourth-order moments or cumulants, or non linear functions of the observations). In a general blind source separation problem, the observed data vector $\underline{r}(t)$ may be represented in frequency-domain by an instantaneous complex mixture for each frequency bin f, which leads to the following model:

(1) $\underline{R}_N^k(f) = A(f)\, \underline{S}_N^k(f) + \underline{B}_N^k(f)$ for each frequency bin f

where $\underline{R}_N{}^k(f)$ is the N-point Discrete Fourier Transform (DFT) of the kth data block of the observation $\underline{r}(t)$. $\underline{S}(f)$ represents the p sources, A is a matrix (M.p) which characterizes the linear propagation from sources to sensors and $\underline{B}k(f)$ represents an additive M-dimensional gaussian noise. The problem consists first in identifying the matrix A(f). Several methods based on the cross-bispectra or the trispectra of the estimated sources have been proposed [5] [6] [7]. [8] and [9] propose an adaptive approach in the time-domain. In this paper, the identification lays on the factorization of the matrix A(f) as a product of three matrices :

(2) $$A(f)=V(f)\,D(f)\,P(f)$$

The matrices V(f) (a unitarian matrix) and D(f) (a diagonal matrix) are identified thanks to second order statistic criteria and P(f) (a unitarian matrix) thanks to fourth order criteria by cancelling the different intercumulants of the estimated sources.

2. PRESENTATION OF THE SPECIFIC PROBLEMS

We focus in this paper on the generalization of the sources separation problem to convolutive mixtures of wide-band sources in the frequency domain. However, the latter lays two main difficulties. The first one directly ensues from the frequency expression of the signals. As a matter of fact, the current methods in the field of sources separation rest on the assumption that the emitted signals are non-gaussian, when in fact the N-point discrete Fourier transform of signals generally tends to be gaussian when N tends to infinity according to the central limit theorem. The second one which is inherent in the application of the algorithm to wide-band signals is the reconstruction of the estimated sources spectra from the signals identified at each frequency bin. As a matter of fact, the sources associated to the ith identified signals are not necessarily the same from one frequency bin to another.

3. GAUSSIANITY AFTER A DISCRETE FOURIER TRANSFORM

This part of the paper is devoted to the analysis of the possible convergence to gaussianity of the signals after DFT, for a finite value of N, which is used in practice. Let s(t) be the signal in the time domain and let define the N-point Discrete Fourier Transform of the kth data block of signal s(t) at the frequency bin f :

(3) $$S_N^k(f) = \sum_{i=0}^{N-1} s(k+i)\, e^{-2\pi f \frac{i}{N}}$$

The central limit theorem is proved [10] under a sufficient condition of convergence relative to the duration of the multicorrelations :

(4) $$\sum_{u_1, \ldots, u_{k-1}}^{\infty} \left| C_k^s(u_1, \ldots, u_{k-1}) \right|^2 < \infty$$

with k=2, 3, ...

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$C_k^s(u_1, ..., u_{k-1})$ represents the multicorrelation of s(t) of order k.

It is clear that white processes whose multicorrelations are reduced to impulses are asymptotically gaussian in the frequency-domain. Nevertheless, in the cases where this condition is not satisfied, the possible convergence towards gaussianity is not proved. We study the distance to gaussianity thanks to a specific criterion : the spectral kurtosis which is defined as a section of the general trispectrum of the normalized sources.

Let $K(S_N^k(f))$ be the kurtosis of $S_N^k(f)$, defined by :

$$(5)\quad K(S_N^k(f)) = \frac{\mathrm{Cum}(S_N^k(f)^*, S_N^k(f), S_N^k(f)^*, S_N^k(f))}{\mathrm{Cum}(S_N^k(f)^*, S_N^k(f))}$$

where Cum represents the cumulants of second and fourth orders and * the complex conjugate.

3.1 Case of bounded multicorrelations

In this section, the usual case where the multicorrelations are bounded is considered. The evolution of the kurtosis (which represents a measurement of the speed convergence to gaussianity) is theoretically established in function of N and the duration of the tricorrelation. Considering the property of circularity specific to signals in the frequency domain, the expression of the kurtosis in the case of second order white signal is developed. It leads to :

$$(6)\quad K(S_N^k(f)) =$$

$$\frac{1}{N\sigma_s^2}\left[\sum_{i=0}^{T}\sum_{t1,t2,t3=-i}^{T} C_4^s(t1,t2,t3)\cos\left(\frac{2\pi f}{N}(t1-t2+t3)\right)\right] + \frac{1}{N\sigma_s^2}\left[\sum_{i=T+1}^{N-2-T}\sum_{t1,t2,t3=-T}^{T} C_4^s(t1,t2,t3)\exp\left(\frac{2\pi f}{N}(t1-t2+t3)\right)\right]$$

where T represents the duration of the tricorelation and σ_s^2 the power of s(t). The value of the spectral kurtosis $K(S_N^k(f))$ generally depends on T and f. It is clear from (6) that its value is close to zero for a fixed value of N and a little value of T (T<N). However, a large value of T is not always sufficient to obtain a significant value of $K(S_N^k(f))$ as the shape of the tricorrelations is also important. To illustrate this, let us consider the simulated second order white signal whose single non zero tricorrelations are the symmetric ones :

$$(7)\quad C_4^s(0, t, t) = g(t) \text{ for } t < T$$

with T: the duration of the tricorrelation.

In that case, if we develop the expression (5), the kurtosis is equal to:

$$(8)\quad K(S_N^k(f)) = \frac{Ng(0) + 2\sum_{i=1}^{T}(N-i)\,g(i)\,(1+2\cos(2\pi if/N))^2}{N^2\sigma_s^2}$$

As an example, let us consider the following signal s(t) :

$$(9)\quad s(t)=u(t).u(t-1).....u(t-T+1)$$

where u(t) is a white normalized process with zero mean. The single non zero tricorrelations of s(t) are the symmetric ones $C_4^s(0,t,t)$. In that case, the kurtosis of s(t), K(s(t)), is equal to :

$$(10)\quad K(s(t)) = (K(u) + 3)^T - 3$$

Considering a particular signal u(t) of kurtosis close to (-2), it leads to :

$$(11)\quad K(u(t)) = -2 + \varepsilon$$

$$(12)\quad K(s(t)) = (1 + \varepsilon)^T - 3 \# -2$$

$$(13)\quad g(i) = E\{u(t)^4\} - 1 = (1 + \varepsilon)^{T-i} - 1$$

If we choose $\varepsilon>0$, then $g(i)>0$ and $|K(S_N^k(f))|$ is increasing in function of T.

$$(14)\quad K(S_N^k(f)) > \frac{2\sum_{i=1}^{T}(N-i)\,g(i)}{N^2\sigma_s^2}$$

Consequently, for a fixed value of N, it exists a value of T so that the spectral kurtosis be significant enough (not too close to zero). As a conclusion, it exists wide-band signals with bounded multicorrelations whose DFT (for a fixed value of N) is not close to a complex normal variable. For that type of signals, the DFT of the sources are independent and an algorithm of blind separation in the case of instantaneous complex mixture may be used [1] [6] [11]. The conception of such signals is important in active radar or sonar applications in order to eliminate internal noises. Two solutions are available : noise cancelling or blind separation methods. In the case where it is impossible to record a right noise-reference, blind separation methods are usable if the emitted signal can be separated from internal noises after reception.

3.2 Case of periodic signals

Let s(t) be a periodic signal with a period T. In that case, the statistical kurtosis (5) has non sense. However, its estimation K can always be computed by L averages on s(t). For example, let s(t) be a sinusoid of deterministic frequency ω and phase ϕ :

$$(15)\quad s(t) = A\sin(2\pi\omega t + \phi)$$

Let us compute the DFT of s(t), $S_N^k(f)$, for f close to ω :

$$(16)\quad S_N^k(f) = \frac{A}{2j}\exp\left(j\left(f+2\pi\omega k+\pi(N-1)\left(\omega-\frac{f}{N}\right)\right)\right)\frac{\sin\left(\pi\left(\omega-\frac{f}{N}\right)N\right)}{\sin\left(\pi\left(\omega-\frac{f}{N}\right)\right)}$$

Let us define :

(17) $S_N^k(f) = (A/2j) \exp(j\psi(f,k)) F(f)$

The estimation of K is computed by :

$$(18)\quad K = \frac{\frac{1}{L}\sum_{k=1}^{L}\left|S_N^k(f)\right|^4 - \left|\frac{1}{L}\sum_{k=1}^{L}\left(S_N^k(f)\right)^2\right|}{\left(\frac{1}{L}\sum_{k=1}^{L}\left|S_N^k(f)\right|^2\right)^2} - 2$$

By replacing the expression (16) of $S_N^k(f)$ in (18), it leads to :

$$(19)\quad \frac{1}{L}\sum_{k=1}^{L}\left|S_N^k(f)\right|^4 = \left(\frac{A}{2}F(f)\right)^4$$

$$(20)\quad \frac{1}{L}\sum_{k=1}^{L}\left|S_N^k(f)\right|^2 = \left(\frac{A}{2}F(f)\right)^2$$

$$(21)\quad \left|\frac{1}{L}\sum_{k=1}^{L}\left(S_N^k(f)\right)^2\right|^2 = \left(\frac{A}{2}F(f)\right)^4\left|\frac{1}{L}\sum_{k=1}^{L}\exp(4\pi\omega k)\right|^2$$

$$(22)\quad K = -1 - \left|\frac{1}{L}\sum_{k=1}^{L}\exp(4\pi\omega k)\right|^2 = -1 - \frac{1}{L^2}A$$

As the term A is bounded, the kurtosis tends towards -1 when L is large enough for f close to ω. In the case of periodic signals, they may be decomposed in Fourier series and K tends towards -1 for the harmonic frequency bins.

As a conclusion, this point theoretically proves the feasibility of the sources separation for rotating machines signals.

4. SEPARATION OF WIDE-BAND SOURCES

When the spectral kurtosis is close to zero, the information of independent sources is not usable in the frequency-domain. We then search a relation between $\underline{R}_N^k(f)$ for f fixed and the p time sources si(t), i=1, .., p. We remark :

$$(23)\quad y(k,f) = e^{-2\pi j\frac{f}{N}} S_N^k(f) - S_N^{k-1}(f) = s(k) - s(k-N)$$

which directly ensues from the expression (3) of the DFT. Consequently, it exists a specific MA filtering of $\underline{S}_N^k(f)$ for each frequency bin f which is relied to a function of the time sources $(\underline{s}(t) - \underline{s}(t-N))$.

4.1 Case of an instantaneous mixture

Considering now the case of an instantaneous mixture in the time-domain, the expression of $\underline{R}_N^k(f)$ is equal to:

(24) $\underline{R}_N^k(f) = A(f)\, \underline{S}_N^k(f) + \underline{B}_N^k(f)$

Apply the previous MA filtering at frequency bin f.

(25) $\underline{y}(k,f) = MA(\underline{R}_N^k(f)) = A(f)\, [\underline{s}(t) - \underline{s}(t-N)] + [\underline{b}(t) - \underline{b}(t-N)]$

The new vector of the observations, $MA(\underline{R}_N^k(f))$, is then an instantaneous complex mixture of non-gaussian sources $[\underline{s}(t) - \underline{s}(t-N)]$. If N >T, where T is the duration of the tricorrelation of each source, the kurtosis of the new sources $[\underline{s}(t) - \underline{s}(t-N)]$ is exactly the same as that of the previous sources $[\underline{s}(t)]$. Consequently, any sources separation method can be applied in the case of an instantaneous complex mixture [1] [6] [11], which provides an estimation of the matrix A(f).

4.2 Case of a convolutive mixture

Consider now the case of a convolutive mixture in the time-domain. As $\underline{R}_N^k(f)$ is computed on a finite duration N, it is not exactly equal to an instantaneous complex mixture in the frequency-domain. It is corrupted by an additive residual term which cannot be neglected after the MA filtering. Suppose that the duration of the convolutive mixture is lower than L. Then we have :

$$(26)\quad \underline{R}_N^k(f) = \sum_{g=0}^{N+L-1} \Omega\; A(g)\, \underline{S}_N^k(g) + \underline{B}_N^k(f)$$

where A(g) represents the DFT on N+L-1 points and $\underline{S}_N^k(g)$ represents the DFT of the sources sequences : [$\underline{s}$i(k-N-L+1), ..., $\underline{s}$i(k)] at frequency bin g.

$$(27)\quad \Omega(f,g) = \frac{1}{N+L}\;\frac{\exp(2\pi j L g/(N/L)) - 1}{1 - \exp(-2\pi j((f/N) - (g/(N+L)))}$$

A solution to the linear system of equations (26) is given with L (or more) extra constraints. If the processes are defined in a limited bandwidth, L frequency bins of the vector $A(g)\underline{S}_N^k(g)$ are supposed to be negligible. If not, the observations are filtered in order to verify this condition. It leads to a complex instantaneous mixture which can be treated as previously in $4.1.

5. RECONSTRUCTION OF THE SOURCES SPECTRA

The crucial point consists in the time sources si(t) reconstruction (i=1, ..., p). After separation, p independent components of $S_N^k(f,i)$ (i=1, ..., p) are identified for each frequency bin f. As the methods independently treat each frequency bin, the ith identified signal $S_N^k(f,i)$ is not necessarily associated to the same time source si(t), from one frequency bin to another. In order to re-establish the continuity of the sources spectra, the procedure developed aims at recovering the statistic relationship between the estimated sources from one frequency-bin to another. Suppose now that $S_N^k(f,io)$ and $S_N^k(f+1,jo)$ are associated to the same time source s1(t). The previous MA filtering of $S_N^k(f,io)$, $(H(S_N^k(f,io)))$, and of $S_N^k(f+1,jo)$, $(H(S_N^k(f+1,jo)))$ will be equal to the same quantity (s1(k+N-1)-s(k-1)). Then a criterion based on the second order moments allows to associate the N frequency components of s1(t) :

$$(28)\quad E\left\{ H\left(S_N^k(f,i)\right), H\left(S_N^k(f+1,j)\right)^* \right\} \neq 0$$

$$\text{for } (i, j) = (io, jo)$$

$$(29)\quad E\left\{ H\left(S_N^k(f,i)\right), H\left(S_N^k(f+1,j)\right)^* \right\} = 0$$

$$\text{for } (i, j) \neq (io, jo)$$

6. SIMULATION RESULTS

The simulation here after illustrates the results of a complete implementation of the algorithm in the frequency-domain, in the case of a convolutive mixture of two sources. The two processes are a rotating machine signal and a non-gaussian noise. The filters of the mixture are MA filters of order 10. We present the spectral density of the right sources and the reconstructed ones after separation, using the proposed technique in §4 and 5. The good correspondence between the spectra reveals a good quality of separation for each frequency bin and a good quality of reconstruction of the sources.

7 CONCLUSION

We focus in this paper on the generalization of the sources separation problem to convolutive mixtures of wide-band sources in the frequency domain. As the N-point discrete Fourier transform of signals generally tends to be gaussian when N tends to infinity according to the central limit theorem, we replace the independence criterion of the sources with the independence of a specific MA filtering of their DFT. The result is then an instantaneous complex mixture in time-domain and the new sources are proved to be non-gaussian.

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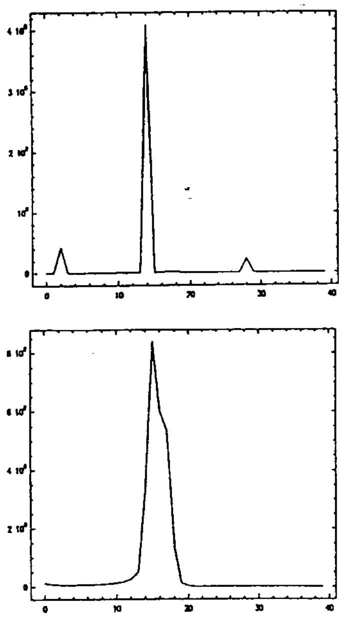

Fig1-2. Spectral densities of the sources

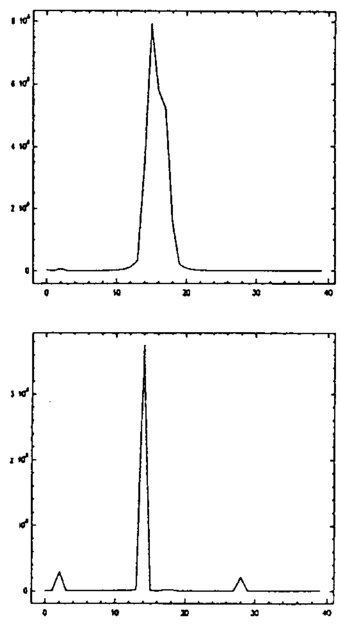

Fig.3-4. Spectral densities of the estimated sources