OPTIMAL FILTERING IN THE SUBBAND DOMAIN

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ABSTRACT

An analytical framework for the implementation of optimal filters in the multiresolution (i.e., subband) domain is presented. In particular, we concentrate on filter bank based on the notions of *wavelets* and *wavepackets*. We show how the notion of *sparse estimation* can lead to significant reduction in computational cost, with only a minor degradation in performance. The combination of a wavelet-based filter bank with a sparse estimation scheme results in a configuration with five design parameters: (i) resolution level, (ii) degree of subband channel overlap, (iii) subband utilization ratio, (iv) estimation sparsity, and (v) filter order. We demonstrate the effect of each one of these design parameters on the over-all cost-performance trade-off.

1. INTRODUCTION

Multiresolution analysis decomposes a single record into a hierarchy of signals at different scales, i.e., into a multichannel configuration (Fig. 1). This enables the application of efficient multiresolution estimation techniques to construct optimal (Wiener) and adaptive filters that operate recursively from coarse to fine scale. Such filters use the subband-domain components $\xi_i(m)$ of a received signal $x(n)$ to construct estimates of either the subband-domain components $\delta_i(m)$ of some desired signal $d(n)$, or of the desired signal itself.

In this paper we present an analytical framework for the multiresolution implementation of such optimal filters in the subband domain. The configuration we propose is motivated by the consideration that $\hat{\delta}_i(\cdot)$, the estimated subband-domain components of the desired signal $d(\cdot)$, are to be used as an input for further processing, such as target detection or classification, which is to be done in the subband domain as well.

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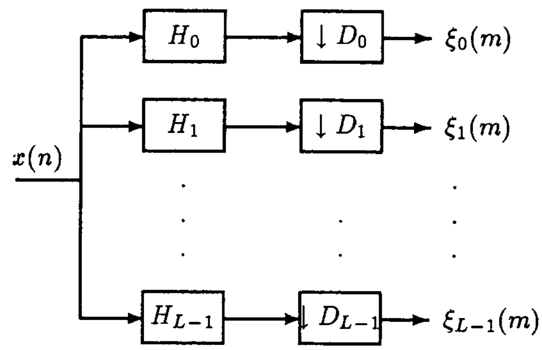

Figure 1: Subband-domain decomposition of a signal $x(\cdot)$.

Our approach to multiresolution optimal filtering is based on carrying out the filtering operation in the subband domain, using a selected *subset* of the subband-domain signals $\xi_i(m)$. This approach allows to trade-off performance for computational cost: using a fraction of the number of subband channels we can achieve estimation errors that are close to the theoretical minimum, and at a fraction of the cost associated with using the complete set of subband channels. The trade-off between performance and cost is controlled by several distinct factors, including:

- Resolution level – total number of available subband channels.
- Channel overlap – degree of overlap between the frequency responses of subband channels.
- Utilization ratio – the fraction of available subband channels that are involved in the estimation scheme.
- Estimation sparsity – the number of neighboring channels used in forming the estimate $\hat{\delta}_i(\cdot)$.
- Filter-order – the number of samples from each of

the subband channels used in estimation of $\delta_i(n)$ for every time instant n.

The first two factors are determined by the details of the subband decomposition scheme itself. For instance, the level of resolution in wavelet- and wavepacket-based decomposition is determined by the depth of the binary tree that characterizes the filter bank, while the channel overlap depends also on the frequency characteristics of the prototype lowpass filter: lower overlap can be achieved by higher quality filters, but at the expense of increased computational cost.

The remaining three factors depend on the details of the (optimal) estimation scheme used. The utilization ratio characterizes the subset of subband channels that are actually involved in the estimation process: ignoring (i.e., not estimating) some of the subband components $\delta_i(\cdot)$ results in a reduction in computational cost, combined with some degradation in overall performance. The notion of estimation sparsity offers a further refinement of the some cost-performance trade-off: relying on the relatively low overlap between non-adjacent subband channels we can significantly reduce the cost of computing each individual subband estimate $\delta_i(\cdot)$, which suffering only minor increase in the attendant estimation error. This index may range from a value of zero, which corresponds to the diagonal estimation scheme, through a value of 1 (tridiagonal scheme), to the full-complexity variant, which uses all subband channels. Finally, selecting the order of the estimation filter (including the possibility of channel-dependent order) is yet another way to control the cost-performance trade-off. While order selection is the only degree of freedom available in classical optimal filtering, here it combines with the other four factors to offer a much broader range of design scenarios.

We compare the cost-performance trade-off of the various suboptimal schemes. We show that utilization ratio and estimation sparsity tend to be the dominating factors in the sense that they allow the most significant reduction of computational cost for a given level of estimation error (Fig. 2). The (lesser) effect of the remaining three factors is described in Sec. 4. Though some results on matching wavelet-based filter bank,ie, to specific signals has already been published [1, 2, 3], previous research has not addressed the cost-performance issue in optimal estimation.

2. SUBBAND DOMAIN (SD) FILTERING

Our first step is to establish the equivalence between input-domain and subband-domain optimal filtering. The optimal ***input-domain*** filter uses the received signal $x(n)$ to construct an estimate $\hat{d}(n) = T \circledast x(n)$ of some desired signal $d(n)$. The optimal subband-domain filter $\mathcal{T}(z)$ accomplishes the same task but in terms of the subband domain representations $\xi(\cdot), \delta(\cdot)$ of the signals $x(\cdot), d(\cdot)$ (Fig. 3).

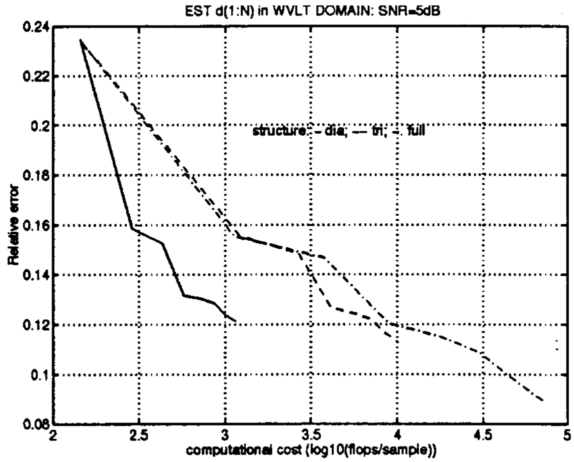

Figure 2: Cost-Performance Trade-Off.

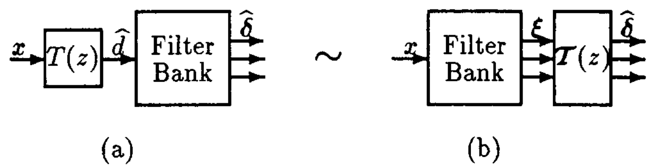

Figure 3: Linear filtering in: (a) input-domain vs. (b) subband-domain

The subband-domain filter is considered equivalent to a given input-domain filter $T(z)$ when both configurations in Fig. 3 have the same input-output relation. A necessary and sufficient condition for equivalence is

$$\mathcal{T}(z^L) = \mathcal{H}(z)\,\mathcal{D}_L\{T(z)\}\,\mathcal{H}^{-1}(z) \qquad (1a)$$

where $\mathcal{H}(z)$ is the so-called alias component matrix [4] associated with the filter bank, viz.,

$$\mathcal{H}(z) = \begin{bmatrix} \mathbf{H}(z) & \mathbf{H}(w_L z) & \mathbf{H}(w_L^2 z) & \dots & \mathbf{H}(w_L^{L-1} z) \end{bmatrix} \qquad (1b)$$

$$\mathbf{H}(z) = \begin{bmatrix} H_0(z) & H_1(z) & H_2(z) & \dots & H_{L-1}(z) \end{bmatrix}^T \qquad (1c)$$

and where $\mathcal{D}_L\{T(z)\}$ is the diagonal matrix.

$$\mathcal{D}_L\{T(z)\} = \; diag\{T(z), T(w_L z), \dots, T(w_L^{L-1} z)\} \qquad (1d)$$

The fundamental equivalence mapping also implies that

$$S_{\xi\xi}(z) = \frac{1}{L}\mathcal{H}(z^{1/L})\mathcal{D}_L\{S_{xx}(z^{1/L})\}\mathcal{H}_*(z^{1/L}) \qquad (2a)$$

$$S_{\delta\xi}(z) = \frac{1}{L}\mathcal{H}(z^{1/L})\mathcal{D}_L\{S_{dx}(z^{1/L})\}\mathcal{H}_*(z^{1/L}) \qquad (2b)$$

This leads to the conclusion that the optimal (non-causal) input-domain Wiener filter $T_{opt}(z) = S_{dx}(z)S_{xx}^{-1}(z)$ (see, e.g. [5] maps via (1) into the optimal (non-causal) subband-domain Wiener filter $\mathcal{T}_{opt}(z) = S_{\delta\xi}(z)S_{\xi\xi}^{-1}(z)$. Moreover, a similar conclusion also holds for FIR causal Wiener filters:

The input-domain FIR causal optimal filter of order M_T maps via (2) to the optimal subband-domain FIR causal filter of order

$$M_{\mathcal{T}} = \frac{M_T + 2M_H}{L} \tag{3}$$

where we have assumed that the filter bank $\mathcal{H}(z)$ is paraunitary and FIR of order M_H.

Since most commonly-used wavelet-based decompositions involve paraunitary filter bank [4], this result applies, in particular, to wavelet-based estimation.

The significance of the equivalence between input-domain and subband-domain is that is allows us to use the (full-complexity) subband-domain optimal filter $\mathcal{T}_{opt}(z)$ as a reference point for comparison with other schemes. In the following section, we discuss the principles of constructing (suboptimal) subband-domain estimation schemes with significantly reduced implementation cost.

3. STRUCTURE OF SD FILTERS

The construction of a (suboptimal) subband-domain estimation scheme involves the selection of five design parameters, corresponding to the five factors that affect the cost-performance trade-off discussed in the introduction. In particular, we focus on filter bank associated with the notion of wavelets and wavepackets. Such filter bank are constructed using a binary tree configuration which involves a single undetermined prototype lowpass filter $P(z)$ [4]. Thus, the resolution level of such filter bank is determined only by the depth of the binary tree, while the degree of channel overlap depends only on the response of the prototype filter $P(z)$.

Since the degree of channel overlap play a central role in determining the performance degradation associated with sparse estimation schemes, one should attempt to reduce overlap within the cost constraints imposed on the design. For instance, we have shown that a wavelet-packet configuration is preferable to a wavelet configuration in the sense that it results in an improved cost-performance trade-off when used in conjunction with sparse estimation schemes.

We use a block-processing scheme, so that subband-domain processing is carried out at the block-rate which, in our example is $\frac{1}{128}$ of the input signal rate. As a consequence the total number of samples per channel (per one block of output data) is $\frac{N}{2^r}$ where r is the number of layers of our wavelet-based filter bank (depth of binary tree), and N is the length of the input data block (see Fig. 4). The use of a block-processing scheme introduces another reduction in the overall implementation cost, since the number of computations per one sample of the input data is thereby reduced by a factor of N. We use two sparse estimation schemes:

- Diagonal estimation, in which the estimate of $\delta_i(\cdot)$ is constructed from samples of $\xi_i(\cdot)$ only.
- Tridiagonal estimation, in which the estimate of $\delta_i(\cdot)$ is constructed from samples of $\xi_i(\cdot)$, $\xi_{i-1}(\cdot)$ and $\xi_{i+1}(\cdot)$.

Together with non-sparse estimation (which uses all available channels) this gives rise to three choices of estimation sparsity, which we call "full", "tridiagonal", and "diagonal" (as in Fig. 2). Also, we demonstrate the effect of the utilization ratio by processing only the subset of all available channels corresponding to $1 \leq i \leq L$, where L can vary in the range $1 \leq L \leq 2^r$. As for the filter order, we consider only two cases: (i) full, which uses all $\frac{N}{2^r}$ samples available from a single channel, and (ii) memoryless, which uses only a single sample.

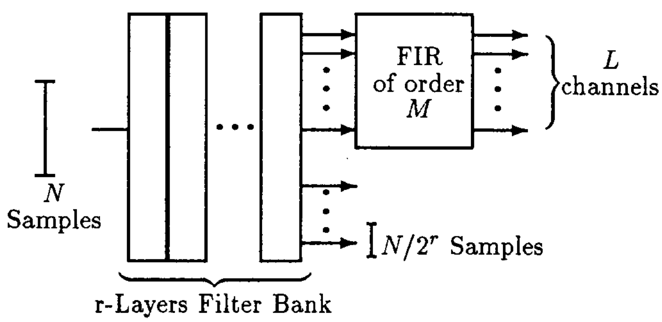

Figure 4: Structure of Subband-Domain Processing.

4. COST-PERFORMANCE TRADE-OFF

We now turn to evaluate the estimation error as a function of the various design parameters for an example involving the estimation of a signal $d(n)$ from a noise-corrupted version $x(n) = d(n) + w(n)$. We scale the estimation error by the (square root of) the power of the desired signal $d(n)$, so that the resulting relative error can be compared across examples.

The effect of the utilization ratio η alone is shown in Fig. 5, which involves three different levels of the signal-to-noise ratio (SNR) in the received signal $x(n)$.

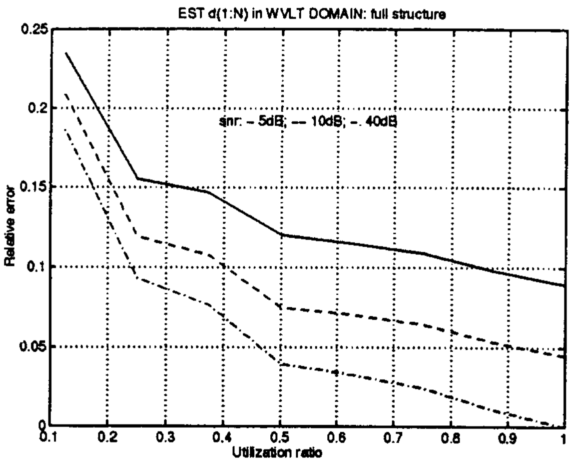

Figure 5: Effect of utilization ratio (η)

The error reduces with increasing η, the effect being more pronounced for high SNRs.

The introduction of sparsity into our estimation schemes results in very minor degradation in performance (except when η is close to unity). However, since sparse estimation requires fewer computations, the cost-performance trade-off curves (Fig. 2) show a dramatic advantage for sparse schemes: one can achieve a reduction of up to an order of magnitude (i.e., a factor of 10) in computational cost for a given level of estimation error.

The effect of filter order is similar to that of estimation sparsity. Again, using less samples results in an improved cost-performance trade-off (Fig. 6), with up to 2 order of magnitude reduction in computational cost.

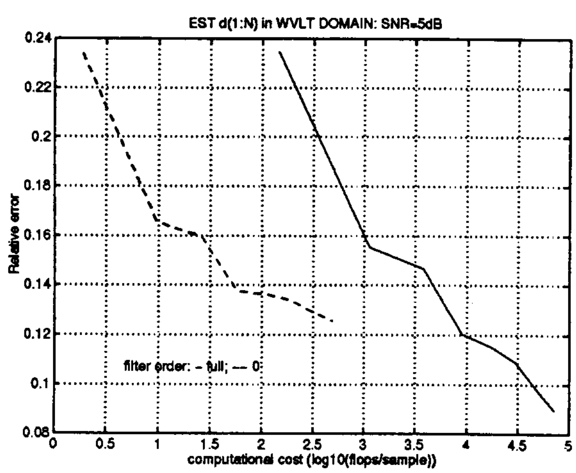

Figure 6: Effect of filter order

Finally, the effects of filter bank selection are shown in Fig. 7. While our example shows a definite preference for simpler filter banks, fewer layers and lower-order filter prototypes, more research is required in order to determine the best choice of a filter bank for a given estimation problem.

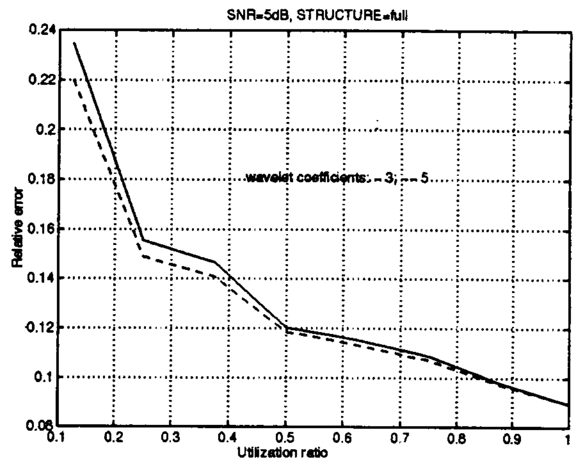

Figure 7: Effect of wavelet coefficients selection

5. REFERENCES

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