SENSOR ARRAY APPROACH TO NONWAVE FIELD PROCESSING

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ABSTRACT

The application of sensor array processing methods for estimation and localization of wavefield sources is well known. In this paper we extend the sensor array processing approach to estimating the parameters of the fields of a nonwave nature (the so-called nonwave fields). Considering the static and diffusion fields as typical examples of nonwave field, we derive the Cramer-Rao bounds of source parameter estimation errors. These theoretical results are completed by the experimental results of localization of diffusion sources in distilled water by a chemical sensor array, showing potentially high performance of sensor array approach. A modified version of the well-known CLEAN deconvolution algorithm has been used for experimental data processing. The nonwave field sensor array processing can find various applications such as localization of pollution sources and another types of admixtures, detection of metallic masses and wandering currents, etc.

1. INTRODUCTION

The use of sensor array processing for the estimation of the parameters of wavefields is known to give excellent results, and it was studied in detail through last decades [1], [2], [3]. In fact, the typical assumption for wavefield signal processing techniques is the assumption that the signal model can be represented as a set of point sources, and that the array is well calibrated. The presence of model errors essentially reduces the sensor array processing performance and requires to involve some additive calibration technique in the processing scheme [3].

In this paper we apply the sensor array processing approach to the case of the so-called nonwave fields (i.e., the fields of nonwave nature). The nonwave fields are well known to differ from the wavefields by the absence of spatial periodicity, i.e., wavelength. An extension of sensor array processing approach to the nonwave field case seems to be promising [4], [5] due to the fact that sensor array processing methods can be successfully employed in the case of exact characterization of the array response. Fortunately, in large number of situations where the nonwave field is measured by a sensor array, *a priori* knowledge of the spatio-temporal array response (the so-called array manifold) is available. Hence, sensor array processing approach can be applied to nonwave field case. However, such an extension often is nontrivial because of the specific features of the considered nonwave field. The proposed sensor array processing approach to the localization of nonwave sources is also cardinally different as compared with the approach which is often used for solving the inverse problems. Actually, the inverse approach provides a smoothed solution which is not applicable to the problem of separation of the point and closely spaced sources. On the contrary, the sensor array approach exploits the point source model and, therefore, it is applicable to the specific problem of localization and estimation the closely spaced sources.

Considering the static and diffusion fields as typical examples of nonwave fields, and assuming that the measurements are carried out by an antenna array, we derive in this paper the Cramer-Rao bounds of source parameter estimation errors. These theoretical results are completed by the experimental results of localization of diffusion sources in distilled water by chemical sensor array, showing potentially high performance of the sensor array processing approach. A modified version of the well-known CLEAN deconvolution algorithm has been used for experimental data processing.

In fact, we suggest a new type of a sensor array (chemical array) for spatial signal processing. In particular, such a type of sensor array can be successfully employed for the localization of pollution sources as well as other types of admixtures.

2. SPATIAL MODEL OF NONWAVE FIELD

Consider an uniform linear array (ULA) of m sensors, measuring nonwave field from q point sources, where $q < m$. Without loss of generality, let us assume that the sources are localized at the (x, y) plane, and let the

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l-th source has the coordinates (x_l, y_l). Let us assume as well that the sensors are localized at the x axis and the coordinate of n-th sensor is $(\tilde{x}_n, 0)$ (the "tilde" sign means below that coordinate corresponds to the point of receiver) where

$$\tilde{x}_n = d\left(n - \frac{m+1}{2}\right), \quad n = 1, 2, \ldots, m \qquad (1)$$

and d is the spacing between neighboring sensors.

The sensor outputs can be generally modeled as:

$$r_n = \sum_{l=1}^{q} s_l g\left(\frac{\tilde{x}_n - x_l}{\rho_l}\right) + \epsilon_n, \quad n = 1, 2, \ldots, m \quad (2)$$

where s_l is the nonrandom amplitude of l-th source, $g(\xi)$ is a function, describing the nth sensor response corresponding to the lth source, ρ_l is the characteristic scale of source field slump with the distance, ϵ_n is the random measurement noise. For example, for electrostatic or gravitation field, the potential can be represented as

$$g(\xi) = (1 + \xi^2)^{-1/2}, \quad \rho_l = y_l \qquad (3)$$

For the diffusion field, raised by single carrying of a portion of substance in a thin layer of liquid, we have

$$g(\xi) = \exp(-\xi^2), \quad \rho_l = 4D^2 t_l \qquad (4)$$

where D^2 is the diffusion coefficient, t_l is the time passed since the appearance of the source in the liquid at point (x_l, y_l).

Let us assume that the random noise ϵ_n is statistically independent and Gaussian with zero mean and variance σ^2. Such a model of noise can be caused, for example, by random measurement errors in sensors. Rewriting (2) in a vector notation, we have

$$\boldsymbol{r} = \boldsymbol{G}\boldsymbol{s} + \boldsymbol{e} \qquad (5)$$

where

$$\boldsymbol{r} = (r_1, r_2, \ldots, r_m)^T \qquad (6)$$

is the $m \times 1$ vector of array outputs,

$$\boldsymbol{s} = (s_1, s_2, \ldots, s_q)^T \qquad (7)$$

is the $q \times 1$ vector of the amplitudes of the sources,

$$\boldsymbol{G} = (\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_q) \qquad (8)$$

is the $m \times q$ matrix of the $m \times 1$ array propagation vectors:

$$\boldsymbol{g}_l = \left(g\left(\frac{\tilde{x}_1 - x_l}{\rho_l}\right), g\left(\frac{\tilde{x}_2 - x_l}{\rho_l}\right), \ldots, g\left(\frac{\tilde{x}_m - x_l}{\rho_l}\right)\right)^T \qquad (9)$$

for $l = 1, 2, \ldots, q$,

$$\boldsymbol{e} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_m)^T \qquad (10)$$

and T denotes transpose. Note, that in this Section we consider for simplicity the single snapshot case; the extension for the spatio-temporal case will be made below. Therefore, under (5), the array output vector has the real multivariate Gaussian distribution with nonzero mean:

$$\boldsymbol{r} \sim RN(1, \boldsymbol{G}\boldsymbol{s}, \sigma^2 \boldsymbol{I}) \qquad (11)$$

where $\boldsymbol{I}$ is the $m \times m$ identity matrix.

3. CRB OF NONWAVE SOURCE PARAMETER ESTIMATION

A useful tool for evaluating the potential accuracy of parameter estimation is the CRB, showing the lower bound of parameter estimation errors. It is well known that for any unbiased estimate of vector parameter $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_K)^T$ (K is the total number of unknowns), the CRB is given by the diagonal elements of inverted Fisher information matrix $\boldsymbol{J}^{-1}$, i.e.,

$$\text{CRB}(\hat{\theta}_i) = [\boldsymbol{J}^{-1}]_{ii} \qquad (12)$$

For multivariate real Gaussian vector $\boldsymbol{r} \sim RN(1, \boldsymbol{\mu}, \boldsymbol{R})$ the (i,j)th element of Fisher information matrix can be written as

$$J_{ij} = \text{Tr}\left(\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{R}}{\partial \theta_i}\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{R}}{\partial \theta_j}\right) + \text{Re}\left(\frac{\partial \boldsymbol{\mu}^T}{\partial \theta_i}\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{\mu}}{\partial \theta_j}\right) \qquad (13)$$

Consider the general case of nonwave field described by array output model (2), (5) and assume that the single source is localized at the point (x_1, y_1) of (x, y) plane. The vector of unknown parameters (σ^2 excluded, because assumed to be known *a priori*) is $\boldsymbol{\theta} = (s_1, \rho_1, x_1)^T$. Using (13), it is easy to show that the diagonal elements of the 3×3 inverted Fisher information matrix

$$[\boldsymbol{J}^{-1}]_{ii} = \left(\frac{d}{\rho_1}C_{ii}\right)\begin{cases} \sigma^2 & \text{if } i = 1 \\ (\sigma^2/s_1^2)\rho_1^2 & \text{if } i = 2, 3 \end{cases} \qquad (14)$$

where $\boldsymbol{C}$ is the matrix, defined via the inverted one $\boldsymbol{S} = \boldsymbol{C}^{-1}$:

$$[\boldsymbol{C}^{-1}]_{ij} = S_{ij} = (d/\rho_1)\sum_{n=1}^{m}\psi_i(\xi_n - \beta)\psi_j(\xi_n - \beta), \qquad (15)$$

for $i, j = 1, 2, 3$. Matrix (15) is different for each type of field. Here the following parameters must be introduced

$$\xi_n = \tilde{x}_n/\rho_1, \quad \beta = x_1/\rho_1,$$

$$\psi_1(\xi) = g(\xi), \quad \psi_2(\xi) = -\xi g'(\xi), \quad \psi_3(\xi) = -g'(\xi)$$

Let now derive the form of the matrix C for different types of received signal model, namely, for the static field (3) and the diffusion field (4). Let us assume for simplicity that the distance d is much less than the characteristic scale ρ_1. This assumption means that the sum in (15) can be replaced by a corresponding integral with the limits $[-\alpha, \alpha]$, where $\alpha = md/2\rho_1$. Fortunately, the assumption $d \ll \rho_1$ can be easily satisfied in practice because it can be taken into account when choosing the construction of the receiving array. Thus, we find that

$$S_{ij} = \int_{-\alpha-\beta}^{\alpha-\beta} \psi_i(\xi)\psi_j(\xi)\, d\xi \tag{16}$$

3.1 Static Field Case

For static field case the CRB's can be derived straightforwardly using (16) and (3). The final expressions are complicated and the reader can find them in [5].

In order to get more simple relations for CRB's let us assume that $md \gg \rho_1$, i.e., $\alpha \gg 1$. The comparison of this condition with $d \ll \rho_1$ shows that these two inequalities can be satisfied both when the number of sensors is large $m \gg 1$. The assumption $\alpha \gg 1$ enables to take the infinite limits in the integral in the right part of (16). In this case, we have

$$C = \frac{1}{\pi}\begin{pmatrix} 3 & -4 & 0 \\ -4 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} \tag{17}$$

The structure of matrix (17) implies that the asymptotic CRB's of $\hat{s}_1$ and $\hat{\rho}_1$ are independent of whether the parameter x_1 is known or unknown, and vice versa. Equation (17) also shows that the asymptotic CRB's of $\hat{\rho}_1$ and $\hat{x}_1$ coincide exactly.

Substitution of (17) into (14) yields

$$\mathrm{CRB}(\hat{s}_1) = \frac{3}{\pi}\frac{d}{\rho_1}\sigma^2 \tag{18a}$$

$$\mathrm{CRB}(\hat{\rho}_1) = \mathrm{CRB}(\hat{x}_1) = \frac{8}{\pi}\frac{d\,\rho_1}{s_1^2}\sigma^2 \tag{18b}$$

where we must recall that in accordance with (3) the parameters ρ_1 and y_1 are the same.

3.2. Diffusion Field Case

Making the same assumption as in the last subsection ($md \gg \rho_1$), i.e., considering the asymptotic case of large number of sensors, we have

$$C = \sqrt{\frac{2}{\pi}}\begin{pmatrix} \frac{3}{2} & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{19}$$

The structure of matrix (19) implies that the asymptotic CRB's of $\hat{s}_1$ and $\hat{\rho}_1$ are independent of whether the parameter x_1 is known or unknown. By the substitution of (19) into (14) we can write the CRB's as follows:

$$\mathrm{CRB}(\hat{s}_1) = \frac{3}{\sqrt{2\pi}}\frac{d}{\rho_1}\sigma^2 \tag{20a}$$

$$\mathrm{CRB}(\hat{\rho}_1) = 2\sqrt{\frac{2}{\pi}}\frac{d\,\rho_1}{s_1^2}\sigma^2 \tag{20b}$$

$$\mathrm{CRB}(\hat{x}_1) = \sqrt{\frac{2}{\pi}}\frac{d\,\rho_1}{s_1^2}\sigma^2 \tag{20c}$$

4. SPATIO–TEMPORAL EXTENSIONS

The important feature of nonwave fields is that the various types of them essentially differs by the temporal dependence. The most simple temporal dependence is inherent in the static field which array output vector can be modeled as

$$\boldsymbol{r}(t) = \boldsymbol{G}\boldsymbol{s} + \boldsymbol{e}(t), \quad \boldsymbol{r} \sim RN(M, \boldsymbol{G}\boldsymbol{s}, \sigma^2\boldsymbol{I}) \tag{21}$$

Hence, the CRB for static field can be trivially reformulated for the multisample case by taking into account the number of samples M in final expressions. Furthermore, the model (21) enables to apply the deterministic (conditional) maximum likelihood (ML) technique.

The extension on the spatio-temporal case for the diffusion field sources is more complicated due to the nonstationary character of temporal dependence. For two-dimensional diffusion (for example, when the substance is dissolved in the thin layer of liquid), the nth sensor output can be modeled as:

$$r_n(t) = \sum_{l=1}^{q}\int_{t_l}^{t} n_l(\tau)\,\frac{e^{T_{l,n}/(t-\tau)}}{t-\tau}\,d\tau + \epsilon_n(t) \tag{22}$$

where $T_{l,n} = (y_l^2 + (\tilde{x}_n - x_l)^2)/4D^2$, t_l is the time of appearance of the lth source in a liquid, $n_l(t)$ is the temporal dependence for the lth source ($n_l = 0$ for $t < t_l$), while $\epsilon_n(t)$, D^2, $\tilde{x}_n$, and (x_l, y_l) are the introduced above sensor noise, diffusion coefficient, coordinate of nth sensor, and lth source coordinates, respectively. We limit our consideration below by two typical models of the function $n_l(t)$:

$$n_l(t) = n_l\delta(t - t_l) \tag{23a}$$

$$n_l(t) = N_l, \qquad t > t_l \tag{23b}$$

corresponding to the so-called "ideal drop" and "ideal crystal" sources, respectively.

For the model (23a) after conversion to a vector notation we have such type of temporal behavior:

$$r(t) = G(t)s(t) + e(t) \qquad (24a)$$

In turn, for the model (23b)

$$r(t) = G(t)s + e(t) \qquad (24b)$$

Hence, in the diffusion field case the array output vectors can be modeled as the real multivariate Gaussian vectors with the mean, depending on time. In special cases, where either temporal only (a single sensor) or spatial only (a single snapshot) measurements are available, the ambiguities exist, which do not allow one to estimate the full number of parameters. These ambiguities are considered in [5].

5. EXPERIMENTAL RESULTS

5.1. Description of Experiment

The experiments with the diffusion sources and the chemical sensor array were carried out as follows. We placed the ULA of 12 sensors in the bath with the thin layer of distilled water. Each sensor of the array was used for the measurement of the conductivity and was constructed as a pair of closely spaced electrodes. In the fixed moments the crystals of $KMnO_4$ or $NaCl$ were placed in the water and results were recorded in digital form.

5.2 Algorithm of experimental data processing

Below we describe the modification of well known CLEAN algorithm, which is adopted here for diffusion field processing. The modification of the algorithm includes least-squares additive estimation of unknown parameters of the array response function at each step of the algorithm. The output of the nth sensor can be represented as noisy convolution of the unknown spatial distribution $P(x)$ of the sources and of the known (within a width ρ) Gaussian function g:

$$r_n = \int_{\Omega_x} P(x) g\left(\frac{\tilde{x}_n - x}{\rho}\right) dx + \epsilon_n, \quad n = 1, 2, \ldots, m \qquad (25)$$

where Ω_x is the area of observation along the x axis.

The $i+1$th step of the effective deconvolution algorithm includes the following substeps:

1).Determine the number of sensor corresponding the maximum of $R_i(\tilde{x}_n)$. Denote the coordinate of maximum as $\tilde{x}_{max,i}$.

2).Calculate the least-square estimates $\hat{\rho}_i$, $\tilde{x}_{pmax,i}$ and $\hat{R}_i(\tilde{x}_{pmax,i})$ of the width ρ, of the precised coordinate of maximum, and of the value of function $R_i(\tilde{x}_{pmax,i})$ corresponding to coordinate $\tilde{x}_{pmax,i}$, respectively. Employ here *a priori* knowledge of the Gaussian form of function g.

3).Carry out the iteration

$$R_{i+1}(\tilde{x}_n) = R_i(\tilde{x}_n) - w\hat{R}_i(\tilde{x}_{pmax,i}) g\left(\frac{\tilde{x} - \tilde{x}_{pmax,i}}{\hat{\rho}_i}\right) \qquad (26)$$

where w is a weight constant ($w \simeq 0.1$).

4). Compare the global maximum of function $R_{i+1}(\tilde{x}_n)$ with the threshold Π, which is determined before processing using *a priori* information about the noise variance. If $\max R_{i+1}(\tilde{x}_n) > \Pi$, go to the next ($i+2$th) step of algorithm. If vice versa, then stop.

The initialization of the algorithm 1).-4). includes the choosing of a weight constant w and: $R_0(\tilde{x}_n) = r_n$.

The reconstructed distribution of sources (the so-called "cleaned" image) after final step of the algorithm 1).-4). is constructed as

$$F(\tilde{x}) = \sum_{i=0}^{M_t - 1} w\hat{R}_i(\tilde{x}_{pmax,i}) \delta(\tilde{x} - \tilde{x}_{pmax,i}) \qquad (27)$$

where M_t is a total number of iterations.

5.3 Results of Data Processing

The use of modified CLEAN algorithm for diffusion data processing allows to essentially improve the performance of source localization. The estimation errors of reconstructed images are less than that without processing (see [5] for details).

Moreover, the modified CLEAN algorithm performs well enough in the situations, when nothing can be said about the number and the coordinates of sources. The results of source reconstruction allow to detect the number of sources as well.

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