CYCLIC DETECTION IN A NONWHITE GAUSSIAN NOISE

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ABSTRACT

This paper deals with detection of weak cyclostationary signals embedded in colored gaussian noise. We consider the normalized correlation function of the noise to be known and the noise power to be unknown. We propose a temporal structure of the single cycle detector which includes a prewhitening filter. We compare performances of this detector to the classical radiometer and the modified radiometer. The performances are quantified in terms of Receiver Operating Characteristics for two different noise power spectral densities. We compute the theoretical deflection, this measure gives a means to choose the best cyclic frequency used in the single cycle detector. We conclude that cyclic methods outperform radiometric methods when the noise and the signal power spectral densities strongly overlap and for a unknown noise power.

1. INTRODUCTION

The solution of weak random signal detection, when the random signal is represented as a stationary process, leads to the well known *radiometer* [1]. In many applications, like communication or underwater acoustic, the cyclostationary model is more appropriate for modulated signals (PAM, BPSK, MSK, ...) in order to exploit all the a priori knowledge about the signal. So in the sequel, we suppose that the signal is wide sense almost cyclostationary i.e. its autocorrelation contains multiple additive periodicities, and these periodicities are incommensurate.

Recent works have studied the detection of weak cyclostationary signals embedded in a white noise [2], [3], [4]. Because the white noise assumption is often violated in practice, Gardner [5] has suggested a cyclic detector written in the frequency domain, using only one cyclic frequency and taking into account the noise Power Spectral Density (PSD). The cyclic detector is called the *single cycle* (SC) in the white noise case and *prewhitened single cycle* (PSC) in the colored noise case.

In the present paper, we study in detail a time domain implementation of this detector and particularly the effect of the prewhitening operation on the detection performances. To do this, we assume an a priori knowledge of the normalized noise correlation function except for the noise power. More precisely, we model the noise as an autoregressive moving average (ARMA) process excited by an independent identically distributed gaussian sequence with unknown variance.

We study the theoretical deflection of the cyclic detector versus all cyclic frequencies. So, we can choose the cyclic frequency that maximizes this deflection. For this one and others cyclic frequencies, we compare the performances, via computer simulations, of the PSC detector and the *prewhitened modified radiometer* detector (PMR) in terms of Receiver Operating Characteristics (ROC) curves for a BPSK signal. We particularly study the case of strong overlap between noise and signal power spectral densities.

2. PROBLEM STATEMENT

The problem of detecting a random signal in additive noise can be represented by the following hypotheses test :

$$\begin{array}{lll} H_0 & : & \mathbf{x} = \mathbf{z} \\ H_1 & : & \mathbf{x} = \mathbf{s} + \mathbf{z} \end{array} \tag{1}$$

where x and z denote N-dimentionnal column vectors whose components are samples of the received signal and noise. The vector s denotes the signal to be detected, which is modeled as a zero-mean almost cyclostationary real process. The vector z denotes the stationary zero mean gaussian noise, assumed to be statistically independent with the vector s, described by its positive-definite symmetric covariance matrix, R_z, such that $R_z{}^{-1} = C^t C/\sigma^2$, where C is a N×N unique invertible lower triangular matrix. The vector z is related to n, also a gaussian vector with independent, identically distributed components (of zero-mean and σ^2 variance), via the linear transformation :

$$\mathbf{z} = C^{-1}\mathbf{n} \tag{2}$$

Using relation (2), the detection problem (1) now becomes:

$$\begin{array}{lll} H_0 & : & \tilde{\mathbf{x}} = \mathbf{n} \\ H_1 & : & \tilde{\mathbf{x}} = \tilde{\mathbf{s}} + \mathbf{n} \end{array} \tag{3}$$

In (3), $\tilde{\mathbf{x}}$ is the transformed observation vector $\tilde{\mathbf{x}} = C\mathbf{x}$ and $\tilde{\mathbf{s}} = C\mathbf{s}$ is the transformed zero-mean signal vector which covariance matrix is given by $R_{\tilde{s}} = C R_s C^t$.
This leads to the receiver structure (4), which is a prewhitened version of the locally optimum detector (LOD's) in the independent case :

$$Z = \sum_{i,m=1}^{N} \left(\frac{\tilde{x}_i \tilde{x}_m}{\sigma^4} - \delta_{i,m}\frac{1}{\sigma^2}\right) R_{\tilde{s}}(i,m) \tag{4}$$

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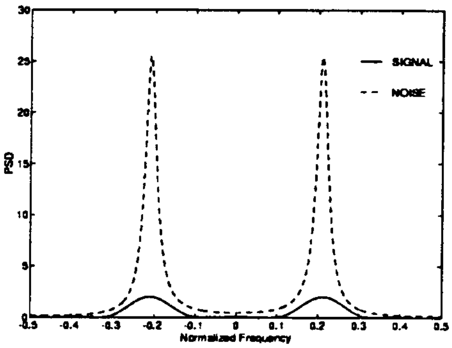

Figure 1: Signal and noise Power Spectral Densities (PSD) SNR=-7db (example 1)

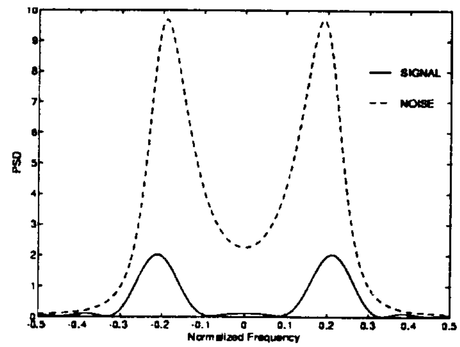

Figure 2: Signal and noise Power Spectral Densities (PSD) SNR=-7db (example 2)

where $\delta_{i,m}$ is the Kronecker delta and $R_{\tilde{s}}(i,m)$ is the i-mth element of the covariance matrix of the transformed signal:

$$R_{\tilde{s}}(i,m) = E[\tilde{s}_i\tilde{s}_m] = \sum_{k,l=1}^{N} C_{i,k}C_{m,l}R_s(k,l) \qquad (5)$$

In the sequel, we suppose that N is sufficiently large ($N \rightarrow \infty$), thus, the matrix C becomes the impulse response matrix of the inverse filter $C(z)$:

$$C(z) = \sum_{k=0}^{N-1} C_k z^{-k} \qquad (6)$$

3. CYCLIC DETECTORS

The almost cyclostationarity assumption allows us to express $R_s(k,l)$ in the Fourier-series form :

$$R_s(k,l) = \sum_{\alpha} R_s^\alpha(k-l)\exp[j\pi\alpha(k+l)] \qquad (7)$$

where

$$R_s^\alpha(r) \stackrel{\text{def}}{=} \lim_{L\rightarrow\infty} \frac{1}{2L+1}\sum_{m=-L}^{L} R_s(r+m,m)\exp[-j2\pi\alpha(m+\frac{|r|}{2})]$$

is the cyclic autocorrelation function, and the sum α is, in general, over integer multiples of fundamental frequencies of cyclostationarity.

By substituting (7) and (5) in (4) under assumption (6) and assuming :

- coefficients C_k vanish for large values of k (which is a valid assumption for stable filter);
- the sample size N greatly exceeds the largest lag cyclic autocorrelation function ($N \gg |r^\alpha_{\max}|$) such that the cyclic autocorrelation function $R_s^\alpha(r) \neq 0$;
- the sample size N greatly exceeds the largest period of cyclostationarity of the signal s, ($N \gg 1/\alpha_{\min}$);

we obtain the following *prewhitened multicycle* (PMC) statistic:

$$Z^{PMC} = \sum_{\alpha} Z^\alpha \qquad (8)$$

$$Z^{PMC} = \sum_{\alpha}\sum_{r=1-N}^{N-1} R_s^\alpha(r)^*\left[R_{\hat{x}}^\alpha(r) - \delta_{\alpha,0}NR_c(r)\hat{\sigma}^2\right]$$

where $\hat{\mathbf{x}} = C^t C\mathbf{x}$ and $R_{\hat{x}}^\alpha(r)$ is an estimate of the cyclic autocorrelation function of the signal $\hat{\mathbf{x}}$ defined by :

$$R_{\hat{x}}^\alpha(r) = \frac{1}{N}\sum_{i=1}^{N-|r|} \hat{x}_i\hat{x}_{i+|r|}\exp[-j2\pi\alpha(i+|r|/2)] \qquad (9)$$

where $R_c(r)$ is the autocorrelation of the impulse response of the inverse filter defined by :

$$R_c(r) = \frac{1}{N}\sum_{k=0}^{N-|r|-1} C_k C_{k+|r|} \qquad (10)$$

where $\hat{\sigma}^2$ is an estimate of the parameter σ^2 which is approximately the same under both hypotheses and is given by :

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{k=1}^{N}\tilde{x}_k^2 \qquad (11)$$

The statistic (8) for $\alpha = 0$ leads to the *prewhitened modified radiometer* (PMR) which is expressed as :

$$Z^{PMR} = Z^{PR} - N\sum_{r=1-N}^{N-1} R_s^0(r)R_c(r)\hat{\sigma}^2 \qquad (12)$$

where Z^{PR} is the *prewhitened radiometer* evaluated when the noise power is known.

Note that :

- The PMC detector cannot be implemented without knowledge of the phase of the signal to be detected.

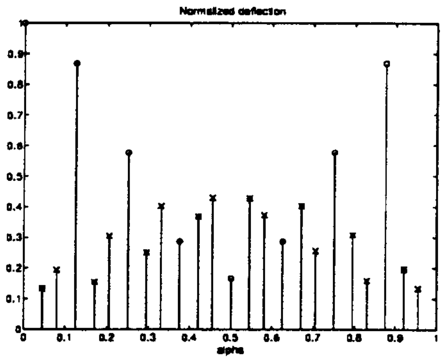

Figure 3: Normalized deflection (example 1) via all the cyclic frequencies for the BPSK signal ($(o) : \alpha = k/T_c$, $(*) : \alpha = 2f0 + k/T_c$, $(\mathrm{x}) : \alpha = -2f_0 + k/T_c \;\; k \in Z$).

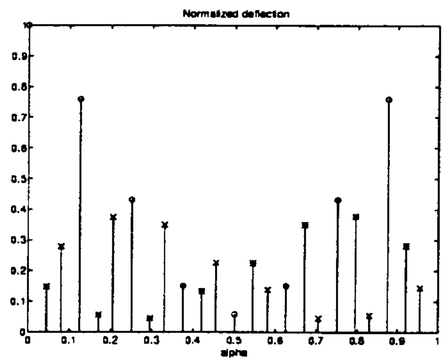

Figure 4: Normalized deflection (example 2) via all the cyclic frequencies for the BPSK signal ($(o) : \alpha = k/T_c$, $(*) : \alpha = 2f0 + k/T_c$, $(\mathrm{x}) : \alpha = -2f_0 + k/T_c \;\; k \in Z$).

A suboptimum structure can be used, referred to as *prewhitened single-cycle detector* (PSC), which employs only one cyclic frequency contained in the signal and then, takes the magnitude of the statistic.

$$Z^{PSC} = |Z^\alpha| \underset{H_1}{\overset{H_0}{\lessgtr}} \gamma \quad (\alpha \neq 0) \tag{13}$$

- The corrective factor (the second term in (8)) due to the estimation of the parameter σ^2 appears only for $\alpha = 0$.
- According to the above remarks, the PSC detector ($\alpha \neq 0$) still remains the same when the noise power is known or unknown. This remark is important to understand the robustness of the cyclic detector in comparison with the PMR detector ; the radiometric method needs a good estimate of σ^2 (11), that is not the case for the cyclic method.

4. DEFLECTION

The squared deflection D, which is a measure of the output SNR, particularly appropriate in weak signal assumption, is defined by :

$$D(Z) = \frac{|E(Z|H_1) - E(Z|H_0)|^2}{\mathrm{VAR}(Z|H_0)} \tag{14}$$

The evaluation of the deflection for the statistic (4), assuming the noise parameter σ^2 to be known is :

$$D = \frac{1}{2\sigma^4} \sum_{i,j=1}^{N} R_{\tilde{s}}^2(i,j) \tag{15}$$

For cyclostationary processes, under the same assumptions used to obtain the statistic (8) we obtain the following expression :

$$D = \sum_\alpha D^\alpha = \frac{N}{2\sigma^4} \sum_\alpha \left\{ \int_{-1/2}^{1/2} |S_{\tilde{s}}^\alpha(f)|^2 df \right\} \tag{16}$$

where $S_{\tilde{s}}^\alpha(f)$ is the cyclic spectrum of $\tilde{s}$ defined as the discrete Fourier transform of the cyclic autocorrelation function

and $$S_{\tilde{s}}^\alpha(f) = C^*(f - \alpha/2) C(f + \alpha/2) S_s^\alpha(f) \tag{17}$$

where $C(f)$ is the discrete Fourier transform of the impulse response of the inverse filter.

D is the PMC deflection except for $\alpha = 0$. By considering the noise power to be known for the evaluation of (16), D^0 corresponds to the PR deflection instead of the PMR deflection.

Equation (17) shows the effect of filtering on cyclostationary processes. This linear time invariant operation can modify, and even destroy cyclostationarity properties. However, the almost cyclostationary assumption (the autocorrelation $R_s(k, m)$ contains multiple additive periodicities, and these periodicities are incommensurate) allows us to find cyclic frequencies such that the cyclic spectrum is not identically zero ($S_{\tilde{s}}^\alpha(f) \neq 0$). The evaluation of the deflection will allow us to choose the cyclic frequency which maximizes D^α. Notice that D^α is not exactly the PSC deflection because of the modulus operator. However, Gardner has shown that the deflection based on the complex detection statistic (D^α) is less than the deflection based on the magnitude (D^{PSC}) for a sufficiently large sample size [4]. Thus, D^α, is a reliably conservative measure of the PSC detector and the evaluation of D^α provides a convenient theoretical means for choosing the best cyclic frequency.

5. SIMULATIONS

In order to compare the performances of PSC and PMR detectors working in a nonwhite gaussian noise with an unknown noise power, simulations have been carried out. The signal to be detected is a BPSK signal embedded in an auto-regressive process (AR).

The signal characteristics are : the time shift keying $T_c = 8$ (eight samples per keying interval) and the reduced carrier frequency is $f_0 = 0.21$.

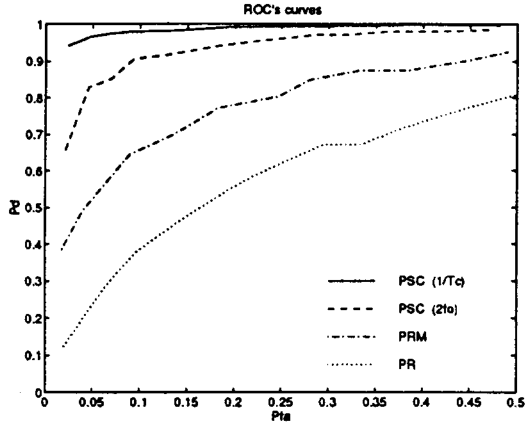

Figure 5: ROC's curves (example 1)

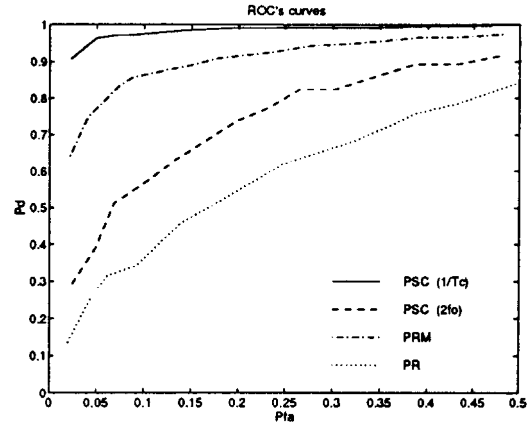

Figure 6: ROC's curves (example 2)

Two examples of auto-regressive (AR) noise processes are considered :
Example 1 : The poles of the AR(2) noise process are at $0.9\exp(\pm j2\pi(0.21))$ (figure 1).
Example 2 : The poles of the AR(4) noise process are at $0.7\exp(\pm j2\pi(0.21))$ and $0.5\exp(\pm j2\pi(0.15))$ (figure 2).

The noise power is randomly distributed according to a normal distribution (mean=μ, variance=$0.1\mu^2$). The parameter μ is fixed by the signal to noise ratio (SNR=-7db) which is evaluated in the signal bandwidth.
We define the normalized deflection as :

$$d^\alpha = \frac{D^\alpha}{D^{PR}} \tag{18}$$

where D^α can be deduced from (16) and D^{PR} is the PR deflection.

In figures 3 and 4, we plot the normalized deflection versus all the cyclic frequencies of the BPSK signal. For the two AR noise processes and the BPSK signal, we find $\alpha = 1/T_c$ or $\alpha = 7/T_c$ as the best cyclic frequencies (figures 3 and 4). Figures 5 and 6 plot ROC's curves for sample size N=512 (64 keying intervals are considered). We compare four detectors for the two examples : the PSC with ($\alpha = 1/T_c$) the best cyclic frequency according to the deflection, the PSC with ($\alpha = 2f_0$) the best cyclic frequency for a white noise model [3], the PMR which is optimum in the stationary signal model case, the PR which is optimum in the stationary signal model and known noise power case.

In both examples of noise PSD, ROC's curves illustrate the superiority of the PSC with $\alpha = 1/T_c$ over others detectors.

From Figure 5, for a strong correlation between noise samples and a strong overlap between noise and signal PSD, we can conclude that :
$PSC(\alpha = 1/T_c) > PSC(\alpha = 2f_0) > PRM > PR.$
From Figure 6, for a larger bandwidth of the noise PSD, we have :
$PSC(\alpha = 1/T_c) > PRM > PSC(\alpha = 2f_0) > PR.$

From these results, we can draw several conclusions :

- The noise PSD interacts with the choice of the cyclic frequency of the most powerful PSC detector.
- Deflection is a reliable means for choosing this cyclic frequency.
- In our simulations, the PSC ($\alpha = 1/T_c$) gives better performance than the expected PSC ($\alpha = 2f_0$).

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