On Detection of Cyclostationary Signals

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Abstract

This paper presents a detector for cyclostationary signals that uses an estimate of spectral autocoherence as a detection statistic. The detector's probability of false alarm is analytically derived and verified with computer simulations. Receiver operating characteristic curves are determined from the simulation data and the analytical expression for the probability of false alarm. A detector for polycyclic signals based on generalized coherence estimation is also described, but is not analyzed.

1 Introduction

A simple *single-cycle* detector for cyclostationary signals based on estimation of spectral correlation has been proposed by Gardner [1]. This paper introduces a modified version of Gardner's single-cycle detector in which magnitude-squared coherence (MSC) estimation is used as a measure of the spectral correlation in a signal. The key advantage of using MSC estimation is that the distribution function of the MSC estimate used in the detector can be determined analytically when no spectral correlation (and hence no cyclostationary component) is present at a particular frequency and cyclic rate. The behavior of this detector in a noise-only (H_0) environment can thus be determined and detection threshold values corresponding to desired false alarm probabilities can be computed. Computer simulations with signal present can then be used determine the receiver operating characteristics of the detector.

For the sake of brevity, this paper will adopt the notation and terminology of [1, 2] for describing a wide-sense cyclostationary stochastic process $\mathbf{x}(\cdot)$. The reader is referred to these sources for the definitions of the cyclic autocorrelation function $R_{\mathbf{x}}^{\alpha}$, the spectral correlation density (SCD) function $S_{\mathbf{x}}^{\alpha}$, and the spectral autocoherence function $\rho_{\mathbf{x}}^{\alpha}$.

2 Description of the Detector

One of the fundamentals of cyclostationary signal analysis is the observation that a process is cyclostationary if and only if certain of its spectral components are correlated. Specifically, $\mathbf{x}(\cdot)$ exhibits cyclostationarity at spectral frequency f and cyclic frequency α if and only if $\rho_{\mathbf{x}}^{\alpha}(f) \neq 0$. The cyclostationary feature detector described in this section determines the presence or absence of a signal having spectral coherence at a spectral frequency f and cyclic frequency α based on a detection statistic which is an estimate of the magnitude squared of the spectral autocoherence function $|\rho_{\mathbf{x}}^{\alpha}(f)|^2$.

The single-cycle detector is shown in figure 1. The spectral shifters and one-sided bandpass filters produce two spectral components of $\mathbf{x}(\cdot)$. The first component is centered at frequency $f = f_c + \alpha/2$ and has bandwidth B, the bandwidth of the one-sided bandpass filter. The second spectral component is centered at frequency $f = f_c - \alpha/2$. These two signals are downsampled and shifted to baseband. The middle leg of the detector correlates them over a time interval T seconds (N independent samples) long to produce an estimate $|\hat{S}_{\mathbf{x}}^{\alpha}(f_c)|^2$ of the magnitude squared of the spectral correlation density function at f_c and α. This section is identical to the detector described in [1]. The upper and lower legs were added to obtain estimates of the time-averaged spectra of $\mathbf{x}(\cdot)$ at $f = f_c + \alpha/2$ and $f = f_c - \alpha/2$, which are denoted as $\left\langle \hat{S_{\mathbf{x}}} \right\rangle (f_c+\alpha/2)$ and $\left\langle \hat{S_{\mathbf{x}}} \right\rangle (f_c-\alpha/2)$. The addition of

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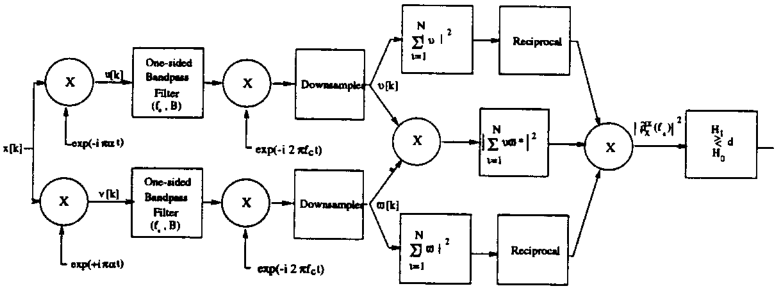

Figure 1: Discrete-time version of cyclostationary feature detector.

the upper and lower legs results in a detection statistic which is an estimate of the magnitude squared of the autocoherence function. The detection statistic is a function of the length of the correlation interval T and the bandwidth B of the bandpass filters, as encompassed in the number of independent samples N used in the correlation sums. The relation of the detection statistic to N is the subject of the next section.

3 Probability of False Alarm

The detection statistic $|\hat{\rho}_{\mathbf{X}}^{\alpha}|^2$ is an estimate of the magnitude-squared coherence of the two signal sequences (ν and ϖ in figure 1) that are the outputs of the downsamplers. The cumulative distribution of $|\hat{\rho}_{\mathbf{X}}^{\alpha}|^2$ can thus be drawn from results previously obtained in the study of MSC estimators.

Magnitude-squared coherence estimators have been studied extensively in connection with sonar and radar applications. If the true MSC $|\gamma|^2$ of two complex processes is zero and one of them is a white complex gaussian process, then the cumulative distribution function of the MSC estimate $\xi = |\hat{\gamma}|^2$ is [3, 4]

$$F_{\underline{\xi}}(\xi) = 1 - (1-\xi)^{N-1} \qquad 0 \le \xi \le 1 \qquad (1)$$

where N is the number of samples used in calculating ξ.

The detection statistic $|\hat{\rho}_{\mathbf{X}}^{\alpha}(f)|^2$ used in the single-cycle detector is an estimate of the MSC of ν and ϖ, the signals at the outputs of the downsamplers. The mathematical calculation made by this MSC estimator from N complex samples $\{x_n + iy_n\}$ of ν and N complex samples $\{u_n + iv_n\}$ of ϖ is,

$$|\hat{\gamma}|^2 = \frac{|\sum_{n=1}^{N}(x_n+iy_n)(u_n-iv_n)|^2}{\sum_{n=1}^{N}|x_n+iy_n|^2 \sum_{n=1}^{N}|u_n+iv_n|^2}, \quad \mathrm{N} \ge 2 \qquad (2)$$

If only WSS noise is present at the input to the detector then the true value of the autocoherence function will be zero. This corresponds to the situation for the MSC estimator where the true value of the MSC is $|\gamma|^2 = 0$. If the noise at the input is white gaussian, then the noise after the bandpass filters will be gaussian as well. It should be noted here that the output of the bandpass filters will be approximately gaussian for a broad class of stochastic input signals [6]. Although the noise at the outputs of the bandpass filters will be correlated, if the downsampling is done correctly the noise will be whitened once again. As shown above, the remaining section of the single-cycle detector is exactly an MSC estimator. The input to the MSC section of the detector is white gaussian, therefore the results in [4] can be applied. Thus, for the case where only white gaussian noise is present at the input of the detector, the detection statistic $\psi = |\hat{\rho}_{\mathbf{X}}^{\alpha}|^2$ will have cumulative distribution function

$$F_{\underline{\psi}}(\psi|\,H_0) = 1-(1-\psi)^{N-1} \qquad 0 \le \psi \le 1 \qquad (3)$$

The probability of false alarm can be determined easily from the cumulative distribution function of the detection statistic for the H_0 case. With detection threshold d,

$$\begin{aligned} P_{FA} &= \Pr(\psi > d|\,H_0) \\ &= (1-d)^{N-1} \end{aligned} \qquad (4)$$

The threshold values that will give specific P_{FA}'s for various values of N are given in Table 1.

4 Simulations and Results

Computer simulations were performed to verify the analytical expressions for the H_0 distribution of the detection statistic and P_{FA} given above and also to determine receiver operating characteristic (ROC) curves for the detector. For the H_0 case, white gaussian noise was input to the detector. Curves were generated for various values of N, the number of independent samples used in the summations. A graph of the

P_{FA}	N				
	8	16	32	64	128
10^{-1}	0.280	0.142	0.0716	0.0359	0.0180
10^{-2}	0.482	0.264	0.138	0.0705	0.0356
10^{-3}	0.627	0.369	0.200	0.104	0.0529
10^{-4}	0.732	0.459	0.257	0.136	0.0700
10^{-5}	0.807	0.536	0.310	0.167	0.0867
10^{-6}	0.861	0.602	0.360	0.197	0.103

Table 1: Detection threshold values for various values of N.

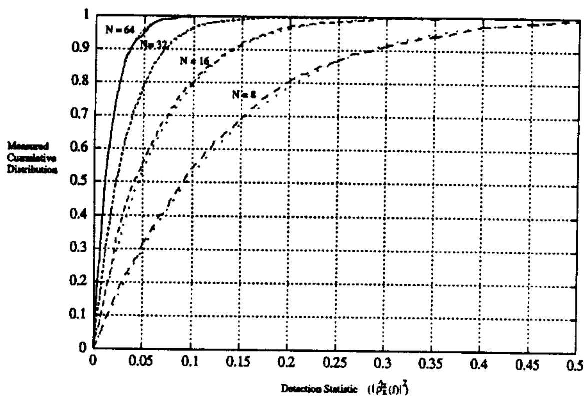

Figure 2: Measured and theoretical cumulative distributions of the detection statistic for various values of N for the H_0 case.

measured cumulative distributions is given in figure 2. The theoretical distributions are also shown for the sake of comparison. It can be seen that the theoretical and empirical results match very closely. From these results it can be concluded that the theoretical analysis was accurate and that equation (3) provides a valid representations of the cumulative distribution of the detection statistic and that (4) provides a valid representation of the probability of false alarm for the single-cycle detector.

Following the verification of the theoretical analysis of the probability of false alarm, simulations were performed to measure the ROC curves for the single-cycle detector with a cyclostationary signal present. A binary-phase-shift-keyed (BPSK) signal was used as the cyclostationary signal. The spectral components centered about the carrier frequency f_0 and separated by and amount $\alpha = R$, where R is the data rate, have complete correlation. The single-cycle detector was set to these values. At these values the autocoherence function for the BPSK signal has unity magnitude. For these simulations N was kept at 32 and the bandwidth of the bandpass filters was set to $0.075R$. The

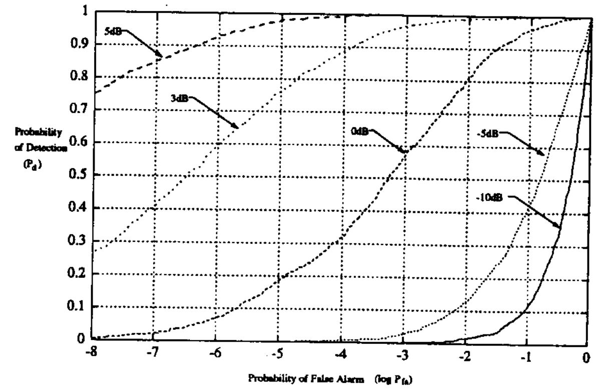

Figure 3: Measured receiver operating characteristics (-10dB to 5dB.)

resulting ROC curves are shown figure 3. From these figure it can be seen that the detector, with the settings just described, performs well for SNR's in excess of 5dB, adequately for SNR's in the range of 0dB to 3dB, and poorly for SNR's below 0dB. The performance can be improved by decreasing the bandwidth of the detector's single-sided bandpass filters. It was shown that decreasing the bandwidth 15 times (from $0.075R$ to $0.005R$) led to an improvement in performance of approximately $11dB$, as was expected since $10\log(15) \approx 11dB$. Also, using larger values for N will improve the performance. The price paid for the improvement in performance in both of these cases is that a longer collection time is required.

In other simulations, a strong signal that had spectral coherence at a (f, α) pair other than at the one to which the detector was set was input to the detector. The measured cumulative distribution curves showed that the detector's performance for this situation was similar to its performance for the noise-only case. This indicates that the single-cycle detector could be used to determine the absence or presence of a certain cyclostationary signal in an environment that is filled with interfering signals, so long as none of the interferers was spectrally coherent at the same (f, α) pair as the signal of interest.

Additional simulations were run with a direct sequence spread spectrum (DS SS) signal and with a scale division multiple access (SDMA) signal. The results of these simulations showed the single-cycle detector is viable for detecting signals with either of these modulation types.

5 Polycyclic Detection

The generalized coherence (GC) estimate is introduced in [7] as a statistic for measuring the coherence of several signals simultaneously. Given M complex data sequences represented by vectors $x_1, ..., x_M$, the M-channel GC estimate $\gamma_M^2(x_1, ..., x_M)$ is defined by

$$\gamma_M^2(x_1, ..., x_M) \triangleq 1 - \frac{g(x_1, ..., x_M)}{||x_1||^2 \cdots ||x_M||^2}$$

where $g(x_1, ..., x_M)$ is the determinant of an $M \times M$ Gram matrix whose $(i, j)^{\text{th}}$ element is the inner product $\langle x_i, x_j \rangle$. In the case of two signals, the GC estimate reduces to the MSC estimate which is the basis for the single-cycle detector described above. As with the MSC estimate, the statistical behavior of the GC estimate has been determined under H_0 hypotheses that the M channels contain uncorrelated complex gaussian signals. It is thus well suited to testing for correlation among the outputs of several narrow band-pass filters on a single signal.

Research currently in progress is investigating the use of the GC estimate for simultaneous detection of non-harmonically related cyclostationary components in a signal – as may occur in passive sonar signals where acoustic signatures are produced by mechanically coupled machinery components. The possibility of improving detector performance by exploiting cyclic harmonics with a GC-based detector is also being investigated.

6 Conclusions

The detection statistic ($\psi = |\hat{\rho}_{\mathbf{X}}^{\alpha}|^2$) of the proposed cyclostationary feature detector is an estimate of the magnitude squared of the spectral autocoherence function. Theoretical analysis has shown that the cumulative distribution of ψ in the H_0 case is given by equation (3). Computer simulations verified the validity of this equation. The probability of false alarm (P_{FA}) for a given threshold d was determined to be

$$P_{FA} = (1 - d)^{N-1}$$

Receiver operating characteristic curves were determined from the analytical expression for P_{FA} and data from the computer simulations. The simulations shown in this paper indicate that the detector is viable for detecting BPSK signals and that its performance can be improved by increasing the collection time. Promising results have also been obtained with other modulation types. The simulations also indicate that the detector is able to distinguish between cyclostationary signals that have different cyclostationary components (e.g., different cyclic frequencies α).

The possibility of using the generalized coherence estimate as the basis of a detector for signals with multiple cyclostationary components has also been briefly discussed.

References

[1] W.A. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Processing Magazine*, vol. 8, no. 2, pp. 14-36, April 1991.

[2] W.A. Gardner, *Introduction to Random Processes with Application to Signals and Systems.* New York: McGraw-Hill, 1990.

[3] A.H. Nuttall, "Invariance of distribution of coherence estimate to second-channel statistics," *IEEE Transactions of Acoustics, Speech, and Signal Processing*, vol. ASSP-29, no. 1, pp. 120-122, February 1981.

[4] H. Gish and D. Cochran, "Invariance of magnitude-squared coherence estimate with respect to second-channel statistics," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. ASSP-35, no. 12, pp. 1774-1776, December 1987.

[5] A. Papoulis, "Narrow-band systems and gaussianity," *IEEE Transactions on Information Theory*, vol. IT-18(1), January 1972.

[6] M. Rosenblatt, "Some comments on band-pass filters," *Quarterly of Applied Mathematics*, vol. 18, pp. 387-393, 1961.

[7] D. Cochran, H. Gish, and D. Sinno, "A geometric approach to multiple-channel signal detection," *IEEE Transactions on Signal Processing*, to appear.