KNOWLEDGE BASED IDENTIFICATION OF FRACTIONALLY SAMPLED CHANNELS

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Abstract: *Blind channel identification has been a popular research subject in recent years. In this paper, we introduce the concept of knowledge-based blind channel identification. By relying on known information such as the pulse shaping filter and the anti-aliasing filter responses, the performance of channel identification and equalization can be significantly enhanced in digital communication systems. We present two simple methods: one in time-domain and one in frequency domain. Our simulation results will demonstrate the performance of these two approaches.*

1 Introduction

In many data communication systems such as the digital mobile systems and digital HDTV systems, data signals are often transmitted through unknown channels which may introduce severe linear distortion. In order to improve the system performance, it is important for the receiver to remove the channel distortion through equalization or sequence estimation. Because the available channel input training sequence may be too short or even non-existent for channel identification, blind channel identification can play useful roles in these systems.

The algorithm by Tong, Xu, and Kailath [1], known as the TXK algorithm is one of the first known methods for fractionally sampled channel identification. Several modifications of the TXK method and other algorithms have since been proposed with good performance (see, e.g., [2]). However, most of these methods are designed to identify the entire discrete channel including the pulse-shaping and receiver filters. In practice, the overall channel is not completely unknown. It is typical that the only unknown part of the channel is the multipath. The focus of this paper is to explore the knowledge-based channel identification when part of the overall channel is known. Our results will show that based on the known pulse-shaping and receiver filters, channel identification can be significantly simplified with improved performance.

2 Problem Formulation

A typical QAM (quadrature amplitude modulated) data communication system can be simplified into a baseband representation. Given that the channel is linear and causal with impulse response $h(t)$, its input/output relation can be written as

$$x(t) = \sum_{k=-\infty}^{\infty} s_k h(t-kT-t_0)+w(t), \quad a_k \in \mathcal{A}, \quad (2.1)$$

where T is the symbol baud period. The noise $w(t)$ is stationary and independent of channel input s_k, but not necessarily Gaussian. Note that $h(t) = c(t) \circledast p(t)$ is the "composite" channel impulse response that includes pulse shaping filter $p(t)$ and the channel impulse response $c(t)$.

Let the sampling interval be $\Delta = T/p$. The oversampled discrete signals are

$$x_i \triangleq x(i\Delta), \quad h[i] \triangleq h(i\Delta) \quad \text{and} \quad w_i \triangleq w(i\Delta). \quad (2.2)$$

Suppose $h(t)$ has finite support $[0, T_h)$, which spans $m_0 + 1$ integer periods. Defining the following vectors

$$\begin{aligned}
\mathbf{x}[k] &\triangleq [x_{kp} \ \ x_{kp+1} \ \cdots \ x_{kp+p-1} \\
&\qquad x_{(k-1)p} \ \ x_{(k-1)p+1} \ \cdots \ x_{kp-Mp+1}]' \\
\mathbf{s}[k] &\triangleq [s_k \ \ s_{k-1} \ \cdots \ s_{k-m_0-M+1}]' \\
\mathbf{w}[k] &\triangleq [w_{kp} \ \ w_{kp+1} \ \cdots \ w_{kp-Mp+1}]' \\
\mathbf{h}_i &\triangleq [\ h[ip] \ \ h[ip+1] \ \cdots \ h[ip+p-1]\ ]',
\end{aligned}$$

we can form a $Mp \times (m_0 + M)$ block Toeplitz matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{m_0} & 0 & \dots & 0 \\ 0 & \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{m_0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{m_0} \end{bmatrix}. \quad (2.3)$$

Consequently the channel output covariance matrix can be written as

$$R_{\mathbf{x}} = \mathbf{H} R_s \mathbf{H}^H + \sigma_w^2 I, \quad (2.4)$$

given the covariance matrix $R_s = E\{\mathbf{s}[k]\mathbf{s}[k]^H\}$ and white and zero mean noise $\mathbf{w}$ with $R_w = E\{\mathbf{w}[k]\mathbf{w}[k]^H\} = \sigma_w^2 I$.

Our objective is to identify the channel $\mathbf{H}$ from $R_{\mathbf{x}}$ under the identifiability condition that both $\mathbf{H}$ and R_s are full-rank.

3 Main Results

3.1 Subspace Method

Through eigenvalue decomposition, we have

$$[\mathbf{U_s}\ \ \mathbf{U_n}]^H R_{\mathbf{X}} [\mathbf{U_s}\ \ \mathbf{U_n}] = \Lambda = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_{Mp}^2)$$

Because $\mathbf{HH}^H$ has rank $m_0 + M$, $\mathbf{U_s}$ has $m_0 + M$ dimensions and spans the signal subspace while $\mathbf{U_n}$ spans the noise subspace

$$\mathbf{U_n} = [\ U_{m_0+M+1}\ \ U_{m_0+M+2}\ \ \cdots\ \ U_{Mp}\ ].$$

It is apparent that

$$\mathbf{H} = \mathbf{U}_s diag(\sigma_1, \sigma_2, \ldots, \sigma_{m_0+M}) V R_s^{-\frac{1}{2}},$$

where V is an unknown orthonormal matrix. As a result of the subspace separation,

$$\mathbf{U_n}^H \mathbf{H} = 0. \qquad (3.1)$$

It has been shown [2] that the full-rank channel matrix $\mathbf{H}$ can be uniquely determined by solving the above equations subject to the constraint that $\mathbf{H}$ is block Toeplitz.

3.2 Knowledge Based Subspace Method

The identification task can be simplified by only identifying the unknown channel $c(t)$ based on the known filter response $p(t)$ instead of identifying the entire $h(t)$. Let

$$c[i] \triangleq c(i\Delta), \quad i = 0,\ 1,\ \ldots,\ m_1 p - 1; \qquad (3.2)$$

$$p[i] \triangleq p(i\Delta), \quad i = 0,\ 1,\ \ldots,\ n_2; \qquad (3.3)$$

$$h[i] = \sum_{k=0}^{i} c[i-k]p[k]. \qquad (3.4)$$

$$\mathbf{c}_i \triangleq [\ c[ip]\ \ c[ip+1]\ \ \ldots\ \ c[ip+p-1]\ ]'. \qquad (3.5)$$

Denote

$$\mathcal{P} \triangleq \begin{bmatrix} p_0 & 0 & \ldots & 0 \\ p_1 & p_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ p_{n_2} & \ddots & \ddots & p_0 \\ 0 & p_{n_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & p_{n_2} \end{bmatrix} \qquad (3.6)$$

and

$$C \triangleq [\ \mathbf{c}_0\ \ \mathbf{c}_1\ \ \ldots\ \ \mathbf{c}_{m_1-1}\ ] \qquad (3.7)$$

we have

$$H = [\mathbf{h}_0'\ \ \mathbf{h}_1'\ \ \ldots\ \ \mathbf{h}_{m_0}']' = \mathcal{P}C. \qquad (3.8)$$

If we partition

$$U_i = \begin{bmatrix} \mathbf{u}_1^{(i)} \\ \mathbf{u}_2^{(i)} \\ \vdots \\ \mathbf{u}_M^{(i)} \end{bmatrix}, \quad \mathbf{u}_j^{(i)} \text{ is } p \times 1, \quad j = 1,\ 2,\ \ldots,\ M, \qquad (3.9)$$

it is then evident that

$$U_i^H \mathbf{H} = H' \mathcal{U}_i^* = C' \mathcal{P}' \mathcal{U}_i^* \qquad (3.10)$$

where

$$\mathcal{U}_i = \begin{bmatrix} \mathbf{u}_1^{(i)} & \mathbf{u}_2^{(i)} & \ldots & \mathbf{u}_M^{(i)} & 0 & \ldots & 0 \\ 0 & \mathbf{u}_1^{(i)} & \mathbf{u}_2^{(i)} & \ldots & \mathbf{u}_M^{(i)} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \mathbf{u}_1^{(i)} & \mathbf{u}_2^{(i)} & \ldots & \mathbf{u}_M^{(i)} \end{bmatrix}.$$

Thus, we can identify the channel C by minimizing

$$J(C) = C^H \mathcal{P}^H \left(\sum_{i=m_0+M+1}^{Mp} \mathcal{U}_i \mathcal{U}_i^H \right) \mathcal{P} C j \qquad (3.11)$$

To avoid the trivial all zero solution, we consider a quadratic power constraint $||H||^2 = E\{|x_k|^2\}/E\{|s_k|^2\}$. The solution is the minimum eigenvector of $R_{\mathbf{X}}$.

Since the pulse shaping $p(t)$ is known *a priori*, we only need to minimize the cost function with respect to a smaller vector C. In practice, the channel $c(t)$ tends to be multipath with a delay less than one symbol in mobile communication systems. This can significantly simplify the minimization procedure especially when the pulse length is rather long in a low bandwidth channel.

3.3 Frequency Domain Phase Recovery

It is well known that the magnitude of the channel frequency response can be obtained from the power spectrum density of the channel output. Once the amplitude is derived, the more important task is to extract the phase information. Relying on the works in [4], we present a frequency domain knowledge-based method.

The correlation function of x_i is defined as

$$R_x[n+m, n] = E\{x_{n+m} x_n^*\}. \qquad (3.12)$$

Since w_i and s_i are independent, x_i is a cyclostationary process with period p. Although we assumed that

w_i is white, our basic derivation can be easily extended (see Gardner [3]) to colored stationary noises.

The cyclic correlation function of discrete process x_i is defined as

$$R_x^{(l\beta)}[m] \triangleq \sum_{n=0}^{p-1} R_x[n+m,n]e^{-jnl\beta}, \quad \beta = \frac{2\pi}{p}, \quad (3.13)$$

for $l \in \mathbb{Z}$. The cyclic spectrum of x_i is hence obtained as

$$\begin{aligned} S_x^{(l\beta)}[e^{j\omega}] &\triangleq \sum_{m=-\infty}^{\infty} R_x^{(l\beta)}[m]e^{-jm\omega} \quad (3.14) \\ &= \sigma_s^2 H(\omega)H^*(\omega - l\beta) + \sigma_w^2 \delta[l]. \end{aligned}$$

Define $\psi(\omega)$ as the phase of $S_x^{(\beta)}[\omega]$ and $\phi_h(\omega)$ as the phase of $H(j\omega)$. We have

$$\phi_h(\omega) - \phi_h(\omega - \beta) = \psi(\omega). \quad (3.15)$$

Recall that the overall channel $H(\omega) = P(\omega)C(\omega)$ contains a known part $P(\omega)$ and an unknown part $C(\omega)$. Thus the channel phase

$$\phi_h(\omega) = \phi_p(\omega) + \phi_c(\omega)$$

also contains a known phase $\phi_p(\omega)$ and an unknown phase $\phi_c(\omega)$. Defining

$$\psi_c(\omega) \triangleq \psi(\omega) + \phi_p(\omega - \beta) - \phi_p(\omega),$$

our goal is to determine $\phi_c(\omega)$ from

$$\phi_c(\omega) - \phi_c(\omega - \beta) = \psi_c(\omega). \quad (3.16)$$

Since both $\phi_c(\omega)$ and $\psi_c(\omega)$ are periodic with period 2π, we can expand them into Fourier series

$$\phi_c(\omega) = \sum_{n=-\infty}^{+\infty} \phi_n \exp(jn\omega) \quad (3.17)$$

$$\psi_c(\omega) = \sum_{n=-\infty}^{+\infty} \psi_n \exp(jn\omega). \quad (3.18)$$

Since $\{\exp(jn\omega)\}$ form an orthogonal basis, by substituting both (3.17) and (3.18) into (3.16), we have

$$\phi_n(1 - \exp(jn\beta)) = \psi_n, \quad (3.19)$$

which can be used to get ϕ_n for $n \neq kp$. Unfortunately, no information about ϕ_n for $n = kp$ is contained in the cyclic spectra, which indicate the limitation of using cyclostationarity to identify the unknown channel.

To remove the phase ambiguity without any prior information on the channel, the phase $\phi(\omega)$ is determined through

$$\min |\phi(\omega)|^2, \quad \text{subject to} \quad \phi_n(1 - \exp(jn\beta)) = \psi_n.$$

Based on (3.19) and the orthogonal basis, the optimum solution is simply

$$\phi_n = \begin{cases} \psi_n(1 - \exp(jn\beta))^{-1}, & n \neq kp; \\ 0, & n = kp. \end{cases} \quad (3.20)$$

Since the missing information ϕ_{kp} is unknown, this simple approximation naturally gives arise to the phase distortion. By using known information $\phi_p(\omega)$, the missing information becomes less significant. Moreover, our solution only requires an unwrapped phase $\psi_c(\omega)$, which is a much easier task compared to the unwrapping of $\psi(\omega)$.

4 Simulation Results

A. Modified Subspace Method

We consider a raised-cosine pulse $p(t)$ limited in $13T$ with rolloff factor 0.11 and a two ray multipath channel

$$c(t) = \delta(t) - 0.7\delta(t - 0.45T).$$

The data input signal is i.i.d. BPSK. The SNR is 30dB and $p = 3$. Results from 20 independent trials of the subspace method and the modified subspace methods are shown in Figure 1. The normalized MSE of the identified channel impulse response is also shown in Figure 2 for different data lengths. The knowledge-based method clearly outperforms the original subspace approach.

B. Frequency Domain Method

Let $p(t)$ be limited in $6T$. The data input signal is i.i.d. 4-level PAM. The data length is 256 and $p = 3$. The two ray channel is

$$c(t) = \delta(t) - 0.9\delta(t - 0.33T).$$

Under SNR=20dB, results from 20 independent trials of the subspace method and the modified subspace methods were shown in Figure 3. The normalized MSE of the identified channel impulse response is also shown for different SNR levels.

5 Conclusions

We present a knowledge-based subspace algorithm for channel identification. We present also a knowledge-based frequency domain approach for channel phase recovery. Both algorithms significantly simplify existing methods that ignore the known system response. The knowledge-based algorithms result in improved identification performance as shown in our simulation results.

References

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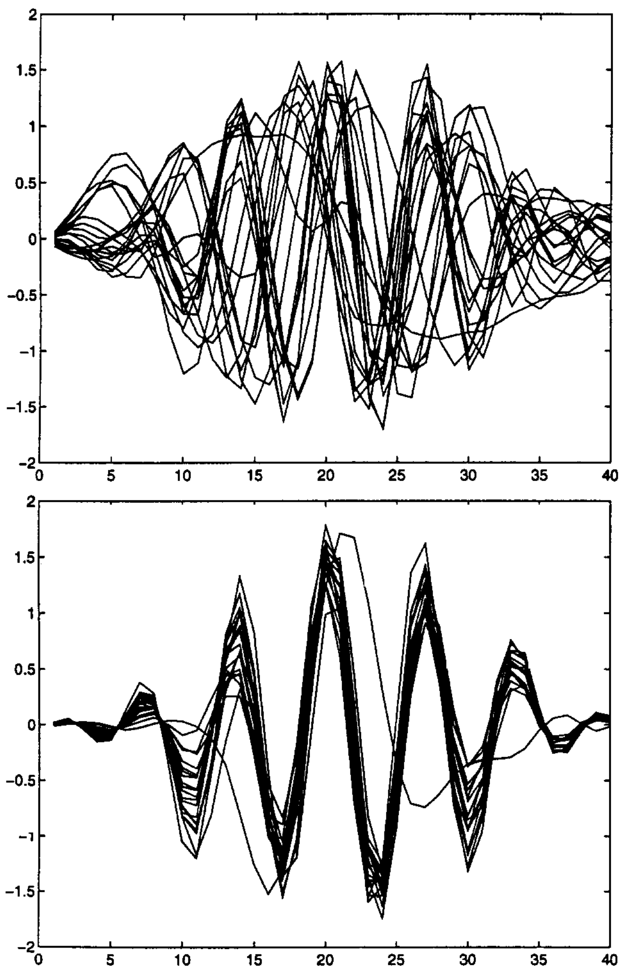

Figure 1: Channel identification from 256 data symbols using the subspace method (top) and the modified method (bottom).

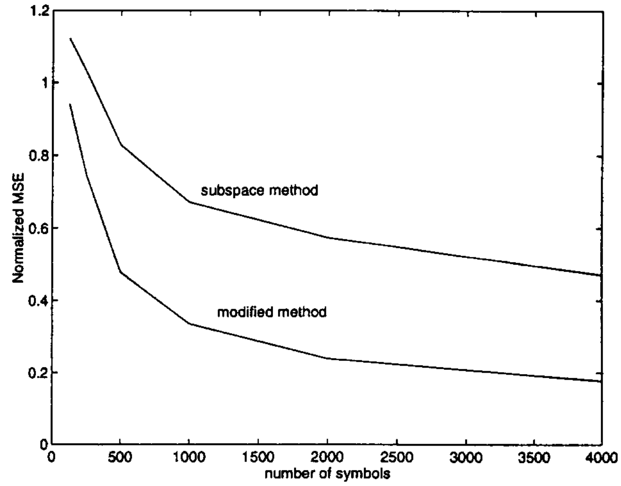

Figure 2: Comparison of normalize MSE.

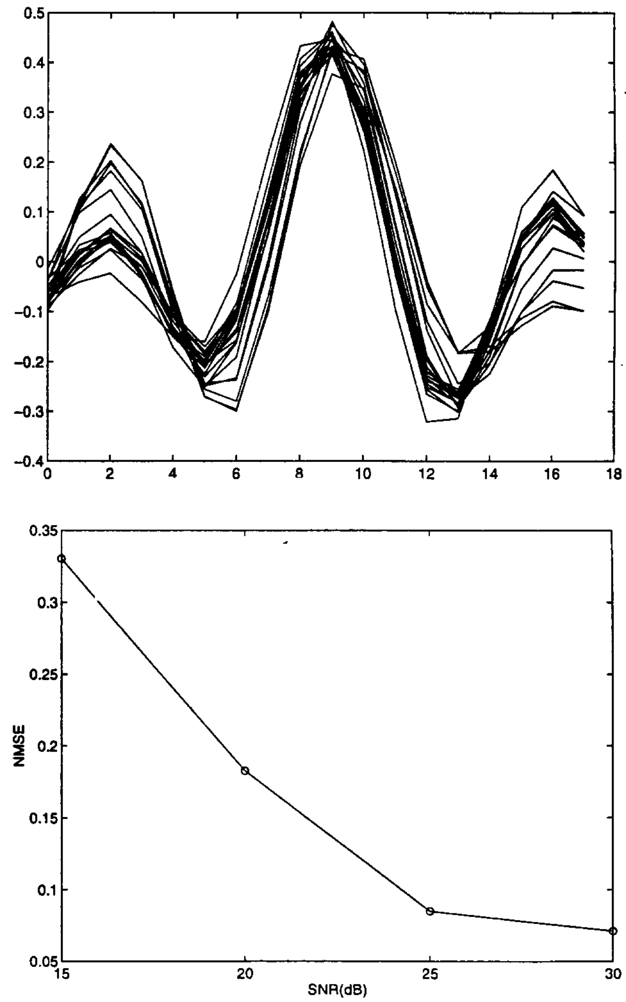

Figure 3: Frequency domain identification.