ON BLIND CHANNEL IDENTIFICATION FOR IMPULSIVE SIGNAL ENVIRONMENTS

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ABSTRACT

New methods for parameter estimation and blind system identification for impulsive signal environments are presented. The data are modeled as stable processes. First, methods for estimating the parameters (characteristic exponent and dispersion) of a symmetric stable distribution are presented. The fractional lower-order moments, both positive and negative order, and their applications are introduced. Then a new algorithm for blind channel identification based on fractional lower-order moments is proposed. The Alpha-Spectrum, a spectral representation for impulsive environments, is developed. Conditions for blind identifiability of non-minimum phase FIR channels are established using the properties of the Alpha-Spectrum.

1. INTRODUCTION

The statistical signal processing framework is incomplete without the study of α-stable ($0 < \alpha \leq 2$) distributions. They are the *only* class of distributions that can be the limit distributions for sums of i.i.d random variables (Generalized Central Limit Theorem). Familiar members of the family are Gaussian ($\alpha = 2$) and Cauchy ($\alpha = 1$) distributions. Many signal/noise processes are impulsive in nature and can be best modeled as α-stable processes [1]. Unlike most statistical models, the α-stable distributions (except Gaussian) have infinite second- or higher-order moments. An alternative tool is the fractional lower-order moment (FLOM). It is known that the p^{th} order FLOM for a symmetric α-stable ($S\alpha S$) random variable is finite for $0 \leq p < \alpha$ [2]. While most blind identification algorithms based on polyspectra [3] fail in the presence of outliers, the FLOMs are the appropriate tools for analysis [4]. In Sec.2, the FLOMs (both positive and negative order) are applied to parameter estimation; in Sec.3, the FLOMs are used to develop a spectral representation for $S\alpha S$ processes: the α-Spectrum . Using the properties the α-Spectrum of We show that any FIR channel (mixed-phase with unknown order) driven by white $S\alpha S$ ($\alpha > 1$) processes can be identified from output measurements only.

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2. $S\alpha S$ PARAMETER ESTIMATION

The most important parameters for a $S\alpha S$ process are the characteristic exponent α and the dispersion γ. We present new estimation methods of these parameters using FLOMs.

2.1. Fractional Negative-Order Moments

We show that a $S\alpha S$ random variable X also has finite fractional negative-order moments. A unified formula is:

$$\mathbf{E}(|X|^p) = C_1(p,\alpha)\gamma^{\frac{p}{\alpha}}, \; -1 < p < \alpha. \qquad (1)$$

where $C_1(p,\alpha) = \frac{2^{p+1}\Gamma(\frac{p+1}{2})\Gamma(-\frac{p}{\alpha})}{\alpha\sqrt{\pi}\Gamma(-\frac{p}{2})}$. X is a real $S\alpha S$ random variable. Eq.(1) can be used for estimation of α and γ. More specifically, α is obtained by solving:

$$\text{sinc}(\frac{p\pi}{\alpha}) = \frac{2\tan(p\pi/2)}{p\pi\mathbf{E}(|X|^p)\mathbf{E}(|X|^{-p})}, 0 < p < \min(\alpha, 1). \qquad (2)$$

γ is obtained by Eq.(1) afterwards.

2.2. Parameter Estimation with $\log|S\alpha S|$

Since the pdf $f(x)$ of a $S\alpha S$ random variable is bounded at $x = 0$, let $Y = \log|X|$, then $\mathbf{E}(|X|^p) = \mathbf{E}(e^{pY})$. By the property of the moment-generating function,

$$\mathbf{E}(Y^k) = \frac{d^k}{dp^k}(C_1(p,\alpha)\gamma^{p/\alpha})|_{p=0}. \qquad (3)$$

$$\mathbf{E}(Y) = \mathbf{C}_e(\frac{1-\alpha}{\alpha}) + \frac{\log\gamma}{\alpha}, \; \mathbf{Var}(Y) = \frac{\pi^2}{6}\left(\frac{1}{\alpha^2}+\frac{1}{2}\right), \qquad (4)$$

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where $\mathbf{C}_e$ is the Euler constant. All the moments of Y are finite and from the second-order and above, only involve α. Eq.(4) provides a simple parameter estimation method. If the samples are not i.i.d, ergodicity is needed to apply the above method. Notice that (i) a $S\alpha S$ random variable U has finite p-th order moments $\mathbf{E}(|U|^p)$ in the neighborhood of $p = 0$, (ii) n jointly $S\alpha S$ random variables $U_1, U_2, \cdots U_n$ have finite joint moments $\mathbf{E}(|U_1|^{p_1}|U_2|^{p_2}\cdots|U_n|^{p_n})$ of orders $p_1, p_2, \cdots p_n$ in the neighborhood of $p_1 = 0, p_2 = 0, \cdots, p_n = 0$ [5]. We can prove that when U is a $S\alpha S$ moving average process, its corresponding $\log|S\alpha S|$ random process $V = \log(|U|)$ is stationary as well as mean- and correlation-ergodic [6].

2.3. Iterative Parameter Estimation Method

To increase estimation accuracy while maintain memory efficiency, an iterative method is proposed. After observing the k^{th} block of data (M samples per block), we update $\alpha(k)$ and $\gamma_y(k)$ from $\alpha(k-1)$ and $\gamma_y(k-1)$ by:

$$\frac{\pi^2}{6}\left(\frac{1}{\alpha^2(k)}+\frac{1}{2}\right) = \frac{k-1}{k^2}\left[\mathbf{C}_e\left(\frac{1}{\alpha(k-1)}-1\right) - \mathrm{Avg}(k) + \frac{\log\gamma_y(k-1)}{\alpha(k-1)}\right]^2 + \frac{k-1}{k}\frac{\pi^2}{6}\left(\frac{1}{\alpha^2(k-1)}+\frac{1}{2}\right) + \frac{\mathrm{Var}(k)}{k}, \quad (5)$$

and,

$$\frac{\log\gamma_y(k)}{\alpha(k)} = \frac{k-1}{k}\left[\mathbf{C}_e\left(\frac{1}{\alpha(k-1)}-1\right) + \frac{\log\gamma_y(k-1)}{\alpha(k-1)}\right] + \frac{\mathrm{Avg}(k)}{k} + \mathbf{C}_e\left(1-\frac{1}{\alpha(k)}\right), \quad (6)$$

where $\mathrm{Avg}(k)$ and $\mathrm{Var}(k)$ are the average and standard deviation of k^{th} block of data $\log(|Y_n|)$. Y_n is a MA process. Monte-Carlo simulation results (Fig.(1)) clearly demonstrate the effectiveness of the algorithm.

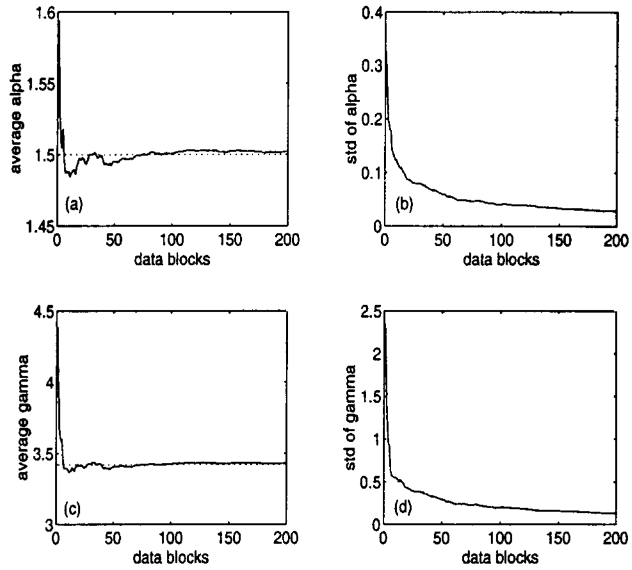

Figure 1: Iterative estimator for α, γ_y, with 200 blocks, 100 samples per block. FIR channel: $Y_n = X_n + 0.5X_{n-1} - 1.3X_{n-2} + 0.7X_{n-3}$. (a). Average of $\hat{\alpha} \to 1.5024$ (true value (dotted) $\alpha = 1.5$). (b). Standard deviation of $\hat{\alpha} \to 0.0289$. (c). Average of $\hat{\gamma_y} \to 3.4317$ (true value (dotted) $\gamma_y = 3.4215$). (d). Standard deviation of $\hat{\gamma_y} \to 0.1316$.

3. BLIND CHANNEL IDENTIFICATION

We present a robust blind channel identification algorithm based on a new frequency domain representation of the output covariation: the α-Spectrum. In the following, the input is assumed to be standard $S\alpha S$ with *known* α.

3.1. Time Domain Covariation Approach

The channel output covariation is related to the impulse response coefficients by [4]:

$$[Y_n, Y_{n+j}]_\alpha = \begin{cases} \sum_{i=0}^{q-j} \frac{h_i}{h_{j+i}}|h_{j+i}|^\alpha, & j = 0, 1, \cdots q \\ \sum_{i=0}^{q+j} \frac{h_{i-j}}{h_i}|h_i|^\alpha, & j = -1, \cdots -q. \end{cases} \quad (7)$$

The covariation is estimated by [7]:

$$[X, Y]_\alpha = \frac{\mathbf{E}(XY^{<p-1>})}{\mathbf{E}(|Y|^p)}\gamma_y, \quad (8)$$

where

$$Y^{<p-1>} = \begin{cases} |Y|^{p-2}Y^* & Y\text{: complex} \\ |Y|^{p-1}\mathrm{sign}(Y) & Y\text{: real,} \end{cases} \quad (9)$$

A least squares method fails to solve Eq.(7) due to the existence of local minima. For $MA(1)$ and $MA(2)$ processes, there are closed form solutions for Eq.(7). For example, for $MA(2)$ process (assuming $h_0 > 0$):

$$h_0 = \left([Y_n, Y_{n+2}]_\alpha \cdot [Y_n, Y_{n-2}]_\alpha^{<1-\alpha>}\right)^{\frac{1}{2\alpha-\alpha^2}}, \alpha \neq 2, \quad (10)$$

$$h_1 = \frac{\frac{[Y_n, Y_{n-1}]_\alpha}{h_2} - \frac{[Y_n, Y_{n+1}]_\alpha}{h_0}}{\frac{[Y_n, Y_{n-2}]_\alpha}{h_2^2} - \frac{[Y_n, Y_{n+2}]_\alpha}{h_0^2}}. \quad (11)$$

$$h_2 = [Y_n, Y_{n-2}]_\alpha \cdot h_0^{<1-\alpha>}, \quad (12)$$

A more general, closed form solution for $MA(q), q > 2$ is unknown. Note that the error propagation in computing the mid-index coefficients is severe with this approach.

3.2. The α-Spectrum

Consider the generalized form of the output covariation: $[Y_n, W_n]_\alpha$, where $W_n = \sum_{i=-q}^{q} a_i Y_{n-i}$, and a_i are arbitrary real or complex coefficients. Hence,

$$[Y_n, W_n]_\alpha = \sum_{k=0}^{q} h_k (\sum_{l=0}^{q} a_{k-l} h_l)^{<\alpha-1>}, \quad (13)$$

where the pseudo-linearity property of the covariation was used, i.e.,

$$[aX_m, bX_n]_\alpha = ab^{<\alpha-1>}\delta(m-n)\gamma_x, \quad (14)$$

where X_m, X_n are i.i.d $S\alpha S$ random variables with dispersion γ_x and $\delta(\cdot)$ is the Kronecker function. a, b are *real or complex* constants. Since the choice of a_i are arbitrary, let $a_i = z^i$, $\forall z \in \mathbf{C}, z \neq 0$. Then Eq.(13) becomes:

$$S_\alpha(z) \triangleq [Y_n, W_n(z)]_\alpha = H((\frac{1}{z})^{<\alpha-1>})\,(H(z))^{<\alpha-1>}, \quad (15)$$

where $W_n(z) = \sum_{i=-q}^{i=q} Y_{n-i} z^i$, which is the windowed z-transform of the channel output Y_n, and $H(z) = \sum_{n=0}^{q} h_n z^{-n}$ is the z-transform of the filter. **Eq.(15) is of fundamental importance**. We name $S_\alpha(z)$ as **the α-Spectrum** . Given the measurement of α-Spectrum , we can identify both the magnitude and phase responses of the channel. More specifically, to identify the channel magnitude response, let $|z| = 1$, then $|H(e^{j\omega})| = (S_\alpha(e^{j\omega}))^{\frac{1}{\alpha}}$; to identify the channel phase response, taking logarithm of both sides of Eq.(15):

$$\begin{aligned}\log S_\alpha(z) &= \log|S_\alpha(z)| + j\Psi(z) \\ &= \log|H(r^{1-\alpha}e^{j\omega})| + (\alpha-1)\log|H(re^{j\omega})| \\ &+ j\{\Phi(r^{1-\alpha}e^{j\omega}) - \Phi(re^{j\omega})\}, \quad (16)\end{aligned}$$

where $|H(re^{j\omega})|$ and $\Phi(re^{j\omega})$ are the channel magnitude and phase responses evaluated on a circle of radius r, $|S_\alpha(z)|$ and $\Psi(z)$ are the magnitude and phase of α-Spectrum, respectively. It is well known [3] for any FIR channel: $H(z) = A_0 z^{-d}\prod_{i=1}^{L_1}(1-a_i z^{-1})\prod_{i=1}^{L_2}(1-b_i z)$, where A_0 is the constant gain, d is the constant time delay, $\{a_i, |a_i| < 1\}$ and $\{1/b_i, |b_i| < 1\}$ are zeros inside and outside the unit circle,respectively. In general, d and the sign of A_0 are known *a prior*. And:

$$\log|H(re^{j\omega})| = -\sum_{m=1}^{\infty} \frac{A^{(m)}r^{-m} + B^{(m)}r^m}{m}\cos(m\omega) + \log(A_0) \quad (17)$$

$$\Phi(re^{j\omega}) = \sum_{m=1}^{\infty} \frac{A^{(m)}r^{-m} - B^{(m)}r^m}{m}\sin(m\omega), \quad (18)$$

where

$$A^{(m)} = \sum_{i=1}^{L_1} a_i^m; \quad B^{(m)} = \sum_{i=1}^{L_2} b_i^m, \quad (19)$$

with the region of convergence (ROC): $\max_i\{|a_i|\} < |z| < \min_i\{1/|b_i|\}$. $\frac{A^{(m)}+B^{(m)}}{m}$ and $\frac{A^{(m)}-B^{(m)}}{m}$ determine the magnitude and phase responses of the channel, respectively. Substituting Eqs. (17) and (18) to Eq.(16), to get:

$$\log|S_\alpha(re^{j\omega})| = -\sum_{m=1}^{\infty} \frac{A^{(m)}\mu_m(r) + B^{(m)}\mu_m(\frac{1}{r})}{m}\cos(m\omega) + \alpha\log(A_0) \quad (20)$$

$$\Psi(re^{j\omega}) = \sum_{m=1}^{\infty} \frac{A^{(m)}\nu_m(r) - B^{(m)}\nu_m(\frac{1}{r})}{m}\sin(m\omega), \quad (21)$$

where $\mu_m(r) = r^{m(\alpha-1)} + (\alpha-1)r^{-m}$ and $\nu_m(r) = r^{m(\alpha-1)} - r^{-m}$, with ROC: $\max_i\{|a_i|, |b_i|^{1/(\alpha-1)}\} < r < \min_i\{1/|b_i|, (1/|a_i|)^{1/(\alpha-1)}\}$. Multiplying both sides of Eq.(20) by $\cos(n\omega)$, $n \geq 1$, and integrating with respect to ω, using the orthogonality of the trigonometric function, we have:

$$\int_0^\pi \log|S_\alpha(re^{j\omega})|\cos(n\omega)d\omega = \frac{A^{(n)}\mu_n(r) + B^{(n)}\mu_n(\frac{1}{r})}{-\frac{2}{\pi}n} \quad (22)$$

Replacing r by $\frac{1}{r}$ in Eq.(22) and then subtracting, we have:

$$\frac{A^{(n)} - B^{(n)}}{n} = \frac{\frac{2}{\pi}\int_0^\pi \log\frac{|S_\alpha(\frac{1}{r}e^{j\omega})|}{|S_\alpha(re^{j\omega})|}\cos(n\omega)d\omega}{\mu_n(r) - \mu_n(\frac{1}{r})}. \quad (23)$$

Similarly, through the *phase* of the α-Spectrum: $\Psi(re^{j\omega})$, we have:

$$\frac{A^{(n)} - B^{(n)}}{n} = \frac{\frac{2}{\pi}\int_0^\pi (\Psi(re^{j\omega}) + \Psi(\frac{1}{r}e^{j\omega}))\sin(n\omega)d\omega}{\nu_n(r) + \nu_n(\frac{1}{r})}. \quad (24)$$

Due to phase wrapping ambiguity associated with Eq.(24), Eq.(23) should be used instead.

It is not surprising that we can extract the channel *phase* information from the *magnitude* of the α-Spectrum. In fact, it is not difficult to get:

$$\frac{A^{(n)} - B^{(n)}}{n} = \frac{\frac{2}{\pi}\int_0^\pi \log\frac{|H(re^{j\omega})|}{|H(\frac{1}{r}e^{j\omega})|}\cos(n\omega)d\omega}{r^n - r^{-n}}, \quad r \neq 1. \quad (25)$$

i.e., we can extract the channel phase response from its magnitude response evaluated on two circles with reciprocal radii.

Since $S_\alpha(e^{j\omega})$ has been evaluated when the channel magnitude response was estimated, Eq.(23) can be further simplified as:

$$\frac{A^{(n)} - B^{(n)}}{n} = \frac{2}{\pi\alpha} \int_0^\pi \big(\zeta_n(r) \log |S_\alpha(e^{j\omega})| - \eta_n(r) \log |S_\alpha(re^{j\omega})|\big) \cos(n\omega) d\omega, \qquad (26)$$

where

$$\zeta_n(r) = \frac{\cosh(n(\alpha-1)\log r) + (\alpha-1)\cosh(n\log r)}{\sinh(n(\alpha-1)\log r) - (\alpha-1)\sinh(n\log r)},$$
$$\eta_n(r) = \frac{\alpha}{\sinh(n(\alpha-1)\log r) - (\alpha-1)\sinh(n\log r)},$$

In conclusion, we can recover the channel magnitude response by evaluating $S_\alpha(z)$ on the unit circle, and then recover the channel phase response by using additional information of $S_\alpha(z)$ evaluated on another circle within ROC.

3.3. Simulation Results

The FLOM covariation estimator (Eq.(8)) for the α-Spectrum is applicable *if and only if* all the random variables are real or *isotropic* complex. When the input X is white isotropic complex $S\alpha S$ process, then any finite linear combination with real or complex coefficients of X_n is also isotropic complex $S\alpha S$ random variable, and thus the FLOM estimator applies. Fig.(2) shows simulation results of blind identification of the channel magnitude and phase responses. 500,000 samples were collected in each of the 20 realizations and $p = 0.3$ in the FLOM estimator. [1] When the input is real $S\alpha S$ process, an appropriate estimator for α-Spectrum $S_\alpha(z)$ (the covariation of a real $S\alpha S$ random variable with a complex one) is yet to be found.

Acknowledgment

We are thankful to Dr. Min Shao for many helpful discussions.

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[1] Most papers claim that p in FLOM estimator should be $1 \leq p < \alpha$. However, we have found estimator with $p < 1$ often has smaller variance. Detailed analysis will be announced later.

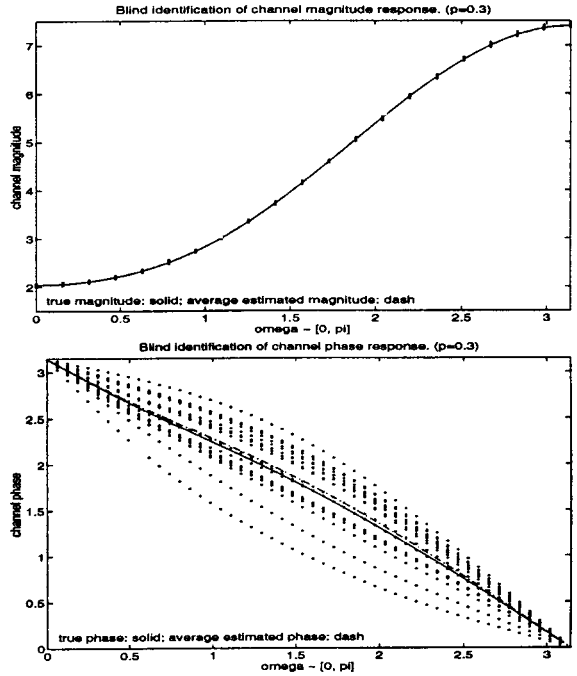

Figure 2: Blind identification of mixed-phase FIR channel: $H(z) = -4z^{-1}(1 - (0.2 \pm 0.2j)z^{-1})(1 - 0.25z)$ driven by white *isotropic complex* $S\alpha S$ process.

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