FRACTIONALLY SPACED BLIND EQUALIZATION: LOSS OF CHANNEL DISPARITY

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ABSTRACT

Under certain conditions on the equalizer length and on the channel dynamics, temporal or spatial diversity allows one to achieve blind equalization perfectly by means of second-order statistics only. Loss of **channel disparity** causes perfect equalization to no longer be achievable. The achievable channel-equalizer combination then depends only on the part of the multichannel transfer function lacking disparity. We show that the Fractionally Spaced Equalizer adapted by the Constant Modulus Algorithm (FSE-CMA) still achieves "reasonable" equalization. Its performance equals that of the non-fractional CMA, with a slightly shorter baud-length equalizer than the FSE, applied to the part of the channel lacking disparity.

Keywords: fractionally spaced equalization, multichannel system identification, adaptive blind equalization.

1. INTRODUCTION

Blind equalization is a crucial topic for digital communications where the distortion in the observed signal results predominantly from Inter-Symbol Interference (ISI) of the unknown (white, non gaussian) input sequence, $(s(n))$. Most linear blind equalization algorithms are performed either by channel identification, e.g., via cumulant matching techniques ([1]) followed by Wiener filtering to recover the data, or directly by an adaptive Bussgang algorithm ([2]).

Digital communication systems use temporal diversity, e.g., oversampling, in order to perform time and phase recovery. Oversampling results in so-called fractionally spaced data, so that the equalizers they drive are called Fractionally Spaced Equalizers (FSEs), see [3]. However, very few studies have been dedicated to the FSEs. Recent results in multichannel identification lead to a better understanding of FSEs based on a multichannel representation of the equalization problem induced by spatial diversity (e.g., using a sensor array). The communication system with temporal or spatial diversity can be seen as a multichannel transfer function with 1-input and L-outputs driven by $(s(n))$, ([4]). Its L entries, $c_k(z)$, $k = 1,..,L$, are assumed to have Finite Impulse Response (FIR) transfer functions. The equalizer is then a L-input/1-output filter, the FIR entries of which are denoted $e_k(z)$, $k = 1,..,L$. The global system is represented in Figure 1.

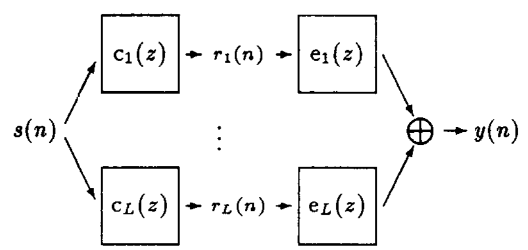

Figure 1: 1-input/L-output Channel, Fractionally Spaced Equalizer

Then, linear equalization consists of pseudo-inverting the multichannel transfer function, or in other terms solving

$$(e_1(z), ..., e_L(z)) \begin{pmatrix} c_1(z) \\ \vdots \\ c_L(z) \end{pmatrix} = z^{-\nu}$$

where ν is a delay. Recent studies have shown that, under some conditions on the degree of the $e_k(z)$ and on disparity between the subchannels, perfect identification/equalization is achievable ([4], [5], [6], [7], [9]).

In this paper, we focus on the case where the disparity condition on the subchannels is no longer met. We show that without enough disparity between the subchannel transfer functions, identification/equalization methods presuming perfect equalization conditions may fail. Nevertheless, the Constant Modulus Algorithm for FSE (FSE-CMA, [9]) is still able to equalize reasonably, even for non-constant modulus

This work was done as I. Fijalkow was a postdoctoral visitor at Cornell University, supported by the French Ministère des Affaires Etrangères (bourse Lavoisier). For this study, C.R. Johnson Jr. was supported by Applied Signal Technology Inc..

input signals. Moreover, in this paper, its asymptotic performance is shown to depend only on the part of the multichannel transfer function lacking disparity. The performance is then shown to be limited by that of non-fractionally spaced CMA applied to the disparity lacking part of the multichannel. Still, the ISI due to the disparity part of the multichannel is perfectly removed, resulting in better performance than non-fractionally spaced CMA applied to any one of the single T-spaced subchannels.

Organization:
The conditions for perfect equalization are recalled in Section 2. Section 3 contains the description of the best achievable equalization when loss of disparity occurs. In Section 4, the asymptotic properties of the FSE-CMA are studied and illustrated by numerical simulations as striving for its best achievable equalizer setting in the absence of channel noise.

2. PERFECT EQUALIZATION

For sake of simplicity, the oversampling factor L is from now on chosen equal to 2.

From the propagation model of Figure 1, one can represent the global (channel + equalizer) transfer function as

$$\mathrm{h}(z) = \mathrm{c}_1(z)\mathrm{e}_1(z) + \mathrm{c}_2(z)\mathrm{e}_2(z) \qquad (1)$$

where $\mathrm{c}_k(z)$ $k = 1,2$ and $\mathrm{e}_k(z)$ $k = 1,2$ are respectively degree Q and degree $(N/2-1)$ FIR filters. (1) is equivalent to:

$$h = \mathbf{C}^{\top}\vec{e} \qquad (2)$$

where the entries of h are the coefficients of $\mathrm{h}(z)$, and the entries of $\vec{e}$ are the coefficients of $\mathrm{e}_1(z)$ and $\mathrm{e}_2(z)$. $\mathbf{C}$ is the $N \times (Q+N/2)$ *convolution matrix* defined by the coefficients of $\mathrm{c}_1(z)$ and $\mathrm{c}_2(z)$ as:

$$\mathbf{C} = \begin{bmatrix} c_1(0) & .. & c_1(Q) & 0 & \cdots & 0 \\ 0 & c_1(0) & .. & c_1(Q) & \cdots & 0 \\ & & \ddots & & & \\ 0 & \cdots & 0 & c_1(0) & .. & c_1(Q) \\ c_2(0) & .. & c_2(Q) & 0 & \cdots & 0 \\ 0 & c_2(0) & .. & c_2(Q) & \cdots & 0 \\ & & \ddots & & & \\ 0 & \cdots & 0 & c_2(0) & .. & c_2(Q) \end{bmatrix}$$

The convolution matrix $\mathbf{C}$ is a Sylvester matrix, the algebraic properties of which have been studied, resulting in the following results.

Lemma 1 *[4]*
If the equalizer total length N is greater or equal to $2Q$ (Q being the maximum degree of $c_k(z)$), the rank of the $N \times (N/2+Q)$ Sylvester matrix $\mathbf{C}$ is equal to $N/2+Q-Z_0$, where Z_0 is the number of common zeros between the subchannels.

Lemma 2 *[5], [4]*
If $N \geq 2Q$, the multichannel is identifiable if the $c_k(z)$ have no common zero. Under these "length and zero conditions", there exist $(e_1(z), e_2(z))$ with degree $N/2-1$, inverting the multichannel.

Consequently, under the length and zero conditions, any global transfer function $\mathrm{h}(z)$ of degree $N/2+Q-1$ is achievable. In particular, any pure delay $z^{-\nu}$ (where $\nu = 0, ..., (N/2+Q-1)$) that performs perfect equalization can be met.

In fact, the previous conditions result in the multichannel $\vec{c}(z) = (\mathrm{c}_1(z), \mathrm{c}_2(z))^{\top}$ being minimum phase, even if each subchannel transfer function is not ([6]). Thus, methods using second order statistics only have been proposed, resulting in very good performance, [4]-[6]. Adaptive algorithms also benefit from $\mathbf{C}$ being full column-rank. Under the length and zero conditions, the FSE-CMA cost-function minima are all global and achieve perfect equalization, [7], [8], [9].

3. LOSS OF CHANNEL DISPARITY

Channel:

We consider now the case where the length condition ($N \geq 2Q$) is still valid, but where the subchannels have common zero(s). The multichannel transfer function can be factored as $\vec{c}(z) = \begin{pmatrix} \underline{c}_1(z) \\ \underline{c}_2(z) \end{pmatrix} \mathrm{c}_0(z)$, where $\underline{c}_1(z)$ and $\underline{c}_2(z)$ are prime (i.e., no common zeros), and where $\mathrm{c}_0(z)$ contains the common zero(s) of the multichannel.

In terms of the channel convolution matrix, Lemma 1 implies that the range of $\mathbf{C}^{\top}$ dimension is $(N/2+Q-Z_0)$ where Z_0 is the number of zeros of $\mathrm{c}_0(z)$. It should be noticed that impulse responses associated to pure delays may no longer belong to this range, and thus that they may not be achievable.

Achievable Equalization:

The combined channel-equalizer transfer function can be written as

$$\mathrm{h}(z) = \mathrm{c}_0(z)(\underline{c}_1(z)\mathrm{e}_1(z) + \underline{c}_2(z)\mathrm{e}_2(z)) = \mathrm{c}_0(z)\underline{e}(z) \qquad (3)$$

A FSE applied to a multichannel lacking disparity can also be viewed as the channel transfer function $\mathrm{c}_0(z)$ equalized by $\underline{e}(z) = \underline{c}_1(z)\mathrm{e}_1(z) + \underline{c}_2(z)\mathrm{e}_2(z)$, where $\underline{c}_1(z)$ and $\underline{c}_2(z)$ are of degree $Q - Z_0$, and $N/2-1$ is the degree of $\mathrm{e}_1(z)$ and $\mathrm{e}_2(z)$. If $N/2 \geq Q - Z_0$, any scalar $(N/2+Q-Z_0-1)$-degree polynomial $\underline{e}(z)$ is achievable, so that the equalization problem is equivalent to the non-fractionally spaced equalization of $\mathrm{c}_0(z)$ by a $(N/2+Q-Z_0-1)$-degree transfer function. Thus, one may think of choosing $\underline{e}(z)$ so to equalize as well as possible the a priori mixed phase $\mathrm{c}_0(z)$.

More precisely, solving (3) to equal a pure delay implies inverting $\mathrm{c}_0(z)$ which is not perfectly achievable with a finite length equalizer. This suggests that the second order statistics based methods ([4]-[6]) proposed to solve the multichannel equalization problem can no longer be used. In fact, since the part of the subchannel with disparity is minimum phase, the multichannel transfer function remains minimum phase if and only if $\mathrm{c}_0(z)$ is minimum phase. Consequently, if $\mathrm{c}_0(z)$ is mixed phase, so is the multichannel, so that methods based on second order statistics can not provide an acceptable answer. The degrees of the different transfer functions being unknown, identification/equalization techniques based on cumulant-matching may be difficult to apply in order to achieve a "good" equalization. We propose to study a particular Bussgang algorithm, CMA [10] applied to fractionally spaced data (FSE-CMA), in order to equalize $c_0(z)$ and to benefit from the existing multichannel disparity simultaneously.

4. FSE-CMA

The FSE-CMA can be expressed as

$$e_k(n+1) = e_k(n) + \mu y(n)(r_2 - y^2(n))R_k(n)$$

where $y(n) = \sum_{k=1,2} e_k(n)^\top R_k(n)$ with $e_k(n)$ the vector which entries are the coefficients of $e_k(z)$ at the iteration n, and with the regressor vector of the observation at the output of the k^{th} subchannel defined as $R_k(n) = (r_k(n), ..., r_k(n - N/2 - Q + 1))^\top$.

Asymptotic Behavior:

In this paragraph, we show how the asymptotic behavior of the FSE-CMA, in terms of its cost-function, is related to that of the non-fractionally spaced CMA for the part of the multichannel without disparity.

Proposal 1 *Consider a 2-channel transfer function of degree Q with Z_0 common zero(s). The minima of the FSE-CMA cost-function (for length N equalizers, with $N/2 \geq Q - Z_0$) are associated with those of the non-fractional CMA cost-function (for length $N/2 + Q - Z_0$ equalizers) applied on the (degree Z_0) part of the multichannel transfer function lacking disparity.*

Proof:
Let $\underline{e}(z)$ be any $(N/2 + Q - Z_0 - 1)$-degree polynomial, we know that there exist $(N/2 - 1)$-degree filters $e_k(z)$ $k = 1, 2$ such as $\underline{e}(z) = (\underline{c}_1(z)e_1(z) + \underline{c}_2(z)e_2(z)$. Writing the global transfer function as $h(z) = c_0(z)\underline{e}(z) = c_1(z)e_1(z) + c_2(z)e_2(z)$, the cost-function of the regular CMA as a function of $\underline{e}(z)$ is equal to the FSE-CMA cost-function with the variable $(e_1(z), e_2(z))$. This is due to both cost-functions being equal to $E[(r_2 - y^2(n))^2]$ with $y(n)$ the output of $h(z)$ driven by $s(n)$. $\triangle$

Studies of the non-fractionally spaced CMA show that, given a "long enough" equalizer and a suitable initialization, a non-minimum phase channel can be effectively inverted, see for example [11]. We will not discuss here the initialization issue and the eventual existence of local minima, since they are strictly related to the non-fractionally spaced CMA with finite length equalizers, on which studies exist, [11] for example.

In the case of a global channel-equalizer transfer function described by (3), the FSE-CMA cost-function extrema are given by the following equation:

$$\mathbf{C}_0\, \Delta(h)\, h = 0, \text{ with } \Delta(h) = (3h^\top h - \rho)I - (3-\rho)\text{diag}(hh^\top)$$

where $h = \mathbf{C}_0\underline{e}$, $\underline{e}$ is any $(N/2+Q-Z_0)$-length vector and $\mathbf{C}_0$ is the $(N/2 + Q - Z_0) \times (N/2 + Q)$ convolution matrix associated to $c_0(z)$. The classification of the extrema is analysed in [12].

Recall that the non-fractional equalization by CMA performance depends mainly on the distance between the zeros of the channel transfer function and the unit circle. A deep null in the channel frequency response corresponding to zeros of $c_0(z)$ close to the unit circle makes the equalization task infeasable. It is important to notice that, in the absence of noise, the equalizer abilities do not depend on the location of zeros of $\underline{c}_1(z)$ and $\underline{c}_2(z)$, however close they are to the unit circle.

However, as in the perfect equalization case ([9]), the gain in performance compared to the non-fractional CMA is somewhat offset by the robustness concerns due to non-uniqueness of the optimal settings. A better understanding of this phenomenon is given by the algebraic relation between $\underline{e}(z)$ and $(e_1(z), e_2(z))$. In terms of convolution matrix, it is equivalent to $\underline{e} = \underline{\mathbf{C}}\vec{e}$ where $\underline{\mathbf{C}}$ is the $(N/2 + Q - Z_0) \times N$ full column-rank Sylvester matrix induced by $\underline{c}_1(z)$ and $\underline{c}_2(z)$. This relation shows that for each vector $\underline{e}$, there exists a subspace of dimension $N - (N/2 + Q - Z_0)$ of settings of $\vec{e}$, all of them having the same equalization performance. In order to overcome this subspace phenomenon that may result in a numerical overflow in the tap values, leakage is usually used, see [13]. It corresponds to adding a regularisation term to the cost function equal to the squared norm of the equalizer.

Finally, notice that the development here is not specific to the FSE-CMA, but that it applies to any algorithm in which the cost-function is a function of the global channel-equalizer transfer function, $h(z)$. For any such algorithm using fractionally spaced data, the equalization will depend only on the non-fractional algorithm performing on $c_0(z)$.

Simulations:

We present simulations of the FSE-CMA for two channels (one with subchannel disparity, one without), and a 4-PAM input signal. The asymptotic behavior is displayed by means of the global channel-equalizer impulse response averaged at steady-state, h_*. The step-size is fixed at 10^{-4}, the global impulse response is averaged over 10000 iterations.

Example (1):
The zeros of $c_1(z)$ are -1.4 and 0.6, the zeros of $c_2(z)$ are 1.1 and -0.4. $Q = 2$, and there is disparity between the subchannels. For $N = 4$, perfect equalization is achieved by $e_1(z) \approx 0.7520 + 0.1408z^{-1}$ and $e_2(z) \approx -0.7520 - 0.2688z^{-1}$.

We compare FSE-CMA and non-fractional CMA with this multichannel. As expected, the FSE-CMA achieves in mean perfect equalization within finite equalizer length ($N = 4$ here). For a 32 equalizer length, the non-fractional

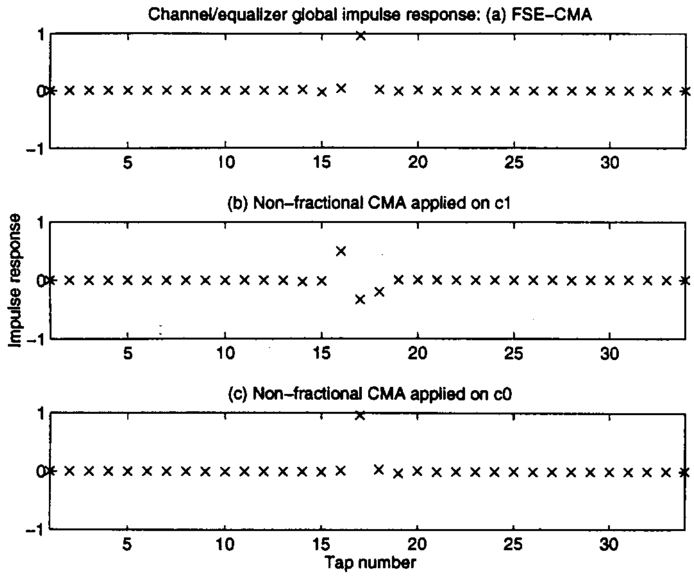

Figure 2: Global channel-equalizer impulse response: (a) FSE-CMA, (b) CMA on c1(z), (c) CMA on c0(z)

CMA applied to $c_2(z)$ gives the global transfer function displayed in Figure 2(b). Perfect equalization is far from being achieved.

Example (2):
The zeros of $c_1(z)$ are -1.4 and -0.4, the zeros of $c_2(z)$ are 1.1 and -0.4. $Q = 2$, and there is a common zero between the two subchannels.

We compare FSE-CMA and non-fractional CMA applied to $c_0(z)$ only with a 34 length global channel-equalizer impulse response in both cases. The equalizer time-span for FSE-CMA was taken as $N/2 = 34 - Q = 32$, and for CMA applied to $c_0(z)$ the equalizer time-span is givem by $N' = N/2 + Q - Z_0 = 33$. Figure 2(a) and (c) show that both algorithms achieve the same global impulse response. Moreover, the achieved impulse response is closer to the desired one (a center spike) than the inpulse response of the non-fractional CMA applied to any of the subchannels only (Figure 2(b)).

5. CONCLUSION

Fractionally spaced equalization (with a long enough equalizer) can be perfectly achieved via second-order statistics based techniques when the entries of the multichannel transfer function have no common zero(s). However, when there are common zeros between the subchannels (or numerically very close zeros), this property does not hold anymore, and the identification/equalization must be performed using high-order statistics. In that case, we have shown that the FSE-CMA is an appropriate algorithm. Its asymptotic performance is limited by that of non-fractional CMA (using a slightly shorter equalizer) applied to a scalar channel transfer function containing only the part with common zero(s).

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