BLIND DECONVOLUTION OF SYSTEM WITH UNKNOWN RESPONSE EXCITED BY CYCLOSTATIONARY IMPULSES

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ABSTRACT

The problem of determining the unknown responses of a system which is continuously excited by cyclostationary signals is considered. By exploiting the periodicity of the input impulses, an approach based on array signal processing techniques is proposed to estimate the system responses. This method generalizes Bresler's idea of resolving overlapping echoes [1] by partitioning the signals into portions which fit the mathematical formulation as in [1]. A compensating algorithm is also devised to compensate the random uncertainties and perturbations of the system and the signals. Once the system responses are determined, standard methods can be used to find the positions and amplitudes of the impulses. Prospective and promising simulation results are obtained.

I. INTRODUCTION

The field of blind deconvolution has arouse much attention in the recent years. Under different situations and assumptions, different methods have been proposed for solving the inverse problems [5]. However, in most of these proposed algorithms, it is usually assumed that the signals of interest are independent and totally uncorrelated. Thus these methods fail once the assumption is violated. Hence in this paper, focus is made on the problem of blind deconvolution of system excited by *correlated* cyclostationary signals which are described below.

Consider the situation as depicted in Figure 1(a) where an unknown system $g(t)$ is excited by a *cyclostationary signal* $s(t)$ consisting of periodic random impulses. By periodic, we mean that the signal can be partitioned into portions of length T (which is known as the period of the signal) with d impulses in each portion. Moreover, the delay factor τ_i of the ith impulse s_i is constant for all portions. Note that in general τ_i will be different for different i, although in most of the cases they may be integral multiples of a constant τ. An example of $s(t)$ is shown in Figure 1(b) with $d = 4$. On the other hand, the system response $g(t)$ consists of d sub-responses $g_i(t)$, $i = 1, \cdots\cdots, d$, with $g_i(t)$ being the impulse response of the ith impulse $s_i(t)$ of each signal portion as shown in Figure 1(a). The described system is indeed a generalized model for many practical engineering problems.

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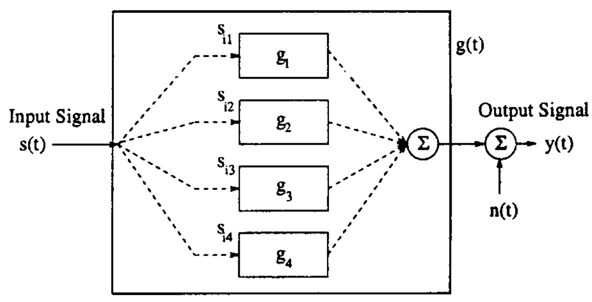
In mobile telecommunications, $g(t)$ may be viewed as the characteristics of a multi-path channel with $g_i(t)$ being the impulse responses of the various propagation paths and $s(t)$ will then represent the transmitted digital signals and their echoes. In biomedical engineering, the heart beat mechanism of the human being can be modeled as the above system with $g(t)$ being impulse responses of the body and $s(t)$ being the impulses generated by the heart muscle. Basically, there exists many methods for solving the problem *if* the system impulses $g_i(t)$ and the delay factors τ_i are known. However, in this paper, we will present a sophisticated algorithm to perform the deconvolution without knowing any of the above parameters.



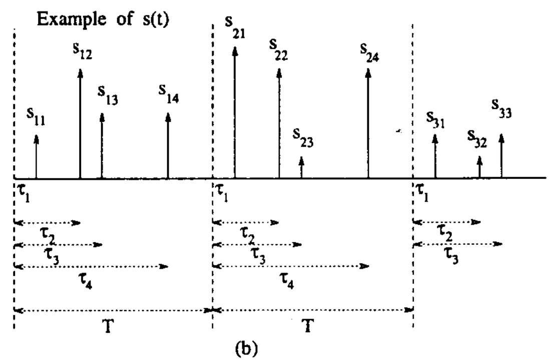


Figure 1: (a) The system under investigation. (b) Example of $s(t)$.

II. PROBLEM FORMULATION

Reconsider the system as described above. Since the impulses are periodic, we can consider a period of time T and

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write that portion of the output as

$$y(t) = \sum_{k=1}^{d} s_k g_k(t - \tau_k) + n(t) \quad 0 \le t \le T, \tag{1}$$

where d is the number of *effective* impulses in the period T with s_k and τ_k being their amplitudes and delay factors respectively. $n(t)$ represents the random noises of the system. Suppose we have N such snapshots, by following similar procedures as in [1] [3], we can reformulate the problem in the frequency domain by taking a $4m$-point **DFT** of the output data (m is the number of digital samples in one portion of output) and (1) is rewritten as

$$\boldsymbol{y}_i = (\boldsymbol{G} \odot \boldsymbol{M})\boldsymbol{s}_i + \boldsymbol{n}_i, \tag{2}$$

where $\odot$ represents the Schur-Hadamard product operator and

$$\boldsymbol{M} = \begin{bmatrix} \phi_1^0 & \phi_2^0 & \cdots & \cdots & \phi_d^0 \\ \phi_1^1 & \phi_2^1 & \cdots & \cdots & \phi_d^1 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \phi_1^{4m-1} & \phi_2^{4m-1} & \cdots & \cdots & \phi_d^{4m-1} \end{bmatrix}, \tag{3}$$

$$\boldsymbol{G} = \begin{bmatrix} G_1^0 & G_2^0 & \cdots & \cdots & G_d^0 \\ G_1^1 & G_2^1 & \cdots & \cdots & G_d^1 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ G_1^{4m-1} & G_2^{4m-1} & \cdots & \cdots & G_d^{4m-1} \end{bmatrix}, \tag{4}$$

$$\boldsymbol{n}_i = [n_{i1} \;\; n_{i2} \;\; \cdots\cdots \;\; n_{i4m-1}], \tag{5}$$

$$\phi_k = \exp(-j\frac{2\pi}{4mt_o}\tau_k) \;, n_{il} = N_i(\frac{2\pi l}{4mt_o}). \tag{6}$$

Here y_i denotes the DFT of the ith snapshots and $G_k^l = G_k(\frac{2\pi l}{4mt_o})$ with $G_k(\Omega)$ being the continuous spectrum of $g_k(t)$. Note that t_o is the sampling period of the digital output.

By performing a permutation on the rows of $\boldsymbol{y}_i$ and assuming that the spectrum is smooth enough, we may rewrite (2) as

$$\begin{aligned} \boldsymbol{x}_i &\approx \begin{bmatrix} (\boldsymbol{G}_o \odot \boldsymbol{M}_o)\boldsymbol{s}_i \\ (\boldsymbol{G}_o \odot \boldsymbol{M}_o\boldsymbol{\Phi})\boldsymbol{s}_i \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_{oi} \\ \boldsymbol{n}_{ei} \end{bmatrix} \\ &= \boldsymbol{A}_o\boldsymbol{s}_i + \begin{bmatrix} \boldsymbol{n}_{oi} \\ \boldsymbol{n}_{ei} \end{bmatrix}, \end{aligned} \tag{7}$$

and the covariance matrix of $\boldsymbol{x}$ is given by

$$\boldsymbol{R_{xx}} = \boldsymbol{A}_o\boldsymbol{S}\boldsymbol{A}_o^* + \sigma^2\boldsymbol{I}, \tag{8}$$

where $\boldsymbol{G}_o$ and $\boldsymbol{M}_o$ contain the odd rows of $\boldsymbol{G}$ and $\boldsymbol{M}$ respectively and $\boldsymbol{\Phi} = \mathrm{Diag}([\phi_1 \;\; \phi_2 \;\; \cdots\cdots \;\; \phi_d])$ with $\mathrm{Diag}(\boldsymbol{a})$ represents a diagonal matrix whose diagonal elements are that of the vector $\boldsymbol{a}$. Note that (7) and (8) has the same formulation as the typical DOA estimation in array signal processing problems and hence we can apply ESPRIT to find $\boldsymbol{\Phi}$ and $\boldsymbol{G}$ as described in [4]. To demonstrate the ability of this algorithm, simulations are carried out with $d = 4$. It is also assumed that all $g_k(t)$ are equal. The signal covariance matrix of the four impulses within one period is given by

$$\boldsymbol{S} = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix}, \tag{9}$$

and the delay factors are given as $\tau_1 = 0$, $\tau_2 = 0.25$, $\tau_3 = 0.39$ and $\tau_4 = 0.625$ (which are expressed as fractions of the total period). The SNR ratio is 20dB and totally 1025 snapshots are used to form the covariance matrix. One hundred of them are shown in Figure 2(a) as reference. The results are shown in Figure 2(b). Since the estimated pulses are shifted and scaled, for the sake of comparison, all the pulses are normalized to have maximum amplitude of one and shifted to the center accordingly before plotting in the figures. The estimated τ are given as $\hat{\tau}_1 = 0$, $\hat{\tau}_2 = 0.289$, $\hat{\tau}_3 = 0.411$ and $\hat{\tau}_4 = 0.643$. Although the estimated delay factors deviate quite a lot from the actual ones, from the figures, it is clear that the deconvolving results are very good as little errors are observed between the original and the estimated impulse responses. The deviations of the estimated delay factors are mainly due to the assumption $\boldsymbol{G}_o \approx \boldsymbol{G}_e$ which introduces unavoidable errors to the algorithm.

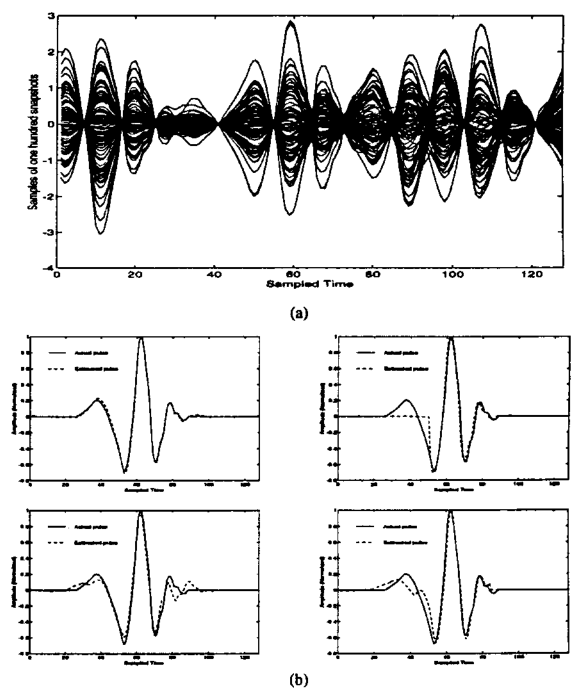

Figure 2: (a) 100 snapshots for signals with precise T and time-invariant τ_k (b) The four deconvolved impulse responses represented by dotted lines. The solid line represents the actual pulse.

III. MODIFIED ALGORITHM

Although excellent results are obtained in the above simulations, this simulated situation is too ideal in practice because we have assumed that τ_k do not change with time. In reality, there will always be some fluctuations in τ_k. Moreover, our a priori knowledge of the period T may not be exact due to imprecise estimation, equipment error or system properties. Large errors and discrepancies will occur as a result, especially when the snapshots are obtained sequentially from a long but single time series. These random perturbations degrade the performance of the algorithm greatly as will be shown in latter simulations.

To circumvent this kind of ubiquitous uncertainties, we have to take into account the deviations in τ_k and T. First of all, notice that the deviations in the delay factors are random in nature. Moreover, though the perturbations introduced by inexact period is not really random, if this error is small and there are a large number of snapshots which are obtained sequentially from single time series, we may assume the perturbations to be uniform. Hence the delay factors may be expressed in the form

$$\tau_{ik} = \tau_k + \theta_{ik}\,, \tag{10}$$

here τ_k is the mean delay factor for the k impulses and θ_{ik} represents the random perturbations from the actual delay. With this, (2) may be rewritten as

$$\bar{\boldsymbol{y}}_i = (\boldsymbol{G} \odot \boldsymbol{M}_i)s_i + \boldsymbol{n}_i\,, \tag{11}$$

where $\boldsymbol{M}_i$ is given by

$$\begin{aligned}\boldsymbol{M}_i &= \begin{bmatrix} \phi_1^0 & \cdots & \phi_d^0 \\ \phi_1^1 & \cdots & \phi_d^1 \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \phi_1^{4m-1} & \cdots & \phi_d^{4m-1} \end{bmatrix} \odot \begin{bmatrix} \Theta_{i1}^0 & \cdots & \Theta_{id}^0 \\ \Theta_{i1}^1 & \cdots & \Theta_{id}^1 \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \Theta_{i1}^{4m-1} & \cdots & \Theta_{id}^{4m-1} \end{bmatrix} \\ &= \boldsymbol{M} \odot \boldsymbol{P}_i\,, \end{aligned} \tag{12}$$

with $\Theta_{ik} = \exp^{-j\frac{2\pi}{4mt_0}\theta_{ik}}$. From this, we can rewrite (7) as

$$\begin{aligned}\bar{\boldsymbol{x}}_i &\approx \begin{bmatrix} (\boldsymbol{G}_o \odot \boldsymbol{M}_o \odot \boldsymbol{P}_{oi})s_i \\ (\boldsymbol{G}_o \odot (\boldsymbol{M}_o\boldsymbol{\Phi}) \odot \boldsymbol{P}_{ei})s_i \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_{oi} \\ \boldsymbol{n}_{ei} \end{bmatrix} \\ &= (\boldsymbol{A}_o \odot \begin{bmatrix} \boldsymbol{P}_{oi} \\ \boldsymbol{P}_{ei} \end{bmatrix})s_i + \begin{bmatrix} \boldsymbol{n}_{oi} \\ \boldsymbol{n}_{ei} \end{bmatrix} \\ &= (\boldsymbol{A}_o \odot \delta\boldsymbol{P}_i)s_i + \begin{bmatrix} \boldsymbol{n}_{oi} \\ \boldsymbol{n}_{ei} \end{bmatrix}\,, \end{aligned} \tag{13}$$

where $\boldsymbol{P}_{oi}$ and $\boldsymbol{P}_{ei}$ contain the odd and the even rows of $\boldsymbol{P}_i$ respectively. To derive a closed form for the covariance matrix of the the perturbed data $\bar{\boldsymbol{x}}$, we assume θ_{ik} to be i.i.d. and treat Θ_{ik} be the outcomes of an uniformly distributed random variable $\boldsymbol{\Theta}$ with zero mean. Moreover, it is assumed that $\boldsymbol{\Theta}$ is statistically independent with the noises and the signals. The justification of these assumptions is obvious from our previous discussions on θ_{ik}. Based on these assumptions, the data covariance matrix is given as

$$\begin{aligned}\boldsymbol{R}_{\bar{x}\bar{x}} &= \mathrm{E}\{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}^*\} \\ &= \mathrm{E}\{[(\boldsymbol{A}_o \odot \delta\boldsymbol{P})\boldsymbol{s} + \boldsymbol{n}]\,[(\boldsymbol{A}_o \odot \delta\boldsymbol{P})\boldsymbol{s} + \boldsymbol{n}]^*\} \\ &= \mathrm{E}\{(\boldsymbol{A}_o \odot \delta\boldsymbol{P})\boldsymbol{s}\boldsymbol{s}^*(\boldsymbol{A}_o \odot \delta\boldsymbol{P})^*\} + \sigma^2\boldsymbol{I} \\ &= \mathrm{E}\{\sum_{i=1}^{d} s_i\,[\boldsymbol{a}_{o_i} \odot \delta\boldsymbol{p}_i] \sum_{j=1}^{d} s_j^*\,[\boldsymbol{a}_{o_j} \odot \delta\boldsymbol{p}_j]^*\} \\ &= \sum_{i=1}^{d}\sum_{j=1}^{d} s_{ij}\mathrm{E}\{[\boldsymbol{a}_{o_i} \odot \delta\boldsymbol{p}_i]\,[\boldsymbol{a}_{o_j} \odot \delta\boldsymbol{p}_j]^*\}\,, \end{aligned} \tag{14}$$

where $s_{ij} = \mathrm{E}\{s_i s_j^*\}$, $\boldsymbol{a}_{o_i}$ and $\delta\boldsymbol{p}_i$ are the ith column vector of $\boldsymbol{A}_o$ and $\delta\boldsymbol{P}$ respectively. Since

$$\begin{aligned} &\mathrm{E}\{[\boldsymbol{a}_{o_i} \odot \delta\boldsymbol{p}_i]\,[\boldsymbol{a}_{o_j} \odot \delta\boldsymbol{p}_j]^*\} \\ &= \mathrm{E}\{\mathrm{Diag}(\boldsymbol{a}_{o_i})(\delta\boldsymbol{p}_i\delta\boldsymbol{p}_j^*)\mathrm{Diag}(\boldsymbol{a}_{o_j}^*)\} \\ &= \mathrm{Diag}(\boldsymbol{a}_{o_i})\mathrm{E}\{\delta\boldsymbol{p}_i\delta\boldsymbol{p}_j^*\}\mathrm{Diag}(\boldsymbol{a}_{o_j}^*) \end{aligned} \tag{15}$$

by substituting (15) into (14), we get

$$\boldsymbol{R}_{\bar{x}\bar{x}} = (\boldsymbol{A}_o\boldsymbol{S}\boldsymbol{A}_o^*) \odot \boldsymbol{V} + \sigma^2\boldsymbol{I}\,, \tag{16}$$

with

$$\begin{aligned}\boldsymbol{V} &= \mathrm{E}\left\{\delta\boldsymbol{p}_i\delta\boldsymbol{p}_j^*\right\} \\ &= \mathrm{E}\left\{\begin{bmatrix} \boldsymbol{\Theta}^1 \\ \vdots \\ \boldsymbol{\Theta}^{4m-1} \\ \boldsymbol{\Theta}^0 \\ \vdots \\ \boldsymbol{\Theta}^{4m-2} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Theta}^{-1} & \cdots & \boldsymbol{\Theta}^{-(4m-1)} & \boldsymbol{\Theta}^0 \cdots & \boldsymbol{\Theta}^{-(4m-2)} \end{bmatrix}\right\} \\ &= \begin{bmatrix} \boldsymbol{V}_1 & \boldsymbol{V}_3 \\ \boldsymbol{V}_2 & \boldsymbol{V}_1 \end{bmatrix}\,, \end{aligned} \tag{17}$$

and

$$\boldsymbol{V}_1(i,j) = \mathrm{E}\{\boldsymbol{\Theta}^{2(i-j)}\} \tag{18}$$

$$\boldsymbol{V}_2(i,j) = \mathrm{E}\{\boldsymbol{\Theta}^{2(i-j)-1}\} \tag{19}$$

$$\boldsymbol{V}_3(i,j) = \mathrm{E}\{\boldsymbol{\Theta}^{2(i-j)+1}\}\,, \tag{20}$$

Notice that (17) follows from the fact that Θ_{ik} are i.i.d. for different i and k. By comparing (16) and (17) with the formulations as in [2], it can be seen that the compensating principle as stated in [2] can be applied, with slight modifications, to compensate for the undesirable perturbations. The main idea of the compensating algorithm is to perform a point by point division of the perturbed data covariance matrix $\boldsymbol{R}_{\bar{x}\bar{x}}$ by the perturbation matrix $\boldsymbol{V}$ before the blind deconvolution procedures. The details of the algorithm can be found in [3].

IV. SIMULATION RESULTS

To illustrate the significance of our modified algorithm described above, simulations are carried out with the same impulse response $g(t)$ as in Section II with fluctuations added to the delay factors of the impulses. The delay factors are set to be uniformly distributed with unknown amplitude about their mean values. The mean delay factors are given as $\tau_1 = 0$, $\tau_2 = 0.25$, $\tau_3 = 0.39$ and $\tau_4 = 0.625$ with the same signal covariance matrix as (9). Moreover, the period of the impulses are inexact and deviated from the actual period by 7%. Though this deviation is not large, it will cause great problems as the snapshots are obtained sequentially from the convolved signals. The deviations will be carried onwards and positions of the pulses will vary from snapshots to snapshots. This situation is depicted in Figure 3 where 100 sample snapshots are shown. It can be seen that they look messy. Although the perturbation parameter is unknown, by intuitive reasoning, we may assume its maximum amplitude to be half of the estimated period. In fact by observing Figure 3, the deviations caused by wrongly estimated period determines the maximum amplitude of the perturbations to be approximately half of the actual period.

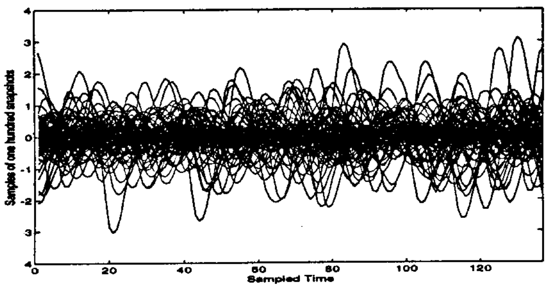

Figure 3: 100 snapshots for signals with estimated T and time-varying τ_k

Results obtained by the unmodified deconvolving method are shown in Figure 4(a) while results for using the modified deconvolving algorithm are plotted in Figure 4(b) By comparing the figures, it is very clear that the modified method outperforms that of the unmodified one when there are errors in estimating the actual period or when the delay factors change with time. This fact is further reinforced by Table 1 which summarizes the results of the estimated delay factors and the errors in the estimated pulse shape in all the three cases (including that from Section II). From the data, the mean squared errors between the estimated and actual pulses (**MSE-PULSE**) for modified deconvolution algorithm are much smaller than that of unmodified method and the estimated delay factors (**EDF**)are more accurate.

	EDF				MSE-PULSE			
	τ_1	τ_2	τ_3	τ_4	**P1**	**P2**	**P3**	**P4**
(a)	0.00	0.29	0.41	0.64	0.08	0.06	0.02	0.09
(b)	0.00	0.34	0.45	0.69	0.15	0.18	0.15	0.17
(c)	0.00	0.32	0.42	0.65	0.10	0.11	0.08	0.09

Table 1: Table showing the estimated delay factors (**EDF**) and mean squared errors (**MSE-PULSE**) between the estimated and actual pulses. **(a)** refers to results in Figure 2(b). **(b)** corresponds to results in Figure 4(a). **(c)** represents the results in Figure 4(b). **P***i* stands for pulse *i*.

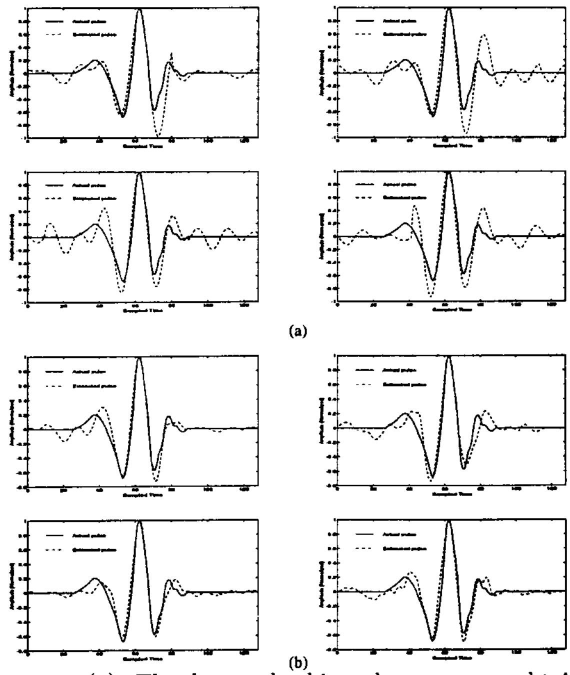

Figure 4: (a). The deconvolved impulse responses obtained without using any compensating method. (b). The deconvolved impulse responses obtained by using the modified algorithm.

V. CONCLUSION

In this paper, we have devised a ***robust*** blind deconvolution algorithm which is based on the principles of array signal processing. The algorithm can be used to find the unknown impulse responses of a system excited by cyclostationary impulses in the presence of signal fluctuations.

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