BLIND IDENTIFICATION USING THE KURTOSIS : RESULTS OF FIELD DATA PROCESSING

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ABSTRACT

In this paper we present a method to estimate non-minimum phase AR or ARMA systems based on maximum kurtosis properties. First the Spectrally Equivalent Minimum Phase (SEMP) filter is estimated from output statistics, then the kurtosis allows us to localise the zeros of the associated transfer function from the zeros of its SEMP filter. Combining kurtosis properties and Singular Value Decomposition (SVD) properties we propose a new ARMA orders determination mehod. On field seismic data we compare the proposed method to Gianakis-Mendel's algorithm and Tugnait's algorithms. On field underwater explosions data we present a new results showing the interest of estimating a non-causal AR filter to modelise the secondary waves. The results obtained on shorth length of data (128 samples) confirm the robustness of the proposed method.

1. INTRODUCTION

The use of Higher Order Statistics (HOS) than two has received attention in the statistics signal processing, and theory literature, for processing non-gaussian linear or non-linear processes [1]. HOS are identically zero for gaussian processes. Furthermore all odd order statistics are identically zero for processes with symmetric probability density functions, for this reason we have chosen to use fourth-order statistics.

The use of parametric approach based on AR, MA, or ARMA models, has provided different solutions [1]. To identify non-minimum phase AR or ARMA filters, we propose first to estimate the spectrally equivalent minimum phase (SEMP) filter, then using the maximum kurtosis property, to recover the true filter. Also we propose using maximum kurtosis properties and singular value decomposition, to determine the orders of filters.

On field seismic data we compare the identification of ARMA filters using the maximum kurtosis properties to Gianakis-Mendel's algorithm and Tugnait's algorithms.

On field underwater explosions data we show the interest of modeling the secondary waves by a non-causal AR.

2. HOS AND BLIND IDENTIFICATION

$$x(t) = \sum_{i=-\infty}^{+\infty} h(i) r(t-i) \tag{1}$$

Let us consider $\{x(t)\}$ the output signal of a linear time invariant filter defined by a real impulse response h(i) and driven by a zero-mean stauonary and fourth order white noise $\{r(t)\}$ such as:

$$C_{2r}(m) = S_{2r}\delta(m) \text{ and } C_{4r}(l,m,n) = S_{4r}\delta(l,m,n) \tag{2}$$

Where $\delta(.)$ is the kronecker delta function, S_{2r} and S_{4r} are real ($S_{4r} \neq 0$).

$C_{2r}(m) = E\{r(t)r(t+m)\}$ is the autocorrelation function., and $C_{4r}(l,m,n)$ is the tricorrelation function defined by :

$$\begin{aligned} C_{4r}(l,m,n) = {} & E\{r(t)r(t+l)r(t+m)r(t+n)\} \\ & -C_{2r}(l)C_{2r}(m-n) - C_{2r}(m)C_{2r}(l-n) \\ & -C_{2r}(n)C_{2r}(l-m) \end{aligned} \tag{3}$$

Assuming the input signal is non-gaussian white noise, the objective is to identify the impulse response h(i) using only the output statistics.

The output second order statistics (ouput spectrum or output autocorrelation) are phase blind (phase of the transfer function), their use only allow to identify the spectrally equivalent zero-phase, minimum-phase or maximum phase filters.

The use of HOS allow to identify the true phase, because the output HOS keep information about the phase of the associated transfer function. The use of HOS has provided different solutions [2]. Based on parametric approach our purpose consists first in estimating the Spectrally Equivalent and Minimum phase filter, then the true filter is given by searching for the zeros location in the transfer function expression giving a deconvolution signal having the maximum kurtosis value.

Assuming the input signal is fourth order white noise, the kurtosis of the output signal $\{x(t)\}$ is :

$$Kurt\{x(t)\} = \frac{C_{4x}(0,0,0)}{\left[C_{2x}(0)\right]^2} = Kurt\{r(t)\} \frac{\sum_i h^4(i)}{\left[\sum_i h^2(i)\right]^2} \tag{4}$$

with :

$$\left|Kurt\{x(t)\}\right| \le \left|Kurt\{r(t)\}\right| \tag{5}$$

Its value indicates "the distance" between the statistics of $\{x(t)\}$ and the gaussianity. Note that the kurtosis of $\{x(t)\}$ is lower than the kurtosis of $\{r(t)\}$, implying that all their filtered versions are "more gaussian". The kurtosis of $\{r(t)\}$ is an upper bound for the kurtosis of any filtred version of $\{x(t)\}$.

As proposed by Donoho [2] and later by Shalvi-Weinstein [3], a criteria of good non-minimum phase blind identification is to maximise the kurtosis absolute value of the estimate of {r(t)}. We use this criteria both to identify the true filter from its SEMP filter, and to determine the order of the filter.

3. IDENTIFICATION OF ARMA FILTER

We consider the case of causal ARMA process :

$$x(t)+\sum_{i=1}^{p} a(i)x(t-i)=r(t)+\sum_{i=1}^{q} b(i)r(t-i) \qquad (6)$$

The associated transfer function in the Z-domain is :

$$H(z)=B(z)/A(z) \qquad (7)$$

The information about non-minimum phase is contained in zero locations of $H(z)$. To identify the ARMA parameters, first the AR parameters are estimated using second order or fourth order statistics. Then the MA part is estimated from the residual time series [1].

3.1) Identification of the AR part :

If H(z) does not contain all-pass factors, the AR part can be estimated from the output autocorrelation function using the Yule-Walker equation [4]. Otherwise the use of fourth order statistics is necessary to identify the AR part, because second order statistics are insensitive to all-pass factors [5].

3.2) Identification of the MA part :

{x(t)} is convolved by $\hat{A}(z)$ the estimate of the AR part, to give a Residual Time Series (RTS) that only contains information about the MA parameters. The minimum phase MA part is estimated using second order statistics, all its zero modulus are lower than the unity. To recover the true MA part we must find the zeros that must be replaced by their inverse conjugates. Using the maximum kurtosis property, the true filter is the one giving a deconvolution signal having the maximum kurtosis value [6].

The interest of this approach is to estimate the numerical values of MA parameters using second order, so with lower estimation variance than if HOS have been used .

3.3) Orders determination :

To determine the true orders of the ARMA(p,q) model, the kurtosis given by ARMA(i,j) is estimated for $i=1,..,p,..,p_{\max}$ and $j=1,..,q,..,q_{\max}$. The true order is the one giving a deconvolution signal having a maximum kurtosis value [6].

Another method to determine ARMA orders consists first in overestimating the order of the MA part, then the AR part order "p" is estimated by applying the SVD to the autocorrelation matrix or tricorrelation matrix [7].

The true MA order is estimated by searching for the ARMA(p,j) that maximises the kurtosis, for $j=1,..,q,..,q_{\max}$ [6].

4. IDENTIFICATION OF AR FILTERS

Suppose that {x(t)} is the output of a non-causal and stable AR system driven by {r(t)}:

$$\sum_{i=-p_{ac}}^{p_c} a(i)x(t-i)=r(t) \qquad (8)$$

with $a(0)=1$

The associated transfer function is :

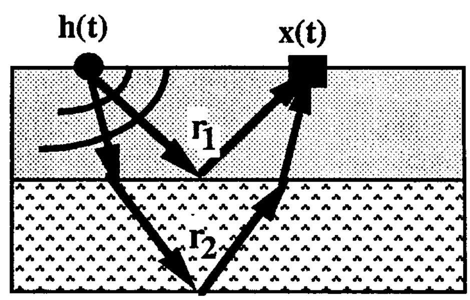
$$H(z)=H_{ac(z)}H_c(z) \qquad (9)$$

where $H_{ac}(z)$ and $H_c(z)$ are respectively the anti-causal part and the causal part. Using the Yule-Walker equation, the second order only allows the identification of $H_{mp}(z)$ the SEMP AR($p_{ac}+p_c$).

All the poles of $H_{mp}(z)$ are inside the unit circle. Like the case of non-minimum phase MA part of the ARMA, $H(z)$ is recovered from $H_{mp}(z)$ by searching for the pole locations (inside or outside the unit circle) maximising the kurtosis [6]. The order can be determined using the same method proposed for the ARMA case.

5. APPLICATION TO FIELD DATA

5.1) Blind identification in seismic :



In seismic, a wave of signature h(t) (Wavelet) is transmitted in the subsurface, unfortunately h(t) is not always known. The propagation is assimilated to a linear time invariant system; the wave is reflected or refracted on every interface 'i', giving a reflection coefficients, of magnitude " r_i "and time position "t_i". The received signal on one sensor (seismic trace) is modelled by

$$x(t)=\sum_i r_i h(t-t_i)=h(t)*\sum_i r_i\delta(t-t_i)=h(t)*r(t) \qquad (10)$$

where$(*)$ is the convolution operator.

{r(t)} is the reflectivity signal, it contains information about the geological structures of the subsurface. Using a statistical-based model and assuming that the reflectivity is non-gaussian white noise, the objective, is to estimate the unknown wavelet h(t) and the reflectivity r(t), only from the statistics of the trace.

On a short length of field seismic data : 128 samples given by "ORSTOM". We compare the blind deconvolution results of different algorithms using an ARMA model for the wavelet

(filter). The data have been pre-processed by Spectral Matrix Filtering to improve the signal-to-noise ratio [8].
The MK2-wavelet and the MK4-wavelet found by the order determination methods are an ARMA(4,5). In figure 2 are presented the trace (the output signal), the deconvolution results by the wavelets estimated using different algorithms :

1) The MK2-wavelet which its AR part is estimated using second order statistics Yule-Walker equation [4] and its MA part using maximum kurtosis approach.
2) The MK4-wavelet which its AR part is estimated using fourth order statistics Yule-Walker equation [5] and its MA part using maximum kurtosis approach.
3) The GM-wavelet which its AR part is estimated using second order statistics Yule-Walker equation [4] and its MA part using Giannakis-Mendel algorithm [9].
4) The T91-wavelet which its AR part is estimated using second order statistics Yule-Walker equation [4] and its MA part using Tugnait's algorithm [10].
5) MP-wavelet which is estimated using a minimum phase approach [4].

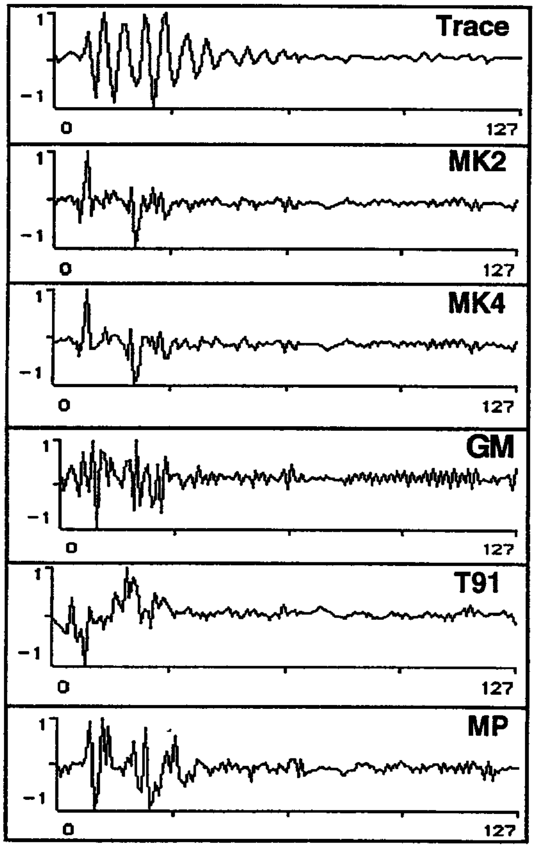


Figure 2- Deconvolution results using the estimated wavelets

In this example of field seismic data, we show the capacity of MK2-algorithm and MK4-algorithm to process short length of data, they given the same results which are close to the geophysical model of the reflectivity. The MP-algorithm gives a coloured reflectivity. GM and T91 use second and fourth order statistics to estimate the MA part. While MK2-algorithm and MK4-algorithm uses only second order statistics, the kurtosis is only used to localise the zeros (inside or outside the unit circle). That may be why GM and T91 do not give the same result than MK2 and MK4.

5.2) Underwater explosions :

An explosion is a chemical reaction in a substance which converts the original material into a gas at very high temperature and pressure, the process occurring with extreme rapidity and evolving a great deal of heat. The first cause of disturbance to the water is the arrival of the pressure wave in the reacting explosive at the water boundaries, the disturbance is propagated radially outward as a wave of compression in the water, the steep fronted wave being described as the "shock wave" [11]. It has a short duration and may be modelled as a minimum-phase AR(p) filter. The initial high pressure in the gas sphere is considerably decreased after the principal part of the shock wave has been emitted. The water in the immediate region of the sphere or "bubble" has a large outward velocity and the bubble does in fact undergo repeated cycles of expansion and contraction, spending most of its time in an expanded condition [11]. We propose to model this secondary waves using a non-causal AR filter, the anti-causal part corresponding to expansion phase and the causal part corresponding to contraction phase. In this paper we are interested by the study of the secondary waves.
The measurement of the impulse response of the different waves is complicated by reflections from the surface, the bottom and the boundaries, the signal observed at the sensor is the sum of direct and reflected waves. We model the signal recorded at the sensor using equation (10).
Where x(t) is the output signal recorded on the sensor, h(t) is the impulse response of the linear time-invariant filter modelling the secondary waves, and r(t) is the input signal containing information about the reflectors time position and amplitudes.
To estimate the non-causal AR filter, first its SEMP AR filter is estimated using the modified covariance algorithm [4].
Then the non-causal AR filter is estimated by searching for poles locations maximising the kurtosis. To estimate the order of the non-causal AR filter we propose two methods.

5.2.1. Orders determination based on a physical approach :

Basing on the physics of the bubble phenomena, we propose to model the secondary wave using a non-causal AR(1,1) filter. The anti-causal part (increasing exponential) corresponds to the expansion phase and the causal part (decreasing exponential) corresponds to the contraction phase. The corresponding AR(1,1) is presented in figure 3.

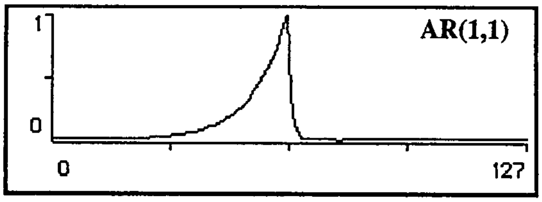


Figure 3- Estimated Non-causal AR(1,1)

5.2.2. Orders determination using the kurtosis :

For i=1,...,12 first we estimate the causal AR(i), then the poles location maximising the kurtosis is given by the AR(i_{nc}, i_c) such as "$i = i_{nc} + i_c$". To make easy the interpretation the AR(i_{nc}, i_c) giving the maximum kurtosis value is indexed by "i" in figure 4. The maximum kurtosis value is obtained for an AR(3,1) (figure 5).

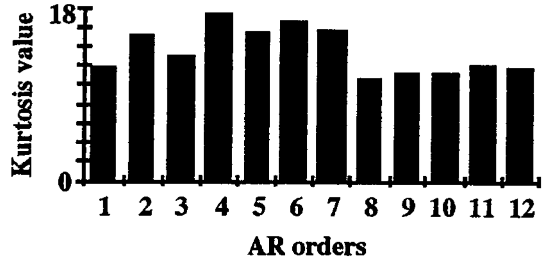


Figure 4- Order determination

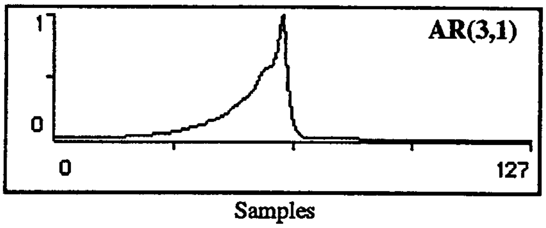


Figure 5- Estimated non-causal AR(3,1)

In figure 6 is presented one sensor record of field underwater explosion given by "LACROIX Society", the result of deconvolution using the non-causal AR(3,1), the deconvolution result using the non-causal AR(1,1), the result of minimum phase deconvolution using a minimum phase AR(4) (MP).

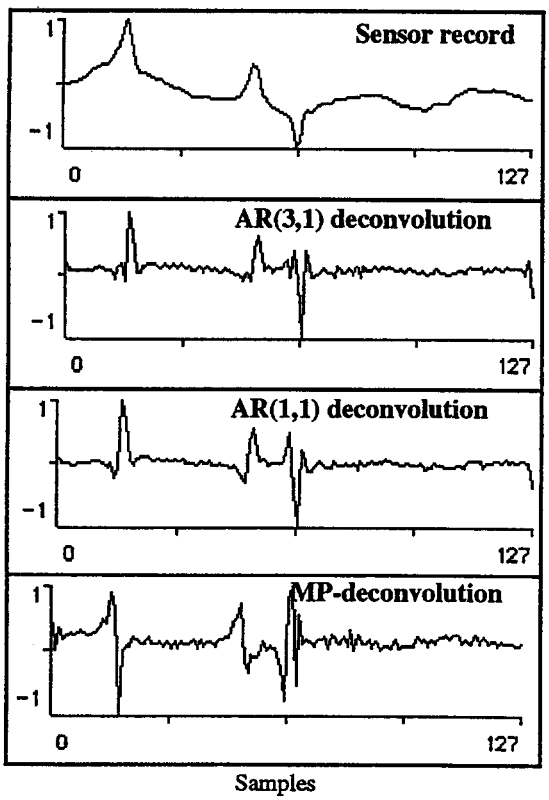


Figure 6- Deconvolution results

This example illustrates the interest of using non-causal AR filter to models the secondary waves with a few parameters. This is possible when the data have been recorded, that allows to use at time "n" the information occurring at "n+m" (m>0).

6. CONCLUSION

We have shown the interest and the capacity of maximum kurtosis property to process field data. On a short length of field seismic data we have successfully tested the method to identify ARMA filter and compare it to others algorithms. Because of the high variance of estimation of HOS, the results obtained confirm the following rule : "For blind identification processes, we must use all the second order statistics information and the strictly required of HOS information".

On field underwater explosions data we have shown the interest of using non-causal AR filter to modelise the secondary waves. Many authors assume that all additive noises are gaussian (are HOS are identically zero), and propose performing blind identification only from the output HOS. We think that is not the best solution since first there is no theoretical or field justification for gaussian noise, and second, the high estimation variance of HOS limits their use. To solve this problem we suggest to pre-process the data by noise subtraction methods before the blind identification.

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