On Fractionally-Spaced Blind Adaptive Equalization Under Symbol Timing Offsets Using Godard And Related Equalizers

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ABSTRACT

The problem of fractionally-spaced (FS) blind adaptive equalization under symbol-timing-phase offsets is considered. It is well-known that in the case of trained (non-blind) equalizers, the performance of FS equalizers is independent of the timing-phase unlike that of baud-rate equalizers. Moreover, trained FS equalizers synthesize optimal filters in MMSE sense, hence are superior to baud-rate trained equalizers. These advantages of trained FS equalizers have not been shown to be true for blind equalizers, rather they have been simply assumed. We present a simulation example where such advantages do not materialize. Then we present a solution based upon a parallel, multimodel Godard adaptive filter bank approach which yields a performance almost invariant w.r.t. symbol-timing-phase. An illustrative simulation example 16-QAM (V22 source) signal is presented where effect of symbol-timing-phase offset is studied via computer simulations.

1 Introduction

It is known that, in the case of trained (non-blind) equalizers under symbol-timing-phase errors, the performance of baud-rate equalizers can seriously degrade if there is an excess bandwidth [3]. Under excess bandwidth, baud-rate equalizers operate on data sampled at a rate lower than the Nyquist rate, hence, they can only equalize an aliased version of the true channel. The aliased channel undergoes changes with timing-phase changes. For some choices of the timing-phase (*bad timing-phase*), severe destructive interference can take place in the aliased-channel transfer function leading to nulls or near-nulls in the transfer function. A baud-rate linear equalizer whose length is fixed a priori independent of the timing phase, may not be able to adequately equalize the severely distorted aliased channel, and moreover, it leads to greater noise enhancement for such channels (because of the presence of near-nulls). In contrast, fractionally-spaced equalizers sample data at least as fast as the Nyquist rate so that they equalize the true channel and their performance is independent of the timing-phase errors. Such advantages of trained fractionally-spaced equalizers have not been shown to be true for blind equalizers, rather they have been simply assumed, as in [5],e.g. We present a simulation example where such advantages do not materialize. Then we present a "fix" based on [8].

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2 Model Assumptions

Suppose that we sample at M-times the baud rate with signal samples spaced T/M seconds apart where T is the symbol duration. Then the sampled version of the continuous-time received signal $y(t)$ is given by [7],[8]

$$y_{ik} = y(t)\,|_{t=kT+t_0+(i-1)\frac{T}{M}} \quad (i = 1, 2, \cdots, M)$$

$$= \sum_{n=-L_{1i}}^{L_{2i}} f_{in} I_{k-n} + \eta_{ik}, \qquad (1)$$

where we have M samples every symbol period, indexed by i, and t_0 denotes some arbitrary time offset. The information sequence I_k is one "sample" per symbol and is a zero-mean, i.i.d. complex sequence. The sequence $\{f_{ik}\}$ (with transfer function $F_i(z)$) is an FIR linear filter (with possibly complex coefficients) that represents the equivalent (sub-)channel. The noise sequence $\{\eta_{ik}\}$ is assumed to be zero-mean Gaussian and independent of $\{I_k\}$. See also Figs. 1 and 2.

3 Fractionally-Spaced Godard Adaptive Blind Equalizer

The fractionally-spaced Godard adaptive blind equalizer (FSGABE) [2],[5] acts on the data $\{y_{ik}, i = 1, 2, \cdots, M\}$ via the FIR linear equalizer $\{c_{ik},\ i = 1, 2, \cdots, M;\ 0 \le k \le K-1\}$ (transfer function $C_i(z)$ $(i = 1, 2, \cdots, M)$) to generate the *symbol-spaced* output

$$\tilde{I}_k = \sum_{i=1}^{M} \left(\sum_{j=0}^{K-1} c_{ij} y_{i(k-j)} \right). \qquad (2)$$

The (fractionally-spaced) equalizer coefficients are adjusted to minimize the cost function $\mathrm{E}[|\tilde{I}_k|^2 - R_2]^2$ where $R_2 = \mathrm{E}\{|I_k|^4\}/\mathrm{E}\{|I_k|^2\}$. Godard has derived an LMS-like stochastic approximation algorithm for the above minimization. Let $\mathbf{c}_n$ denote the M-vector of equalizer tap gains at time sample n given by $\mathbf{c}_n = \{c_{ik}(n),\ i = 1, 2, \cdots, M;\ 0 \le k \le K-1\}$, and let $\tilde{\mathbf{Y}}_n$ denote the KM-vector of data samples on which $\mathbf{c}_n$ operates. Then the equalizer tap gains are updated as

$$\mathbf{c}_{n+1} = \mathbf{c}_n - \lambda \tilde{\mathbf{Y}}_n^* \tilde{I}_n \left[|\tilde{I}_n|^2 - R_2\right], \qquad (3)$$

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where λ, a positive scalar, is the filter step-size. A block-diagram of FSGABE is shown in Fig. 1.

Foschini [4] has analyzed the convergence of this class of blind equalizers under the idealized case of *no noise* and *doubly-infinite linear equalizers.* Under these conditions, asymptotically the equalizer transfer functions satisfy

$$\sum_{i=1}^{M} F_i(z)C_i(z) = Az^{-n_d} \tag{4}$$

where is A is a complex constant with $|A| = 1$ and n_d is an integer. Constants A and n_d reflect the "phase-rotation" and time-delay, respectively, that the Godard equalizer cannot resolve. Notice that (4) does not indicate a unique solution; rather it shows multiple solutions each one of which leads to zero ISI (in the idealized case). In the presence of noise, not all solutions to (4) lead to same noise enhancement; some are much worse than the others. The problem is that there is no inherent mechanism in the Godard equalizer that would "prefer" less noise enhancing solution over more noise enhancing solution! Moreover, in the case of finite-length equalizers, there are additional convergence problems [9]. Let's look at a simulation example.

Simulation Example The basic pulse shape used for transmission is a raised cosine pulse $c(t, \alpha)$ with a roll-off factor α. The equivalent continuous-time channel impulse response is given by

$$h(t) = \sum_{i=0}^{10} d_i c(t - iT, 0.3) W_{4T}(t - iT)$$

where $\{d_i, \ \ 0 \leq i \leq 10\} = \{0.04, -0.05, 0.07, -0.21, -0.50, 0.72, 0.36, 0.00, 0.21, 0.03, 0.07\}$, T denotes the symbol interval and $W_{4T}(t) = 1$ if $|t| \leq 2T$, $=0$ otherwise. That is, the raised-cosine pulse is truncated to four symbol intervals. Let t_0 represent the symbol-timing offset such that for T/M sampled data the channel impulse response is given by $h(t_0 + (kT/M))$. The channel amplitude spectra for T-sampling and various timing offsets t_0 are shown in Fig. 3. Fig. 4 shows the channel amplitude spectra for $T/2$-sampling and various timing offsets t_0's. The dependence on t_0 has markedly decreased in Fig. 4.

Computer simulation experiments were carried out for 16-QAM (V22 source) signals using T- and T/M-spaced data and equalizers for M = 2,3 and 4. Independent records of data with σ_I^2/σ_η^2 = 20dB were generated, where σ_I^2 is the variance of I_k and σ_η^2 is the variance of η_k. A record length of 15000 symbols was used per run. The FSGABE step-size was "optimized" by trial-and-error to achieve the minimum mean-square error at the equalizer output at the end of the run. The equalizer was initialized by setting the center tap to $2 + j0$ with all other taps set to zero [2]. Length of the equalizer was 31 symbols leading to $M \times 31$ taps ($K = 31$). Fig. 5(a) shows the symbol error rate (SER) for FSGABEs corresponding to $M = 1, 2, 3, 4$, based upon 10 Monte Carlo runs. ($M = 1$ correspond to baud-rate equalizer.) It is seen that oversampling does not lead to a performance invariant to timing offset t_0 except for $M = 4$. However, even for $M = 4$, the SER for some t_0's is quite inferior to that for baud-rate equalizer, quite unlike the trained case [3].

4 Multimodel Godard Adaptive Filter Bank Approach [8]

For details refer to [8]. A block diagram is shown in Fig. 2. Now we have a bank of *baud-rate* Godard blind equalizers operating independently on each subchannel. The baud-rate (running) Godard cost is used to separate the "good" subchannels (less noise-enhancing) from "bad" subchannels (having nulls or near-nulls, hence very noise-enhancing). Various ways are discussed in [8] to select and "fuse" various subchannels to obtain the equalized output $\tilde{I}_k$ (see Fig. 2).

Back to the Simulation Example The results averaged over 10 runs are shown in Figs. 5(b) and 5(c) for this approach under two different ways of combining good subchannels. Best subchannel refers to selection of a single subchannel, the one that corresponds to the least baud-rate Godard cost. Direct sum refers to selection of a group of subchannels that are close to the best subchannel, and then aligning (shifting and/or scaling) the respective outputs to simply sum them up. The results shown in Figs. 5(b),(c) were obtained under exactly the same conditions as those for FSGABE, namely, center tap initialization, and length of each baud-rate equalizer equal to 31 symbols. It is seen from Fig. 5 that a substantial improvement has been obtained by use of the approach of [8] over FSGABE, both in terms of insensitivity to symbol-timing-phase and in terms of absolute performance as measured by SER.

We note also that for baud-rate Godard equalizers the problem of mis-convergence due to finite length equalizers[9] can be handled via more educated initialization as discussed in [10]. However, for the considered example center-tap initialization along with an equalizer length of 31 symbols was found to work well.

5 References

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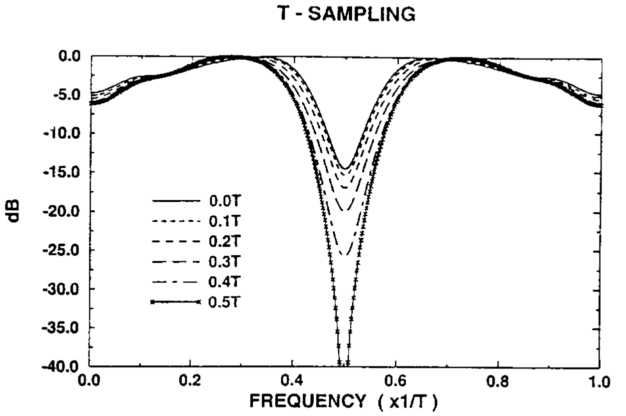


Fig. 3. Channel amplitude spectrum at baud-rate sampling at various symbol-timing-phase offsets t_0.

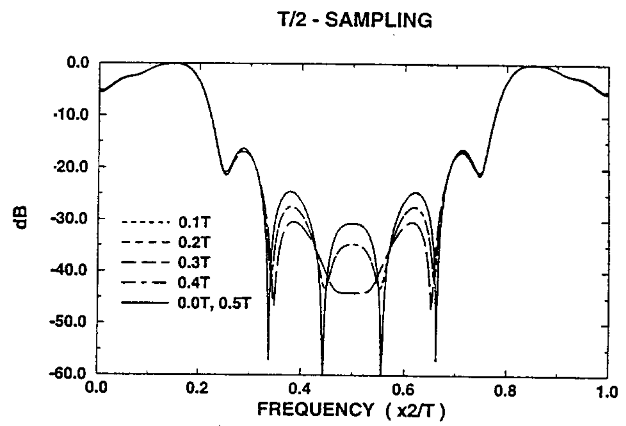


Fig. 4. Channel amplitude spectrum at twice the baud-rate sampling at various symbol-timing-phase offsets t_0.

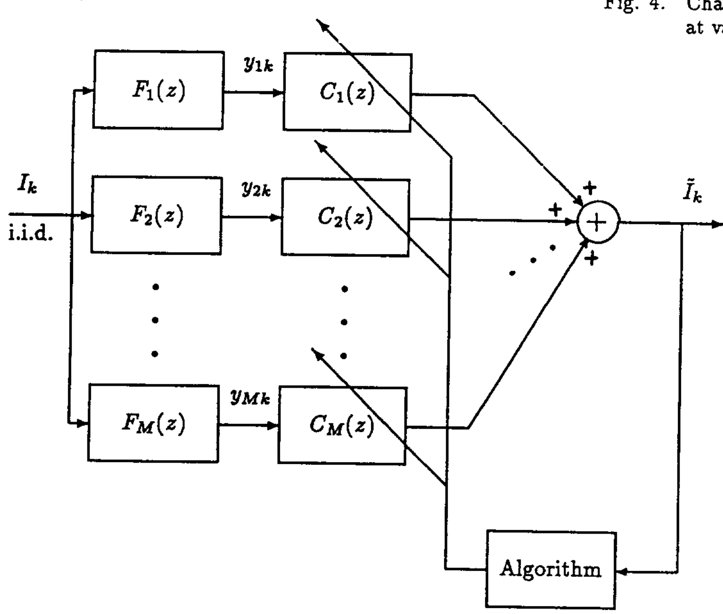


Figure 1: Fractionally-Spaced Godard Adaptive Blind Equalizer

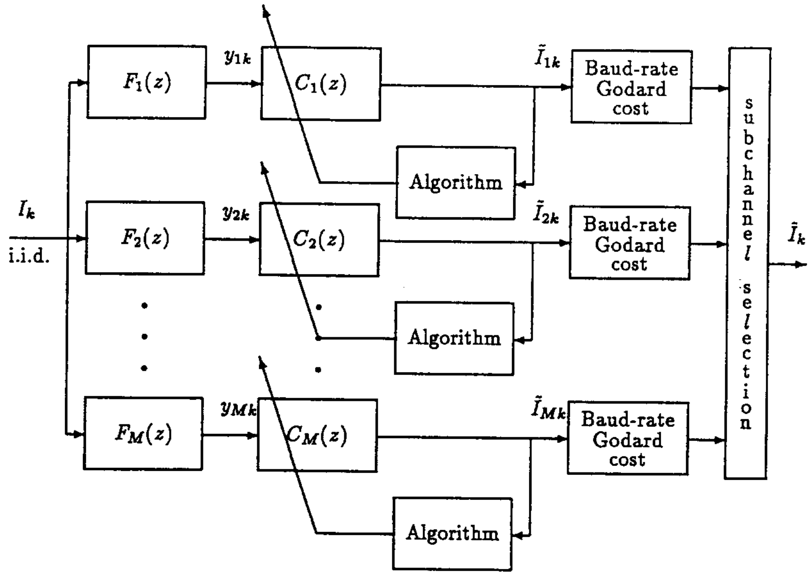
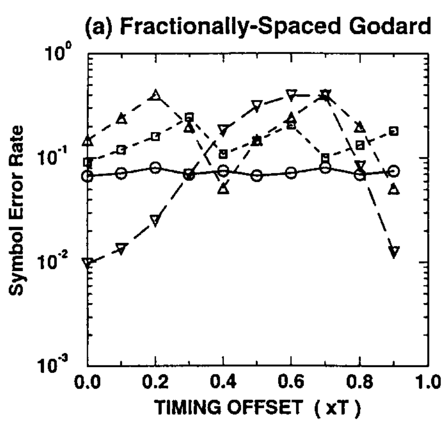


Figure 2: Multimodel Godard Filter Bank Approach



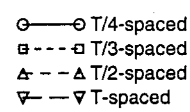


Fig. 5. Symbol error rate as a function of symbol-timing-phase offset t_0 and M (=1,2,3,4), based upon 10 Monte Carlo runs:

(a) Fractionally-spaced Godard equalizer,
(b) direct-sum equalization [8],
(c) best-subchannel equalization [8].

Data record length n = 14950 symbols, 16-QAM (V22 source) signal, σ_I^2/σ_η^2 = 20dB.