

A UNIFIED CLASS OF INVERSE FILTER CRITERIA USING TWO CUMULANTS FOR BLIND DECONVOLUTION AND EQUALIZATION

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ABSTRACT

Cumulant (higher-order statistics) based inverse filter criteria maximizing $J_{r,m} = |C_m|^r / |C_r|^m$, where $m \neq r$ and C_m (C_r) denotes the m th-order (r th-order) cumulant of the inverse filter output, have been proposed for blind deconvolution and equalization with only non-Gaussian output measurements of an unknown linear time-invariant (LTI) system. This paper shows that the maximum of $J_{r,m}$, associated with the true inverse filter of the unknown LTI system, exists only for r to be even and $m > r$, otherwise, $J_{r,m}$ is unbounded. The admissible values for $(r, m) = (2s, l + s)$ where $l > s \geq 1$ include (2, 3), (2, 4) and (4, 6) proposed by Tugnait, Wiggins, Shalvi and Weinstein in addition to more new ones such as (2, 5), (2, 6) and (4, 5). Some simulation results for the inverse filter criteria $J_{r,m}$ with the proposed admissible values of (r, m) are then provided. Finally, we draw some conclusions.

1. INTRODUCTION

Blind deconvolution as well as equalization is a quite known statistical signal processing problem to estimate the desired signal $u(n)$ with a given set of measurements $x(n)$, $n = 0, 1, \dots, N-1$ based on the following convolutional model

$$\begin{aligned} x(n) &= u(n) * h(n) + w(n) \\ &= \sum_{i=-\infty}^{\infty} h(i)u(n-i) + w(n) \end{aligned} \quad (1)$$

where $w(n)$ is measurement noise and $h(n)$ is an unknown linear time-invariant (LTI) system which corresponds to such as the source wavelet in seismic deconvolution, the channel impulse response in channel equalization and the vocal-tract filter in speech processing. A major conventional approach to this problem is the correlation (second-order statistics) based predictive deconvolution. The deconvolved signal is obtained by processing $x(n)$ with a minimum-phase linear prediction error (LPE) filter, which corresponds to an estimate for the inverse filter of $h(n)$ except for phase distortion because $h(n)$ may not be minimum-phase in practice. Recently, higher-order (≥ 3) statistics (HOS) [1,2], known as cumulants, have been considered in various signal processing areas where $x(n)$ is non-Gaussian

and measurement noise $w(n)$ is Gaussian with unknown statistics, partly because cumulants of $x(n)$ can be used to extract not only the amplitude information but also phase information of $h(n)$ and partly because higher-order cumulants of Gaussian noise $w(n)$ are zero.

Cumulant based inverse filter criteria [3-7] have been considered for the estimation of the inverse filter $h_I(n)$ of the unknown LTI system $h(n)$. Assume that $v(n)$ is an estimate for $h_I(n)$ and $e(n)$ is the output signal of $v(n)$ in response to $x(n)$, i.e.,

$$e(n) = x(n) * v(n) \quad (2)$$

and C_m denotes the m th-order cumulant of $e(n)$. A class of cumulant based inverse filter criteria has the following form [3-5]:

$$J_{r,m}(v(n)) = \frac{|C_m|^r}{|C_r|^m} \quad (3)$$

where $m \neq r$, $m \geq 2$ and $r \geq 2$. Wiggins [3] proposed an inverse filter criterion by maximizing $J = E[e^4(n)] / (E[e^2(n)])^2$, which is related to $J_{r,m}$ by $|J - 3|^2 = J_{2,4}$. Shalvi and Weinstein [4] proposed an inverse filter criterion by maximizing $|C_4|$ subject to the constraint $E[e^2(n)] = E[u^2(n)]$. Tugnait [5] also proposed inverse filter criteria by maximizing $J_{2,3}$ or $J_{2,4}$, or $J_{4,6}$. However, for other choices of r and m , it is still unknown whether maximizing $J_{r,m}$ can lead to the true inverse filter of $h(n)$. Chi and Kung [6] estimated the inverse filter by maximizing a single cumulant $|C_m|$ for $m \geq 3$ when $h(n)$ is an allpass system.

In this paper, we show that maximizing the objective function $J_{r,m}$ given by (3), which only uses two cumulants of $e(n)$, is applicable only for some certain choices of r and m which lead to more new inverse filter criteria in addition to the aforementioned existing inverse filter criteria as special cases of the admissible r and m .

2. ADMISSIBLE CUMULANT ORDERS FOR THE INVERSE FILTER CRITERIA $J_{r,m}$

First of all, let us make the following assumptions for measurements $x(n)$ modeled by (1).

- (A1) The unknown LTI system $h(n)$ is causal stable with either minimum phase or nonminimum phase and a stable inverse filter $h_I(n)$ of $h(n)$ exists.

- (A2) The driving input $u(n)$ is real, zero-mean, stationary, independent identically distributed (i.i.d.) non-Gaussian with variance σ_u^2 and m th-order cumulant γ_m where $m \geq 3$.
- (A3) Measurement noise $w(n)$ is Gaussian with unknown statistics.
- (A4) The input $u(n)$ and the noise $w(n)$ are statistically independent.

The admissible cumulant orders for the inverse filter criteria given by (3) are described in the following theorem.

Theorem 1. Assume that $x(n)$ is the noisy signal generated from the model given by (1) under the previous assumptions (A1) through (A4). Then the following two statements are true:

- (S1) $J_{r,m}(v(n))$ is unbounded except the case that $r = 2s$ (i.e., r is even), $m = l+s > r$ where $l > s \geq 1$. Moreover,

$$\max\{J_{2s,l+s}(v(n))\} = \frac{|\gamma_{l+s}|^{2s}}{|\gamma_{2s}|^{l+s}} \quad (4)$$

- (S2) The optimum $\hat{v}(n)$ associated with $J_{2s,l+s}(\hat{v}(n)) = \max\{J_{2s,l+s}(v(n))\}$ where $l > s \geq 1$ satisfies

$$\hat{v}(n) * h(n) = \alpha \delta(n - \tau) \quad (5)$$

where $\alpha \neq 0$ is a scale factor and τ is an unknown integer, for the two cases that $s = 1$ with $SNR = \infty$ and that $s > 1$ with finite SNR .

Two remarks for the inverse filter criteria $J_{2s,l+s}$ are summarized as follows:

- (R1) For $(s, l) = (1, 2)$, $(s, l) = (1, 3)$, and $(s, l) = (2, 4)$, $J_{2s,l+s}$ reduce to Tugnait's inverse filter criteria $J_{2,3}$, $J_{2,4}$ and $J_{4,6}$, respectively. For any other choices of (s, l) , $J_{2s,l+s}$, such as $J_{2,5}$, $J_{2,6}$ and $J_{4,5}$ are new.
- (R2) In practice, the two cumulants C_{2s} and C_{l+s} required by $J_{2s,l+s}$ must be replaced by the corresponding sample cumulants \hat{C}_{2s} and \hat{C}_{l+s} which are well-known consistent estimates for C_{2s} and C_{l+s} [2], respectively. Therefore, the optimum estimate $\hat{v}(n)$ is also a consistent estimate for the inverse filter $h_I(n)$ except for a scale factor and an unknown time delay.

Next, let us present how we find the optimum inverse filter $\hat{v}(n)$ associated with the inverse filter criteria $J_{2s,l+s}$ with finite data set $\{x(0), x(1), \dots, x(N-1)\}$. The inverse filter $v(n)$ is assumed to be a causal FIR filter of order equal to L . Then the inverse filter output $e(n)$ given by (2) can be expressed as

$$e(n) = \mathbf{v}^T \mathbf{x}_n \quad (6)$$

where \mathbf{v} and \mathbf{x}_n are $(L+1) \times 1$ column vectors given by

$$\mathbf{v} = [v(0), v(1), \dots, v(L)]^T \quad (7)$$

and

$$\mathbf{x}_n = [x(n), x(n-1), \dots, x(n-L)]^T, \quad (8)$$

respectively. Note that the inverse filter criteria $J_{2s,l+s}$ where $l > s \geq 1$ are highly nonlinear functions of \mathbf{v} since sample cumulants \hat{C}_{2s} and \hat{C}_{l+s} are nonlinear functions of \mathbf{v} . A gradient type numerical optimization algorithm is used to search for the optimum inverse filter estimate $\hat{\mathbf{v}}$. At the i th iteration $\hat{\mathbf{v}}_i$ is updated with

$$\hat{\mathbf{v}}_i = \hat{\mathbf{v}}_{i-1} + \rho \mathbf{g}_{i-1} \quad (9)$$

where ρ is a positive constant and \mathbf{g}_{i-1} is the gradient of $J_{2s,l+s}$ with respect to \mathbf{v} for $\mathbf{v} = \hat{\mathbf{v}}_{i-1}$. As other numerical optimization algorithms, an initial condition for $\hat{\mathbf{v}}_0$ is needed to initialize the above numerical optimization algorithm. For instance, a minimum-phase LPE filter can be used as the initial condition for $\hat{\mathbf{v}}_0$. Next, let us present some simulation results to justify that the inverse filter criteria $J_{2s,l+s}$ where $l > s \geq 1$ can be used for the estimation of the inverse filter of the unknown LTI system $h(n)$ and for deconvolution.

3. SIMULATION RESULTS

Two simulation examples are to be presented to support the proposed unified class of inverse filter criteria $J_{2s,l+s}$ where $l > s \geq 1$ presented in Theorem 1. The first example is a performance test to the inverse filter criterion $J_{2,3}$ proposed by Tugnait and the new $J_{2,5}$ (i.e., $s = 1$ and $l = 4$ in $J_{2s,l+s}$). The second example is seismic deconvolution using the new criterion $J_{2,6}$ (i.e., $s = 1$ and $l = 5$ in $J_{2s,l+s}$).

Example 1: (Performance test)

The driving input $u(n)$ used was a zero-mean, i.i.d. Exponential random sequence with variance $\sigma_u^2 = 1$, skewness $\gamma_3 = 2$, kurtosis $\gamma_4 = 6$ and fifth-order cumulant $\gamma_5 = 24$. The unknown LTI system $h(n)$ used was a nonminimum-phase second-order autoregressive moving average (ARMA) system with the transfer function (taken from [7]) given by

$$H(z) = \frac{1 - 2.7z^{-1} + 0.5z^{-2}}{1 + 0.1z^{-1} - 0.12z^{-2}} \quad (10)$$

Synthetic noisy data $x(n)$ of length $N = 4096$ were generated for $SNR = 10$ dB and noise $w(n)$ being white Gaussian. The inverse filter $v(n)$ was assumed to be a causal FIR filter of order $L = 16$. An initial condition $\mathbf{v} = [0, \dots, 0, 1, v_b(1), \dots, v_b(8)]^T$ was used to initialize the associated gradient type optimization algorithm for finding the optimum $\hat{\mathbf{v}}$ where $\{1, v_b(1), \dots, v_b(8)\}$ were the coefficients of an eighth-order LPE filter obtained by the well-known Burg's algorithm [8].

The simulation results over 30 independent runs associated with $J_{2,3}$ and $J_{2,5}$ (new criterion) are shown

in Figures 1(a) and 1(b), respectively. Note that each inverse filter estimate \hat{v} obtained from each independent run was normalized by $\|\hat{v}\| = 1$ and the associated unknown time delay was artificially removed. From Figures 1(a) and 1(b), one can see that the estimated inverse filter is basically unbiased for both $J_{2,3}$ and $J_{2,5}$, but it has a smaller variance for $J_{2,3}$ than for $J_{2,5}$. The reason for this is simply that sample cumulant \hat{C}_5 has larger variance than \hat{C}_3 for finite data [2]. These simulation results justify that $J_{2s,l+s}$, where $l > s \geq 1$ can be used to estimate the inverse filter of an unknown nonminimum-phase LTI system.

With no doubt, when $J_{2,3}$ is sufficient in certain practical applications, $J_{r,m}$ for higher admissible r and m such as $J_{2,5}$ are redundant. Nevertheless, we would like to emphasize that when cumulants of measurements are small or close to zero for lower r and m such as $J_{2,3}$ and $J_{2,4}$, the inverse filter criteria $J_{r,m}$ only for higher admissible r and m such as $J_{2,5}$ can be considered for the estimation of inverse filter.

Example 2. (Seismic deconvolution)

Assume that $u(n)$ is a Bernoulli-Gaussian sequence (solid lines in Figures 2(b) and 2(c)) which is input to a third-order nonminimum-phase system (taken from [6]) with the following transfer function

$$H(z) = \frac{1 + 0.1z^{-1} - 3.2725z^{-2} + 1.41125z^{-3}}{1 - 1.9z^{-1} + 1.1525z^{-2} - 0.1625z^{-3}} \quad (11)$$

The synthetic noisy data $x(n)$ were generated for $N = 2048$, $SNR = 27dB$ and noise $w(n)$ being white Gaussian. Because $\gamma_3 = 0$ (skewness) and $\gamma_5 = 0$ but $\gamma_2 = \sigma_u^2 = 0.1$, $\gamma_4 = 0.27$ (kurtosis) and $\gamma_6 = 1.08$ for this case, the new inverse filter criterion $J_{2,6}$ was used to estimate the causal inverse filter $v(n)$ of order $L = 24$. The initial condition $\mathbf{v} = [0, \dots, 0, 1, v_b(1), \dots, v_b(12)]^T$, where $\{1, v_b(1), \dots, v_b(12)\}$ were the coefficients of a twelfth-order LPE filter obtained by Burg's algorithm, was used to initialize the associated gradient type optimization algorithm for finding the optimum \hat{v} .

The obtained inverse filter estimate (dash-dotted line) normalized by $\|\hat{v}\| = 1$ is shown in Figure 2(a) together with the true noncausal stable inverse filter $h_I(n)$ (solid line) also normalized by $\sum_{n=-\infty}^{\infty} |h_I(n)|^2 = 1$ where the unknown time delay between $\hat{v}(n)$ and $h_I(n)$ was artificially removed. For comparison, a conventional minimum-phase LPE filter $v_b(n)$ of order equal to 24 was also obtained by Burg's algorithm, which is depicted by a dashed line in Figure 2(a). Note, from Figure 2(a), that $\hat{v}(n)$ is quite close to the true inverse filter $h_I(n)$ but very different from the LPE filter in waveshape. The data $x(n)$ were then processed by the LPE filter to obtain the predictive deconvolved signal $e_b(n)$ which is depicted by a dotted line in Figure 2(b) for $N = 0 \sim 511$ together with the true input sequence $u(n)$ depicted by a solid line. One can observe, from Figure 2(b), that in addition to a scale factor, each spike in $u(n)$ is associated with a residual wavelet which begins with two opposite peaks and gradually decays. The reason for this is simply

that an allpass distortion remains in $e_b(n)$ because only the amplitude response of nonminimum-phase source wavelet can be equalized by $v_b(n)$. The deconvolved signal $e(n)$ (dotted line) obtained by the optimum inverse filter depicted by a dash-dotted line in Figure 2(a) is shown in Figure 2(c) for $N = 0 \sim 511$ together with the true input sequence $u(n)$ (solid line). One can see, from Figure 2(c), that $e(n)$ approximates $u(n)$ well except for a scale factor. Comparing the deconvolved signal shown in Figure 2(b) with the one shown in Figure 2(c), one can easily see that $e(n)$ is indeed a much better estimate of $u(n)$ than $e_b(n)$ because the phase distortion (allpass distortion) in $e_b(n)$ (dotted line in Figure 2(b)) was almost inexistent in $e(n)$ (dotted line in Figure 2(c)). These simulation results support that the proposed inverse filter criterion $J_{2,6}$ can be used for deconvolution.

As discussed in Example 1, one surely can use Tugnait's criterion $J_{2,4}$ rather than the new criterion $J_{2,6}$ for this example because $\gamma_4 = 0.27 \neq 0$. This example only emphasizes the application of the new $J_{2,6}$ to deconvolution although both of $J_{2,4}$ and $J_{2,6}$ are members of the unified class of $J_{2s,l+s}$, where $l > s \geq 1$ presented in Theorem 1.

4. CONCLUSIONS

We have shown that the cumulant based inverse filter criteria $J_{r,m}$ given by (3) which use an m th-order cumulant and an r th-order cumulant for blind deconvolution and equalization require r to be even and $m > r$ (see Theorem 1). Therefore, these criteria form a family of criteria $J_{2s,l+s}$, where $l > s \geq 1$ and they include not only the existing inverse filter criteria as special cases of (l, s) but also new inverse filter criteria (see (R1)). The optimum inverse filter associated with $J_{2s,l+s}$, can only be obtained by iterative nonlinear optimization algorithms which can only guarantee a local optimum solution. Some simulation results were provided to support that $J_{2s,l+s}$, with $l > s \geq 1$ are effective.

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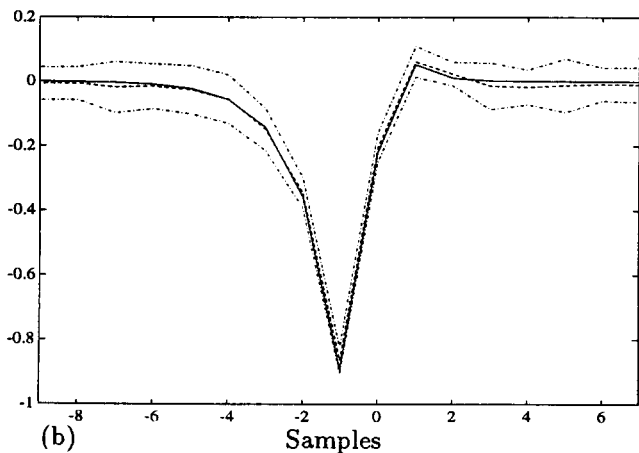
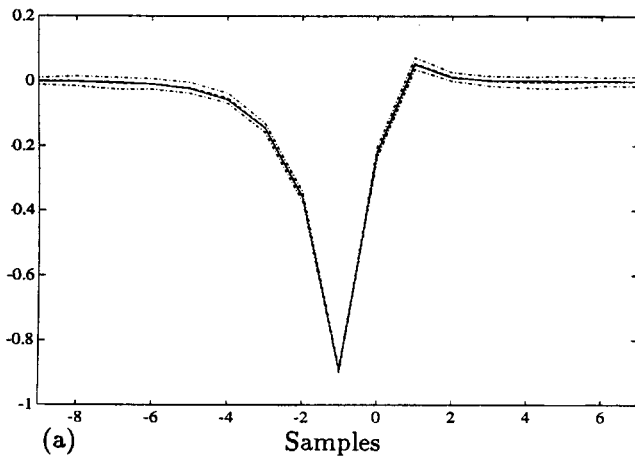


Figure 1. Simulation results ($N = 4096$, $SNR = 10dB$) for Example 1. The average (dashed line) as well as \pm one standard deviation (dash-dotted lines) of 30 independent inverse filter estimates together with the true inverse filter (solid line) associated with (a) $J_{2,3}$ and (b) $J_{2,5}$, respectively.

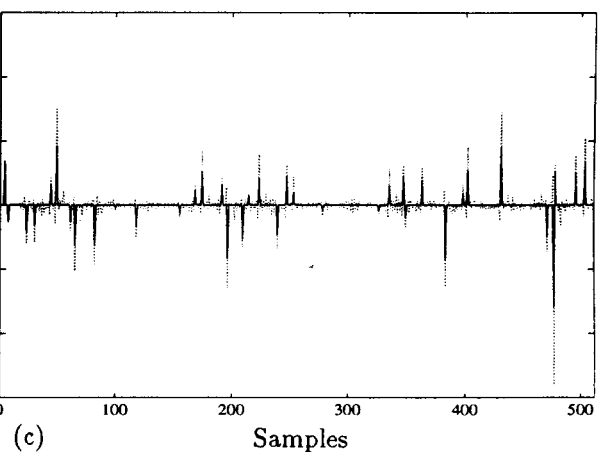
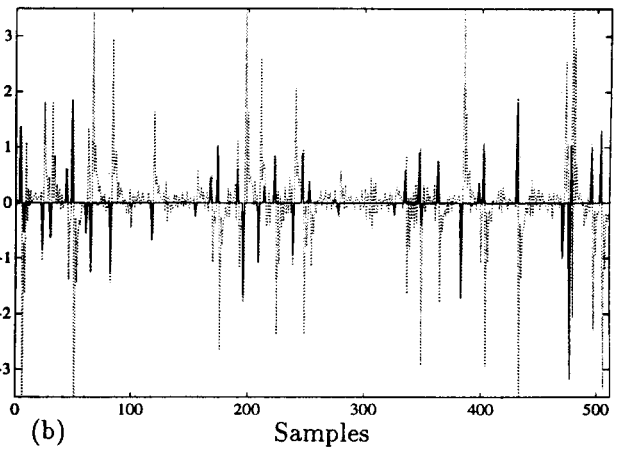
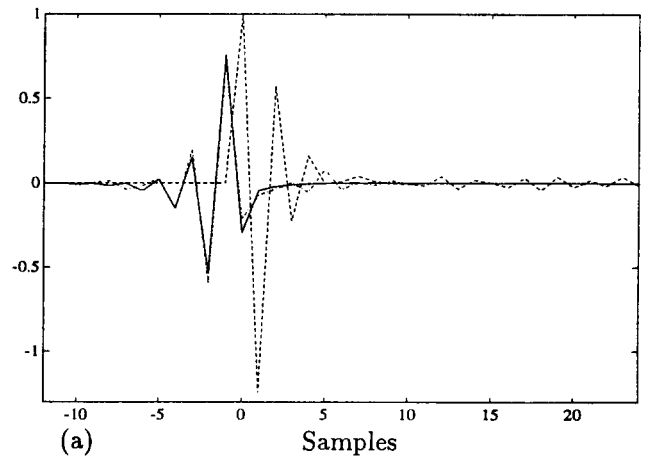


Figure 2. Simulation results ($SNR = 27dB$) associated with $J_{2,6}$ for Example 2. (a) The true noncausal stable inverse filter $h_I(n)$ (solid line), the inverse filter estimate (dash-dotted line) and the LPE filter (dashed line) of order equal to 24 obtained by Burg's algorithm; (b) the predictive deconvolved signal $e_b(n)$ (dotted line) together with the true input signal $u(n)$ (solid line); (c) the deconvolved signal $e(n)$ (dotted line) obtained by the optimum inverse filter together with the true input signal $u(n)$.