SIGNAL SUBSPACE TECHNIQUES FOR DOA ESTIMATION USING HIGHER ORDER STATISTICS

A.R.Leyman and T.S.Durrani

Signal Processing Division
Department of Electronic and Electrical Engineering
University of Strathclyde,Royal College Bldg,
Glasgow G1 1XW, United Kingdom

ABSTRACT

Eigendecomposition based techniques such as MUSIC and its variants constitute effective methods for determining the direction of arrival(DOA) estimates of narrowband sources. In this paper, a new strategy which extends the MUSIC algorithm to higher order statistics(HOS) is proposed for estimation of the DOA. Also, we present a new method for the estimation of the number of multiple narrowband incoherent and coherent non-Gaussian source signals arriving on the array which we consider as a significant contribution. The performance of the technique is compared with other recently suggested HOS-based methods.

1. INTRODUCTION

Many sensor array processing methods which utilize the eigendecomposition of the array spatial covariance matrix have been developed for locating narrowband farfield Gaussian sources embedded in Gaussian noise. Generally, they require a search procedure in the parameter space and have resolving capabilities which are more powerful than coventional beamforming techniques.However, when the arriving source signals are assumed to be non-Gaussian (as it is usually the case in a real-world enviroment), these second-order methods are suboptimal and hence it becomes essential to develop new techniques to cope with this problem. Hence, the application of higher order statistics (HOS) for direction of arrival (DOA) estimation has many advantages over the conventional techniques. A variety of HOS-based techniques have appeared in the open literature which employ higher order spectra in deriving new high resolution direction finding algorithms. The bearing estimation method based on the asymptotic distribution of the cross-bispectrum estimate of the array data has been addressed by Forster and Nikias[1]. They have used the concept of maximum likelihood of the asymptotic distribution in formulating their algorithm, called, cross-bispectrum beamformer. Porat and Friedlander[2] derived two sets of algorithms that employ the fourth order statistics of the array data. One is a MUSIC-like algorithm which is based on the eigendecomposition of the fourth order cumulant matrix of the data. The other is an optimal estimator based on the minimisation of a certain cost function. This cost function is related to a fixed set of cumulants of the array data. The results in [3] are based upon information-theoretic measures in the HOS-domain of the array geometries. In [4], we have presented a set of three new HOS-based algorithms for the localization of non-Gaussian farfield sources. There the exploitation of HOS provides the ability to effectively remove the spatial influences of Gaussian noise, hence yielding improved estimates. These algorithms are based on a new class of performance measures which are related to the bispectral power of the snapshot data. This paper extends the results of [4] by deriving a new cumulant-based MUSIC-like high resolution techniques for the localization of non-Gaussian sources. We show that performing the eigendecomposition of the bispectral power matrix enables us to construct the orthogonal signal and source subspaces in the non-Gaussian domain, and to form the spatial spectrum by employing a search procedure. We also propose a new method for determining the number of source signals arriving on the array. The techniques also shed light on the problem of combating the effects of source coherency. Experimental results are given which demonstrate the improved DOA estimation resolution of the proposed method as compared to those in [4].

2. BACKGROUND PRELIMINARIES

Consider an equally spaced linear array of M identical sensors, on which $P(< M)$ farfield uncorrelated radiation sources are incident. The array sensor output can

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be expressed in terms of Generalised Functions[1][2] as

$$\mathbf{X}(t) = \sum_{p=1}^{P} \int dS_p(\omega) e^{j\omega t} \mathbf{e}(\omega;\theta_p) + \mathbf{V}(t) \quad (1)$$

Here P is the number of complex source signals incident on the array. These signals are assumed to be zero mean, spatially and temporally stationary non-Gaussian random process with $\{dS_p(\omega)\}$ denoting the corresponding innovation representation of the signal arising from the p^{th} source. The representation of Eq.(1) allows consideration of both narrow-band and broad-band signals by employing appropriate models for $\{dS_p(\omega)\}$, e.g. for narrow-band signals, $dS_p(\omega) = a_p\delta(\omega - \omega_0)d\omega$, where a_p represents the signal amplitude. Although in our formulation we have assumed the source signals to be independent, we have also investigated the case for coherent source signals and have shown that estimation inaccuracies can be overcome by employing spatial smoothing in the bispectral domain. $\mathbf{X}(t)$ is an $(M \times 1)$ noisy data vector of the sampled snapshots from the sensor system at time instant t. M denotes the number of sensors. $\mathbf{V}(t)$ is $(M \times 1)$ receiver noise vector that is assumed to be zero mean, spatially, temporally stationary, independent and Gaussian. The noise and source signals are assumed to be spatially and temporally independent from each other. $\mathbf{e}(\omega;\theta_p)$ is an $(M \times 1)$ directional vector for source at bearing θ_p, i.e. it represents the array response to the source signals. This vector is sometimes known as the *transfer vector* (between the signal and sensors) and is denoted as $\mathbf{e}(\omega;\theta_p) = \left[\ 1 \quad e^{\{-j\omega\tau_p\}} \quad e^{\{-j\omega\tau_{2p}\}} \quad \ldots \quad e^{\{-j\omega\tau_{(M-1)p}\}}\ \right]^T$, where $\tau_{pk} = \frac{kd\sin\theta_p}{c}$ is the inter-sensor delay of the wavefront at the k^{th} sensor. d is the sensor spacing and ω is the center frequency of the source signals. As shown in [4], the third order cumulants (and bispectra) of $\mathbf{X}(t)$ (for narrowband independent signals with ω_0 = midband frequency) maybe expressed as

$$\mathbf{C_X}(\tau_1,\tau_2) = e^{j\omega_0(\tau_1+\tau_2)} \sum_p C_p(\tau_1,\tau_2)\mathbf{E}_\otimes(\theta_p) \quad (2)$$

$$\mathbf{B_X}(\omega_1,\omega_2) = \sum_p B_p(\omega_1,\omega_2)\mathbf{E}_\otimes(\theta_p) \quad (3)$$

where $\mathbf{E}_\otimes(\theta_p)$ is the directional vector in the *bispectral* domain, given as the Kronecker product, $\mathbf{E}_\otimes(\theta_p) \triangleq [\mathbf{e}^*(\theta_p) \otimes \mathbf{e}(\theta_p) \otimes \mathbf{e}(\theta_p)]$, $\{C_p(\tau_1,\tau_2)\}$ and $\{B_p(\omega_1,\omega_2)\}$ is the third order cumulant and bispectrum of the p^{th} source respectively.

3. HOS-BASED MUSIC ALGORITHM

Considering the new performance criteria[4], which relates to the *cumulant power* of the array output, via

$$\mathbf{\Gamma_X} \triangleq \int\int \mathbf{C_X}(\tau_1,\tau_2)\mathbf{C_X}^H(\tau_1,\tau_2)d\tau_1 d\tau_2 \quad (4)$$

by substituting Eq.(2) into Eq.(4) we obtain

$$\mathbf{\Gamma_X} = \mathbf{ECE}^H \quad (5)$$

where $\mathbf{E} = [\ \mathbf{E}_\otimes(\theta_1) \quad \mathbf{E}_\otimes(\theta_2) \quad \cdots \quad \mathbf{E}_\otimes(\theta_P)\ ]$ is the $M^3 \times P$ steering matrix in the bispectral domain with its column P steering vectors corresponding to each of the incident wave. It is clear that $\mathbf{E}$ is full rank. $\mathbf{C}$ is a $P \times P$ source cumulant power matrix where the $(p,q)^{th}$ element is given by $\int\int C_p(\tau_1,\tau_2)C_q{}^H(\tau_1,\tau_2)d\tau_1 d\tau_2$. $\mathbf{C}$ is Hermitian, non-diagonal and is full rank if only *uncorrelated* or *partially* correlated source signals are present. Thus from Eq.(5), $\mathbf{\Gamma_X}$ is also Hermitian and it is likely that it may be singular. To acheive inversion for computing purposes we employ diagonal loading which is expressed as

$$\hat{\mathbf{\Gamma}}_\mathbf{X} = \mathbf{ECE}^H + \mathbf{\Gamma_V} \quad (6)$$

where $\mathbf{\Gamma_V}$ is a loading factor, $\mathbf{\Gamma_V} = \epsilon\mathbf{I}$ with ϵ is the Tikhonov's regularisation parameter. This motivates us to view the loading factor, $\epsilon\mathbf{I}$, as a synthetic contribution of *uncorrelated non-Gaussian noise* cumulant power matrix. Consider λ_n and $\mathbf{u}_n (n = 1,2,\ldots,M^3)$ to be the eigenvalues and the corresponding eigenvector of the matrix pencil $(\hat{\mathbf{\Gamma}}_\mathbf{X}, \mathbf{\Gamma_V})$ where the former are ordered in a monotonically non-decreasing[5] fashion i.e. $\lambda_n > \lambda_{n+1}$ then the following observation holds[5]: (1)$\lambda_{P+1} = \lambda_{P+2} = \cdots = \lambda_{M^3} = \epsilon$; (2) If $\mathbf{C}$ is full rank, each of the columns of $\mathbf{E}$ are orthogonal to the matrix $\mathbf{U}_n \triangleq [\mathbf{u}_{P+1} \quad \mathbf{u}_{P+1} \quad \cdots \quad \mathbf{u}_{M^3}]$. Since ϵ is chosen *aprior*, observing the number of eigenvalues in Eq.(6) different from ϵ, yields the number of non-Gaussian sources impinging on the array. Observation (2) suggests that we may estimate the DOA of the source signals by performing the eigendecomposition of the cumulant power matrix

$$\mathbf{\Gamma_X} = \begin{pmatrix} \mathbf{U}_s & \mathbf{U}_v \end{pmatrix} \begin{pmatrix} \mathbf{\Lambda}_s & 0 \\ 0 & \mathbf{\Lambda}_v \end{pmatrix} \begin{pmatrix} \mathbf{U}_s^H \\ \mathbf{U}_v^H \end{pmatrix} \quad (7)$$

where $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_v$ are the $(P \times P)$ and $(M^3 - P)$ diagonal matrices given by $\mathbf{\Lambda}_s = diag(\lambda_1, \lambda_2 \ldots, \lambda_p)$ and $\mathbf{\Lambda}_v = diag(\epsilon, \epsilon, \ldots, \epsilon)$ respectively. $\mathbf{U}_s$ and $\mathbf{U}_v$ are the $(M^3 \times P)$ and $M^3 \times (M^3 - P)$ dimensional matrices of the signal and noise subspaces respectively. The spatial spectrum is then established as :

$$P_4(\phi) = \frac{1}{\|\mathbf{U}_v^H \mathbf{E}_\otimes(\phi)\|^2} \quad (8)$$

where the largest peaks represent the source directions.

4. SOURCE COHERENCY

Source coherency leads to changes in the structure and rank properties of the source bispectral power matrix, $\mathbf{C}$, hence the noise eigenvectors, something, need not necessarily be orthogonal to the source steering vectors. Here we propose *spatial smoothing*[6] in the bispectral domain which gives a preprocessing method for decorrelating highly correlated narrow-band non-Gaussian source signals. Mathematically, this can be expressed as: $\hat{\Gamma}_{\mathbf{X}} = \frac{1}{Q}\sum_{k=1}^{Q}\left(\hat{\Gamma}_{\mathbf{X}}^{k}\right)$, where $\left(\hat{\Gamma}_{\mathbf{X}}^{k}\right)$ denotes the k^{th} subarray and Q is the number of overlapped/non-overlapped subarrays.However, to illustrate the treatment tractably, we will only deal with the case of three source signals, $s_1(t)$, $s_2(t)$ and $s_3(t)$, impinging on the array, where two of the source signals are correlated and the other is independent. Let us now consider that the uniform array with M sensors is divided into overlapping subarrays of size Q with sensors, $\{1,\ldots,Q\}$ forming the first subarray, sensors $\{2,\ldots,(Q+1)\}$ forming the second subarray and so on. The third order cumulant of the k^{th} subarray can easily be shown to be

$$\begin{aligned}\mathbf{C_X}^k(\tau_1,\tau_2) &= e^{j\omega_0(\tau_1+\tau_2)}\sum_p\sum_q\sum_r C_{pqr}(\tau_1,\tau_2) \\ &\quad \{e^{j\omega_0(\tau_p-\tau_q-\tau_r)}\}^{(k-1)}\mathbf{E}_{\otimes}(\theta_p,\theta_q,\theta_r)\end{aligned} \quad (9)$$

The cumulant power matrix of the k^{th} subarray can then be written as

$$\begin{aligned}\mathbf{\Gamma_X}^k &= \int\!\!\int\left[\mathbf{C_X}^k(\tau_1,\tau_2)\right]\left[\mathbf{C_X}^k(\tau_1,\tau_2)\right]^H \\ &= \mathbf{E}\mathbf{D}^{(k-1)}\int\!\!\int \mathbf{C}(\tau_1,\tau_2)\mathbf{C}^H(\tau_1,\tau_2)\bullet \\ &\quad [\mathbf{D}^{(k-1)}]^H\mathbf{E}^H\end{aligned} \quad (10)$$

where the identities are defined as follows for the 3 source signals case :

$$\begin{aligned}\mathbf{E} \triangleq &[\ \mathbf{E}_{\otimes}(\theta_1,\theta_1,\theta_1)\ \ \mathbf{E}_{\otimes}(\theta_1,\theta_1,\theta_2)\ \ \mathbf{E}_{\otimes}(\theta_1,\theta_2,\theta_1) \\ &\mathbf{E}_{\otimes}(\theta_1,\theta_2,\theta_2)\ \ \mathbf{E}_{\otimes}(\theta_2,\theta_1,\theta_1)\ \ \mathbf{E}_{\otimes}(\theta_2,\theta_1,\theta_2) \\ &\mathbf{E}_{\otimes}(\theta_2,\theta_2,\theta_1)\ \ \mathbf{E}_{\otimes}(\theta_2,\theta_2,\theta_2)\ \ \mathbf{E}_{\otimes}(\theta_3,\theta_3,\theta_3)\ ]\end{aligned} \quad (11)$$

$$\mathbf{D} = diag\{\ e^{j\omega_0(\theta_1-\theta_1-\theta_1)}\ \ e^{j\omega_0(\theta_1-\theta_1-\theta_2)}\ \cdots\ e^{j\omega_0(\theta_3-\theta_3-\theta_3)}\} \quad (12)$$

$$\begin{aligned}\mathbf{C}(\tau_1,\tau_2) \triangleq &[\ C_{111}(\tau_1,\tau_2)\ \ C_{112}(\tau_1,\tau_2)\ \ C_{121}(\tau_1,\tau_2) \\ &C_{122}(\tau_1,\tau_2)\ \ C_{211}(\tau_1,\tau_2)\ \ C_{212}(\tau_1,\tau_2) \\ &C_{221}(\tau_1,\tau_2)\ \ C_{222}(\tau_1,\tau_2)\ \ C_{333}(\tau_1,\tau_2)\ ]\end{aligned} \quad (13)$$

Equation (10) can be written as

$$\mathbf{\Gamma_X}^k = \mathbf{E}\mathbf{D}^{(k-1)}\mathbf{C_S}[\mathbf{D}^{(k-1)}]^H\mathbf{E}^H \quad (14)$$

with

$$\mathbf{C_S} \triangleq \int\!\!\int \mathbf{C}(\tau_1,\tau_2)\mathbf{C}^H(\tau_1,\tau_2)d\tau_1 d\tau_2 \quad (15)$$

We now define the spatial smoothing in the cumulant domain as

$$\begin{aligned}\hat{\mathbf{\Gamma}}_{\mathbf{X}} &\triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{\Gamma_X}^k \\ &= \mathbf{E}\left[\frac{1}{K}\sum_{k=1}^{K}\mathbf{D}^{(k-1)}\mathbf{C_S}[\mathbf{D}^{(k-1)}]^H\right]\mathbf{E}^H\end{aligned} \quad (16)$$

where $K = M - Q + 1$ is the total number of subarrays. Equation (16) can be compactly rewritten as

$$\hat{\mathbf{\Gamma}}_{\mathbf{X}} = \mathbf{E}\hat{\mathbf{C}}_{\mathbf{S}}\mathbf{E}^H \quad (17)$$

where the modified source cumulant power matrix is given by

$$\hat{\mathbf{C}}_{\mathbf{S}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{D}^{(k-1)}\mathbf{C_S}[\mathbf{D}^{(k-1)}]^H \quad (18)$$

We shall now prove that $\hat{\mathbf{C}}_{\mathbf{S}}$ must be of rank 9 for the three impinging sources to be localized. Equation (18) can be conveniently written as

$$\hat{\mathbf{C}}_{\mathbf{S}} = \mathbf{G}\mathbf{G}^H \quad (19)$$

where

$$\mathbf{G} \triangleq [\ \mathbf{C}_1\ \ \mathbf{D}\mathbf{C}_1\ \ \mathbf{D}^2\mathbf{C}_1\cdots\ \mathbf{D}^{(K-1)}\mathbf{C}_1\ ] \quad (20)$$

with $\mathbf{C}_1 \triangleq [\ C_{111}\ \ C_{112}\ \cdots C_{333}\ ]$ denoting the Hermitian square root of $\frac{1}{M}\mathbf{C_S}$, i.e.

$$\mathbf{C}_1\mathbf{C}_1^H = \frac{1}{K}\mathbf{C_S} \quad (21)$$

We observe that the rank of $\bar{\mathbf{C}}_{\mathbf{S}}$ is the same as the rank of $\mathbf{G}$, hence we have to prove that $\rho\{\mathbf{G}\} = 9$. But $\mathbf{G}$ can be decomposed as

$$\mathbf{G} = \mathbf{B}\mathbf{V} \quad (22)$$

where $\mathbf{D}$ is the (9×9) diagonal matrix defined as

$$\mathbf{B} = diag\{\ C_{111}\ \ C_{112}\ \cdots C_{333}\ \} \quad (23)$$

and $\mathbf{V}$ is the $(9\times L)$ Vandermonde matrix

$$\mathbf{V} = \begin{bmatrix} 1 & v_1 & v_1^2 & \cdots & v_1^{K-1} \\ 1 & v_2 & v_2^2 & \cdots & v_2^{K-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & v_9 & v_9^2 & \cdots & v_9^{K-1} \end{bmatrix} \quad (24)$$

where the elements if $\mathbf{V}$ are defined as follows: $v_1 = e^{j\omega_0(\theta_1-\theta_1-\theta_1)}$, $v_1 = e^{j\omega_0(\theta_1-\theta_1-\theta_1)}$, $v_2 = e^{j\omega_0(\theta_1-\theta_1-\theta_2)}$, $v_3 = e^{j\omega_0(\theta_1-\theta_2-\theta_1)}$, $v_4 = e^{j\omega_0(\theta_1-\theta_2-\theta_2)}$, $v_5 = e^{j\omega_0(\theta_2-\theta_1-\theta_1)}$, $v_6 = e^{j\omega_0(\theta_2-\theta_1-\theta_2)}$, $v_7 = e^{j\omega_0(\theta_2-\theta_2-\theta_1)}$ $v_8 = e^{j\omega_0(\theta_2-\theta_2-\theta_2)}$ and $v_9 = e^{j\omega_0(\theta_3-\theta_3-\theta_3)}$. From Eq.(24), the rank of $\mathbf{V}$ depends on K, hence if $K \geq 9$, $\rho\{\mathbf{V}\} = 9$ which implies that $\mathbf{G}$ is also of rank 9. Therefore, the rank of $\bar{\mathbf{C}}_S$ is restore and hence can be used to localize the three source signals irrespective of their of coherence.

5. COMPUTER EXPERIMENTS

To demonstrate the performance of the MUSIC-like method and the detection of coherent sources, a linear array of $M = 6$ identical, equispaced sensors is used. The non-Gaussian source signals are generated using the same centre frequency and in all cases we have used 8192 snapshots to estimated the bispectral power matrix. Three equipower sources are incident at $-65°$, $10°$ and $50°$ bearings and the SNR is maintained at -7dB. For comparison purposes, the resolving performance of the algorithms proposed in[4] are also included. Fig.1 shows the results of the proposed HOS-based MUSIC-like method against the methods of [1]. The decorrelating effect of spatial smoothing in the bispectral domain with linear array segmented into 10 overlapped subarrays is shown in Fig.2, where the peaks denote the DOA of 2 fully coherent and 1 uncorrelated sources at $50°$ and $70°$ and $35°$. Note the dynamic range of all the algorithms. Note the dynamic range of all the algorithms.

6. CONCLUSION

In this paper, a HOS-based eigendecomposition method is proposed for the estimation of DOA's of non-Gaussian source signals. A method of determining the number of sources impinging on a linear array is also presented. It is also shown that spatial smoothing in the bispectral domain can be used to combat the effects of source coherency. Simulation results show the effectiveness of the proposed method.

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Key:

CBF : Cumulant Beamformer

CIC : Cumulant Interference Canceller

MECT : Maximum entropy cumulant technique.

MBD : MUSIC-like Algorithm in Bispectral Domain

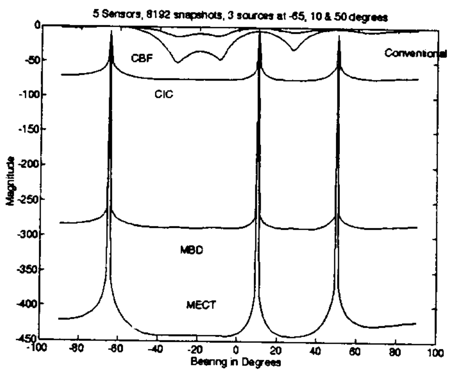

Figure 1: HOS-based algorithms for 3 uncorrelated sources.

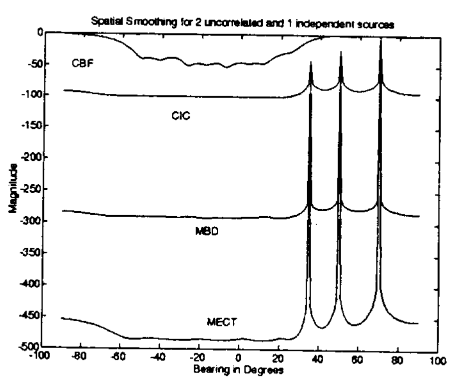

Figure 2: Smoothed HOS-based algorithms for 2 uncorrelated and 1 uncorrelated sources