USING RECURSIVE PARAMETER ESTIMATION FOR SLEEP DISORDER DISCRIMINATION

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ABSTRACT

The human pupillary light reflex has long been studied as a typical biological nonlinear system. We have used a sinusoidal non-harmonic signal as the input light stimulus and pupil diameter as the output of the system. A recursive least square method is then used to estimate the measured pupil diameter in terms of the input light. With a good estimate, the underlying dynamical behavior of the system would be captured by the estimated parameters. Thus we modeled the estimated parameters as ARIMA processes. Then the residual noise associated with the ARIMA models was examined and revealed that people with narcolepsy had considerably lower sum-square-error than people without this sleep disorder (controls). This method turns out to be a relatively simple and fast test procedure for narcolepsy discrimination.

1. Introduction

Narcolepsy is a grave sleeping disorder which includes sleep attacks, hypnagogic hallucinations, sleep paralysis, and cataplexy [1]. At onset it is extremely difficult to diagnose [2] even though for years it has been suspected that pupil activity is sensitive to the presence of narcolepsy [3]. The accepted diagnostic test for narcolepsy requires at least two weeks of patient vigilance and an overnight sleep procedure. For this reason and the purpose of early detection, a more definitive and simpler test would be a significant healthcare screen.

Pupillometry has been demonstrated to have excellent potential for the study of alertness and potential as a screening technique [4]. Pupil light reflex (PLR) has been used for Input-Output modeling of the system. For example Semmlow[5] suggested a nonlinear servo mechanical control system, Usui et al. [6], a homeomorphic model based on iris muscle dynamics and Stanten et al. [7], a stochastic process in a biological system. Apparently there are a few nonlinearities [5] in the system which make the models complicated and so would be the detection of abnormalities from these models. The purpose of our model is to get significant measures from the simplest PLR dynamics capable of discriminating sleep disorders.

2. Instrumentation

The pupillometer records pupil diameter responses to various stimuli [8]. It consists of two infrared-sensitive video cameras that record data from both eyes simultaneously. Both cameras are mounted on a horizontal bar and can be adjusted to match the interpupillary distance of the subject. During data collection in a quiet, dark room, the subject is instructed to sit quietly in a comfortable chair, try to stay awake without moving, and fixate straight ahead on the image of a red dot. The PLR data is collected for 2 minutes after 5 minutes of dark adaptation.

3. Pupil Light Reflex (PLR) Models

We want from our models a significant measure of PLR noise cleansed of any PLR dynamics. To elicit the noise measure we use linear, time-varying parameter models since, theoretically, any neurophysiological difference in noise between controls and narcoleptic should originate in the model parameters, the PLR stimulus being deterministic. It is possible that PLR noise also could be explained in terms of the model residuals. However there is no neurological basis for residual noise and empirically the magnitude and autocorrelation of residual noise depends very strongly on the number of parameters estimated and consequently can be an unreliable measure of noise.

Our generic PLR model has the form

$$p(t)=a_0(t)+\sum_{j=1}^{m} a_j(t)s(t-j)+n(t) \qquad (1)$$

in which
t = time measured in 62.5 ms increments
p(t) = pupil diameter in mm.
s(t) = pupil light stimulus in arbitrary intensity units.

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a(t) = time varying PLR parameters
n(t) = residual disturbance term.

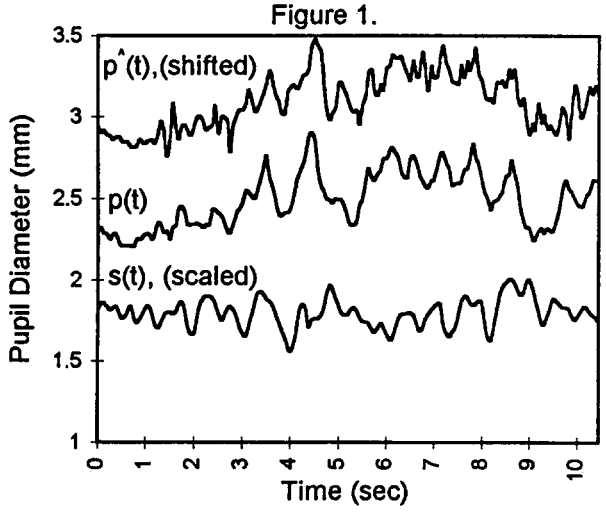
Model parameters are estimated using recursive least squares (RLS) estimation [9] which minimizes the criterion

$$J(\lambda)=\sum_{t=1}^{T}\lambda^{T-t}n^{2}(t) \qquad (2)$$

where λ is an exponential weighing factor and n(t) is from (1). The purpose of λ is to allow the a(t) parameters to follow any noise patterns these parameters may have by weighing early, in-sample, noise by an ever decreasing factor (typically $0 < \lambda < 1$). The exact value of λ used is determined by minimizing J over λ for the entire sample period, T =300. Further, the order of the PLR models, the integer m in (1), is determined by increasing m until there are no decrements in J; as a ratio in m. J is distributed as Snedecor's F statistic when n(t) is normal.

Using increments of $\Delta\lambda = 0.05$ it was found that $\lambda = 0.6$ was optimal and that the optimal m was 4 for all data. A typical stimulus, s(t), pupil response, p(t), and model estimate of p(t), $\hat{p}(t)$, are shown in Figure 1.

Figure 1.



We turn to analysis and modeling of the estimated $a_j(t)$ parameters in order to extract a discrimination measure of pupil noise. The autocovariance functions of the estimated parameters $a_j(t)$, j = 0,1,...,4 have functional forms which are typical of stationary autoregressive (AR) processes [10]. We use a uniform set of criteria specifying the form of the AR models so as to give all models an unbiased structure [11]. This means that every AR model noise estimate results from the same criterion applied to every set of a(t) estimated data. Our interest, of course, is in the estimated variance of the parametric model noise and its distribution. We summarize our noise findings in the following section.

3.1. Maximum Entropy Tests

It is well known [12] that to be maximally noncommittal regarding unobserved data the assumption to make is that the unobserved data is from a uniform distribution. Define e_i ,i = 1, 2,..., 5 as the residual sum of squares observations from a single subject and assume these observations are from a uniform distribution. Then the empirical sample distribution is

$$p_i=0.2\sum_{j=1}^{5}\delta[\hat{e}_i(j)-e_i(j)] \qquad (3)$$

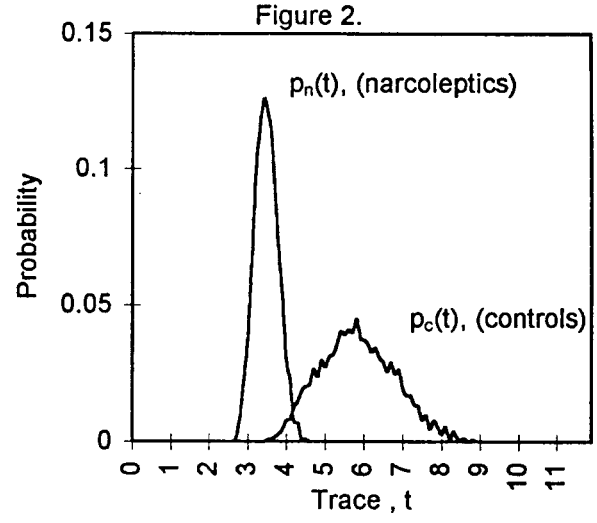
Where $\delta(.)$ is the impulse function. Note that the trace of the residual, SSE covariance matrix , is just

$$t_i=\sum_{j=1}^{5}e_i(j) \qquad (4)$$

Consequently, the sample estimate of the trace distribution is the 5-fold convolution of (3), that is,

$$p(t) = p_1 * p_2 * \ldots * p_5 \qquad (5)$$

Figure 2.



In Figure 2, $p_c(t)$ and $p_n(t)$ denote the estimate in

(5) for the control and narcoleptic subjects respectively. The estimated distributions in Figure 2 make it apparent that the scatter as well as the mean for the controls exceed those for the narcoleptic.

Let H_n be the hypothesis that the test subject is a narcoleptic against the alternative H_c that the subject is a control. We estimate from Figure 2 that

$$\alpha = pr(\text{type I error}) = pr(4.0 \le t \le 4.5 / H_n) = 0.024$$
$$\beta = pr(\text{type II error}) = pr(3.5 \le t \le 4.0 / H_c) = 0.022$$

Based on 10 test subjects the trace has a very small critical region and exhibits excellent power. The detailed analysis of the power of our proposed tests and its dependence on sample size is in the next section

3.2. Test of the Narcoleptic Hypothesis and Sample Size Estimates

The modeling of the PLR data from a given subject results in an estimate of the subject's ARIMA model noise covariance matrix and the sum of the diagonal elements of this matrix is the subject's sample value trace, t_s. We illustrate here how this value is used to test the hypotheses that the subject is in a certain class. We use the maximum entropy discriminator and assume that the empirical distributions $p_n(t)$ and $p_c(t)$ shown in Figure 2, govern the populations from which t_s is drawn.

If H_n is the hypothesis the subject is narcoleptic then t_s should have population distribution $p_n(t)$ while if the alternate hypothesis H_c is true, then t_s should be from the control distribution $p_c(t)$. To test H_n against H_c, the minimum discrimination statistic determines the critical region from the probability statement

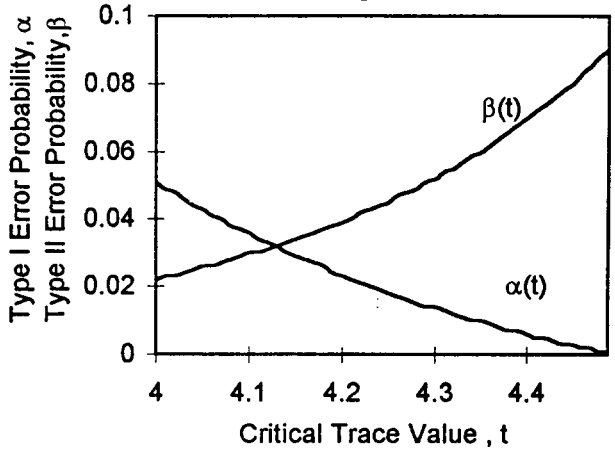
$$pr\,[\, \log(p_c(t)/p_n(t)) \ge c / H_n \,] \le \alpha$$

where α is the probability of a type I error, rejecting H_n when it is true, and c is a threshold for adjusting the size of α. As c varies from 1.4 to infinity, we estimate from $p_n(t)$ and $p_c(t)$ of Figure 2 that t_α, the critical value of the trace t such that if $t_s \ge t_\alpha$ then we reject H_n, is given by the graph in Figure 3. Type II errors, accepting H_n when t_s is actually from the control population, satisfy

$$pr[\, \log(p_n(t)/p_c(t)) \ge c / H_c \,] \le \beta$$

The corresponding estimate of β versus t_β is shown in Figure 3. The narcoleptic hypothesis is accepted for $t_s < t_\beta$ and the exceptionally small β values, based on a sample of only 10 subjects, confirm the most powerful critical region property of this test [13]. We next estimate the subject sample size n required to lower bound the power of our discrimination test.

Figure 3.



For any threshold c we shall require β to be less than 0.05. To estimate n we assume the trace population control distribution is approximated by $p_c(t)$ in Figure 2. From Figure 2, if $\beta<0.05$ for all thresholds then $F_c(4.5)=0.05$ where F_c is the trace cumulative and $t_p=4.5$ is the p=0.05 quantile of F_c. It is well known [14] that the P^{th} quantile has, under the assumption we have made, an asymptotically normal distribution with mean t_p and variance $P(1-P)/P_c^2(t_p)$. We take $p_c(4.5)$ from Figure 2. P=0.05 and require any trace observation to be within 10% of t_p with a probability of 0.95. The 2 standard deviation point on the normal distribution implies

$$2[p(1-p)/p_c^2(t_p)]^{0.5} = 2.44/\sqrt{n} = 0.45 \quad , \quad n = 29.4$$

This suggests we take 30 subjects from each category tested.

4. Summary

We have found that pupillary noise is a robust discriminator between controls and narcoleptics. The pupil light reflex is stimulated for 30 seconds and noise estimates from subsequent models provide the test statistics. Estimates predict that distributions from a sample size of 30 subjects should provide a useful narcoleptic screening test.

ACKNOWLEDGMENT

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REFERENCES

[1] Mitler, M.M., Hajdukovic, R., Erman, M. and Koziol, J.A., "Narcolepsy", J. of Clinical Neurophysiology, 17, No 1, pp. 93-118,1990.

[2] Carskadon, M.A., "Patterns of Sleep and Sleepiness in Adolescents" , Pediatrician, 17, pp. 5-12, 1990.
[3] Yoss,R.E., Moyer, N.J. and Ogle, K.N., "The Pupillogram and narcolepsy: A Method to measure decreased levels of Wakefulness". Neurology, 19, pp. 921-928, 1969.
[4] Newman, J, Broughton, R. " Pupillometric assessment of excessive daytime sleepiness in narcolepsy-cataplexy. Sleep, 14(2), 121-129, 1991.
[5] Semmlow, J., Chen, D. " A Simulation Model of the Human Pupil Light Reflex", Math. Biosci. 33:5-24, 1977.
[6] Usui, S. and Stark, L. " A Model for Nonlinear Stochastic Behavior of the Pupil", Biol. Cybern. 45:13-21, 1982.
[7] Stanten, S.F., and Stark, L. ," A Statistical Analysis of Pupil Noise", IEEE Trans. Biomed. Eng. BME-13:140-52, 1966.
[8] " A New Video Pupillometer", to appear in Ophthalmology, March 1995.
[9] Ljung, L. "System Identification: Theory for the User", Englewood Cliffs, New Jersey: Prentice Hall, 1987.
[10] Box, G.E. P. and Jenkins, G.M. "Time Series Analysis", San Francisco: Holden Day, 1976.
[11] Liu L.M. "Forecasting and Time Series Analysis Using the SCA Statistical System", SCA Corp, 1992.
[12] Jaynes, E.T. "Information Theory and Statistical Mechanics", Physics Reviews, 156:620-630, 1957.
[13] Kullback, S. "Information Theory and Statistics" ,New York: Dover, 1968.
[14] Wilks, S.S. "Mathematical Statistics", New York:Wiley and Sons, 1963.