DESIGN OF OPTIMUM PERIODIC TIME VARYING FILTERS FOR APPLICATIONS IN COMBUSTION DIAGNOSIS OF CAR ENGINES

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ABSTRACT

For adaptive control of modern car engines the combustion process has to be observed. Direct measurement of cylinder pressure is costly and not suitable for implementation. Therefore, we approximate the pressure by appropriate filtering of one or more vibration signals that can be measured easily. It has been shown that the pressure signal can be modelled as second order cyclostationary (SOSC)[1]. The transfer characteristic between pressure and vibration is time–varying due to the motion of the piston during observation. Therefore, for constant rotation speed we assume a linear periodic time–varying model. In this case, pressure and vibration signals are jointly SOCS. Based on this we formulate the optimum filter problem for our application. Solutions for this problem are known from the literature. We choose an appropriate one, adapt it to our problem and estimate the filter parameters. A real data experiment demonstrates the quality of this estimation.

1. INTRODUCTION

Modern car engines have to be optimized to achieve high efficiency, low fuel consumption and a low exhaust level. This optimum depends on fuel quality, temperature, etc. Therefore, the optimization procedure has to be adaptive, and it becomes neccessary to *'look inside the engine'* by measuring relevant information about the actual state of the engine. The best method to observe the combustion process is measurement of pressure inside the combustion chamber. Knowledge of the cylinder pressure as a function of the crank angle allows conclusions about the actual power (low frequency part of the signal) or about knock (high frequency part) [2], [3]. Knock is an abnormal combustion that is characterized by oszillations of the burnt gas. Frequent knock can lead to engine damage and has to be avoided. Unfortunately, direct measurement of the cylinder pressure is costly and not suitable for practical implementation. Therefore, we try to approximate the pressure signal based on observations of structural vibration signals which are often very noisy. In this paper we concentrate on filtering out the high frequency part of the pressure signal that is caused by knock. The methods presented here can also be applied to the low frequency part of the signals, too.

2. PROBLEM

For system identification purposes we restrict our attention to an engine running on a test bed with constant rotation speed. One cylinder pressure sensor is placed in the combustion chamber, P vibration sensors are mounted on the surface of the engine. The sensor output signals are filtered appropriately before they are sampled. We use the same data as in [1], where we have demonstrated that the sampled cylinder pressure signal Y_n can be modeled as second order cyclostationary (SOCS). Due to the fact that for each cylinder of our engine a combustion takes place every 2 rotations, the cycle period is $2N_{\text{rot}}$, where N_{rot} is the number of sampling intervals per rotation of the crankshaft.

Fig. 1 visualizes the basic idea concerning the relationship between cylinder pressure and structural vibration signals. It gets obvious that the transfer characteristic depends on the position of the piston which moves up and down during observation. Therefore, the transfer characteristic depends on crank angle, and we have to assume a time–varying system. For constant rotation speed this system will be periodic time–varying (PTV). The acoustic wave propagation motivates a linear model. The vibration sensor outputs are disturbed by noise which may be stationary (gear, drive–belt, wheels, etc.) or SOCS with period N_{rot} or $2N_{\text{rot}}$ (valves, crankshaft, etc.). We assume the noise to be uncorrelated with Y_n and to be SOCS with pe-

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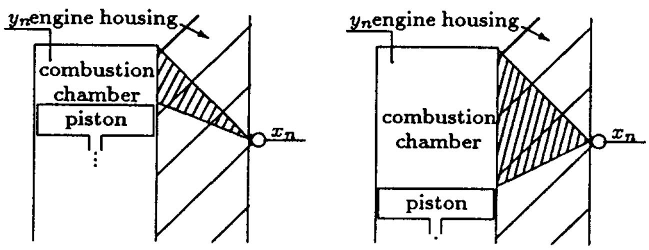

Figure 1: Sectional drawing of engine for different piston positions

riod $2N_{\text{rot}}$, which is the most general case and we use the following linear, PTV model for the output of the i'th vibration sensor:

$$X_n^{(i)} = \sum_{m=-\infty}^{\infty} g^{(i)}(n,m)Y_{n-m} + Z_n^{(i)} \qquad (1)$$

Based on the following periodicities for a SOCS process with period $2N_{\text{rot}}$ and a PTV system with period N_{rot}

$$\begin{aligned} r_{YY}(n,m) &= \text{E}Y_nY_m \\ &= r_{YY}(n+2N_{\text{rot}}, m+2N_{\text{rot}}) \ \forall\ n,m \quad (2) \\ g(n,m) &= g(n+N_{\text{rot}}, m) \quad \forall\ n,m \qquad (3) \end{aligned}$$

it is easy to show that $X_n^{(i)}, (i = 1,\ldots,P)$ are SOCS with period $2N_{\text{rot}}$, too:

$$r_{X^{(i)}X^{(i)}}(n,m) = r_{X^{(i)}X^{(i)}}(n+2N_{\text{rot}}, m+2N_{\text{rot}}) \ \forall\ n,m$$

Additionally, the cylinder pressure signal and the vibration signals are jointly SOCS with period $2N_{\text{rot}}$.

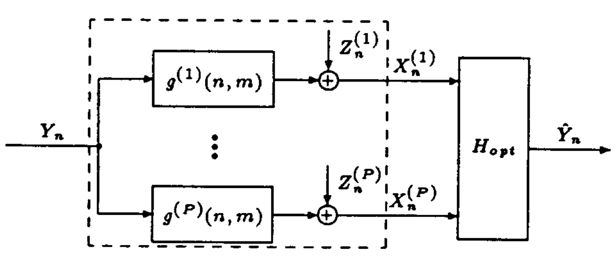

Figure 2: Wiener–Filter problem

Fig. 2 demonstrates the problem that has to be solved. We are looking for an optimum linear filter for Y_n based on $X_n^{(i)}, (i = 1,\ldots,P)$. All processes are SOCS with known period $2N_{\text{rot}}$, and the solution should be optimum in the least squares sense, i.e. $\text{E}(Y_n - \hat{Y}_n)^2$ has to be minimized.

3. SOLUTION

It is well known from the literature that a SOCS process with period N can be converted to a wide sense stationary (WSS) vector process with N components. It has to be distinguished between the harmonic series representation (HSR)[4], where the components of the vector process are the outputs of a filterbank driven with the SOCS process and the translation series representation (TSR)[5], where shifted versions of the SOSC process are subsampled to perform the vector process. We use the second approach because it enables as a time domain method to concentrate on time intervals that are of interest for detection of knock (compare 3.2).

3.1. General Solution

Following [5] we vectorize and subsample each SOCS process with respect to the cycle period

$$\mathbf{Y}_k = \begin{pmatrix} Y_{2N_{\text{rot}}k-n_1} \\ \vdots \\ Y_{2N_{\text{rot}}k-n_0} \end{pmatrix}, \ \mathbf{X}_k^{(i)} = \begin{pmatrix} X^{(i)}_{2N_{\text{rot}}k-n_1} \\ \vdots \\ X^{(i)}_{2N_{\text{rot}}k-n_0} \end{pmatrix} \qquad (4)$$

with $n_0 = 0$ and $n_1 = 2N_{\text{rot}} - 1$ and compose a single vibration vector

$$\mathbf{X}_k = \left(\mathbf{X}_k^{(1)'} \ldots \mathbf{X}_k^{(P)'}\right)'. \qquad (5)$$

Our model

$$\mathbf{Y}_n = \sum_{k=-\infty}^{\infty} \mathbf{H}_k \mathbf{X}_{n-k} + \mathbf{Z}_n \qquad (6)$$

now bases on WSS vector processes. Therefore, the problem reduces to a standard Wiener filtering problem, and we have to solve

$$\mathbf{R}_{YX}(n) = \sum_{k=-\infty}^{\infty} \mathbf{H}_k \mathbf{R}_{XX}(n-k). \qquad (7)$$

3.2. Application–specific Solution

The amplitudes and phases of the oscillations caused by knock are random due to the random nature of the combustion process itself. Low stochastic dependencies between different combustions and long cycle periods (compared with acoustical propagation times) motivate $\mathbf{R}_{XX}(n) = \mathbf{R}_{XX}\delta_n$ and $\mathbf{R}_{YX}(n) = \mathbf{R}_{YX}\delta_n$. This simplifies (7) and leads to the solution

$$\mathbf{H}_k = \mathbf{H}\delta_k = \mathbf{R}_{XX}^{-1}\mathbf{R}_{YX}\delta_k \qquad (8)$$

and the simplified model $\mathbf{Y}_n = \mathbf{H}\mathbf{X}_n + \mathbf{Z}_n$.

For our real data experiment the cycle period is $2N_{\text{rot}} = 3420$. Only a limited interval (≈ 350 samples) is of interest for knock detection. We can adapt our solution to this limited interval by choosing n_0 and n_1 in (4) appropriately.

Now we are able to estimate $\hat{\mathbf{H}} = \hat{\mathbf{R}}_{XX}^{-1} \hat{\mathbf{R}}_{YX}$ with

$$\hat{\mathbf{R}}_{XX} = \sum_{l=0}^{L-1} \mathbf{x}_l \mathbf{x}_l' \quad \text{and} \quad \hat{\mathbf{R}}_{YX} = \sum_{l=0}^{L-1} \mathbf{y}_l \mathbf{x}_l'. \qquad (9)$$

These estimates are consistent for $L \to \infty$, but due to a limited number of observed cycle periods L they are not robust enough. Therefore, we modify the estimates:

- $r_{XX}(n, n-k)$ varies slowly with n [1]. Therefore, we implement some smoothing in the diagonal direction of the covariance matrix estimates.
- We modify the estimate of $\mathbf{R}_{XX}$ by adding a diagonal matrix which corresponds to adding white noise to the input process. This stabilizes the solution by restricting the estimated transfer function mainly to those regions of the time–frequency plane where knock oscillations occur.

Simulations have shown that, in contrast to a simulated data experiment, the quality of the real data approximation can be further improved by limiting the length of the impulse response. For this, the problem has to be split up into $n_1 - n_0 + 1$ subproblems of the principal form given in (8). The covariance matrices are submatrices of those given in (8). This leads to a transfer matrix H with diagonal structure and zero entries in the upper right and lower left corner.

4. REAL DATA EXPERIMENT

4.1. Experimental Basis

For our experiments we take the same data as in [1]. We analyze 149 cycle periods of an engine running on a test bed with 3500 rpm. For the parameter estimation we take 100 cycle periods, 49 remaining combustions are used to check the quality of the estimation by calculating the relative mean spuare errors

$$q = \frac{\sum_{l=101}^{149} ||\mathbf{y}_l - \hat{\mathbf{y}}_l||^2}{\sum_{l=101}^{149} ||\mathbf{y}_l||^2} \quad \text{and} \quad q^{(l)} = \frac{||\mathbf{y}_l - \hat{\mathbf{y}}_l||^2}{||\mathbf{y}_l||^2}. \qquad (10)$$

4.2. Optimization

First of all we try to find an optimum for the design parameters mentioned in the previous chapter. Numerical examinations with different values for the three parameters 'smoothing length', 'stabilization factor' and 'length of impulse response' for a prediction based on two vibration signals lead to the results given in Fig. 3. Obviously, the error function q has a single minimum. Fortunately, this minimum is 'flat', i.e. small variations of the design parameters do not lead to significantly worse approximation results.

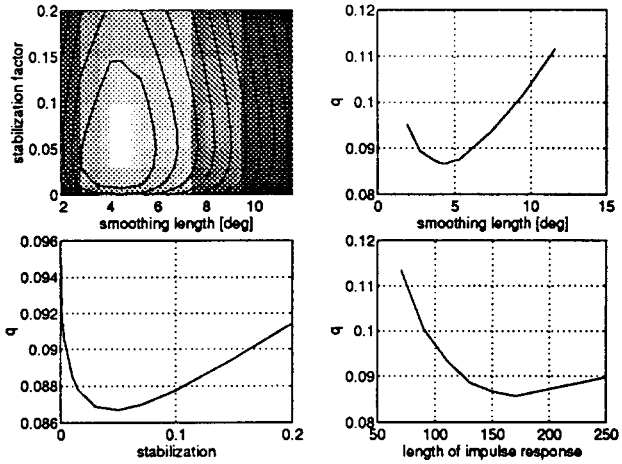

Figure 3: Optimization of design parameters

4.3. Quality of Approximation

For an estimation based on two vibration signals we achieve a mean error of $q = 0.0857$. Fig. 4 shows a **typical** approximation with a relative mean square error of $q^{(132)} = 0.0893$ and the **worst** approximation with $q^{(135)} = 0.358$. Obviously, the prediction is usually very good. Even the worst estimation has small errors within the interval 15° CA up to 50° CA, that is relevant for detection of knock.

Another problem of interest is whether multi channel prediction is advantageous or not. Table 1 shows resulting errors for prediction based on 1, 2 and 3 vibration signals. Compared with the 1 channel experiment, approximation quality significantly increases in the 2 channel experiment and the results still get better when working with 3 different vibration channels.

	number of vibration signals		
	one	two	three
q	0.1623	0.0867	0.0591
$\max_l(q^{(l)})$	0.622	0.3582	0.2505

Table 1: Errors over number of vibration channels

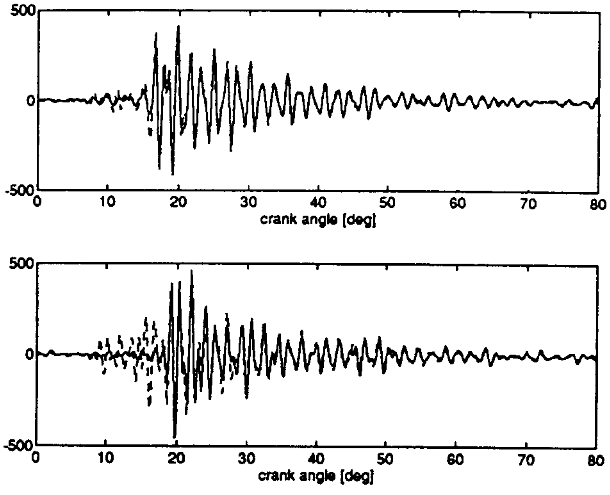

Figure 4: pressure(—) and estimated pressure (- -), top: typical estimation, bottom: worst estimation

5. REDUCTION OF PARAMETER SPACE

For implementation reasons, the number of parameters has to be reduced. Experiments demonstrate, that this can be achieved by suitable parametric models that include the time–variant structure of the engine [6]. In this paper we concentrate on a method based on Singular Value Decomposition (SVD) of the estimated transfer matrix $\hat{\mathbf{H}} = \mathbf{U}\ \mathbf{S}\ \mathbf{V}'$, where $\mathbf{S}$ is a diagonal matrix with the singular values λ_i sorted by size on its diagonal. The SVD enables us to estimate the pressure signal by using only the N largest singular values

$$\hat{\mathbf{y}}_l = \sum_{i=1}^{N} \mathbf{u}_i \lambda_i (\mathbf{v}'_i \mathbf{x}_l). \qquad (11)$$

For prediction based on one vibration signal the resulting prediction error q as a function of the number N of singular values used for the estimation of the pressure signal is plotted in Fig. 5. Obviously, it is sufficient to work with less than 50 singular values which reduces the number of parameters by more than 87 % and the number of flops necessary for calculating the approximation by more than 73 %.

6. CONCLUSIONS

In this paper we present a method for prediction of cylinder pressure signals based on structural vibration signals. Based on the cyclostationary model for these signals and the PTV transfer characteristric we solve the optimum filter problem and adapt the solution to our application. The real data experiments demonstrate the high quality of the approximation that can be achieved.

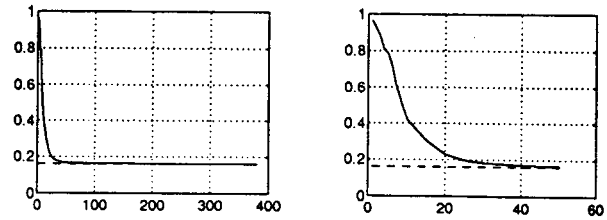

Figure 5: Error q over the number of singular values used for estimation

Vibration sensors placed at different positions on the engine contain supplementary information about the combustion process. Our examinations demonstrate that at least two vibration channels should be used. Finally we show, that the huge number of parameters can be reduced by SVD of the estimated transfer function without leading to significant worse prediction results. This could be relevant in view of a real time implementation of the method presented in this paper.

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