# OPTIMUM CUMULANT-BASED BLIND BEAMFORMING FOR COHERENT SIGNALS AND INTERFERENCES

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## ABSTRACT

We propose an optimum cumulant-based *blind* beamforming method for signal recovery in *coherent* signal environments. Our approach is applicable to any array configuration having *arbitrary* and *unknown* response. There is no need to estimate the directions of arrival. A comparable result does not exist using second-order statistics.

## 1. INTRODUCTION

Optimum beamforming is the process of combining the sensor outputs of an array by a weight vector such that the desired signal is passed with minimum distortion while interfering signals are rejected to the maximum extent. In this paper, we address the *blind* beamforming problem for coherent signal environments assuming no knowledge about the array structure or response, which makes our approach different than present approaches to handle coherency problem. Coherent signal environments are very likely in practice when multipath propagation or smart jammers are present.

There are a number of criteria that have been proposed for obtaining the optimum beamforming weight vector, which all lead to the same general form for the optimum weight vector [1], i.e., $\mathbf{w}_{opt} = c\mathbf{R}^{-1}\mathbf{a}(\theta_d)$ where $\mathbf{R}$ is the spatial covariance matrix of the received signal $\mathbf{r}(t)$, $\mathbf{a}(\theta_d)$ is the array response in the desired direction (look-direction), and $c$ is a constant whose value depends on the criterion used. It is clear that the array response in the desired signal direction must be either known or estimated to implement the optimum beamformer. If the array response or geometry is unknown, as in the blind beamforming problem, it is necessary to calibrate the array; however, array calibration is a very costly procedure. Calibration can be avoided and the array response can be estimated using ESPRIT [2]; however, ESPRIT requires translationally equivalent subarrays, which is usually an impractical constraint. Besides, as other subspace-based methods, ESPRIT fails in the coherent sources case. In addition, the optimum beamformer tends to cancel the desired signal and it fails to perform optimally when there are interferences coherent with the signal [3]. Several methods have appeared [4], [5] (as well as others), to overcome the signal cancellation problem due to coherent interferences. The methods of [4], [5] are limited to uniform linear arrays. None of these methods are directly applicable to the blind beamforming problem due to their implicit constraints on the array structure.

In the cumulant-based array processing framework, the blind beamforming problem was addressed by Dogan and Mendel [6]. Their method can handle multipath propagation; however, in their work it was assumed that the independent interfering signals are Gaussian while the desired signal is non-Gaussian, and cumulants were used to suppress the Gaussian interferences and noise so that one is left only with the desired signal statistics. Here, we assume a more general scenario where there may be multiple desired signals and interferences, all of which may be subject to multipath propagation. The desired signals must have nonzero fourth-order cumulants, but no such an assumption is made about interferences–if their cumulants are zero, they are already suppressed by the virtue of cumulants; if not, they are rejected by an optimum beamformer. As in [6], our method handles multipath propagation. Smart jammers can also be modeled as multipaths. Our approach does not require any knowledge about the array, and relies solely on the measurements. Our earlier work [7] on direction-finding in the coherent sources scenario provides a basis for our approach.

## 2. FORMULATION OF THE PROBLEM AND PROPOSED APPROACH

Consider a coherent signal scenario in which there are several narrow-band sources and interferences. Assume that source signals undergo multipath propagation producing several sets of delayed and scaled replicas. We

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assume an array having arbitrary and unknown response and geometry. Let a total of $P$ wavefronts from $G$ independent and narrow-band sources, with $p_i$ coherent wavefronts for each source $u_i(t)$ ($\sum_{i=1}^{G} p_i = P$), impinge upon the array. The collection of $p_i$ coherent wavefronts, which are scaled replicas of the $i$th source, are referred to herein as the $i$th *group* , and there are $G$ groups. The array measurements are corrupted by additive Gaussian noise whose spatial correlation structure is unknown. Based on these assumptions, the received signal is

$$\mathbf{r}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \qquad (1)$$

where $\mathbf{A}$ is an $M \times P$ *unknown* steering matrix; $\mathbf{s}(t)$ is a $P \times 1$ wavefront vector, and $\mathbf{n}(t)$ is the independent Gaussian measurement noise vector. The coherence among the received wavefronts can be expressed by the following equation:

$$\mathbf{s}(t) = \begin{bmatrix} \mathbf{s}_1(t) \\ \mathbf{s}_2(t) \\ \vdots \\ \mathbf{s}_G(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{c}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{c}_G \end{bmatrix}}_{\triangleq \mathbf{Q}} \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_G(t) \end{bmatrix} \qquad (2)$$

where $\mathbf{s}_i(t)$ is a $p_i \times 1$ signal vector representing the coherent wavefronts from the $i$th independent source $u_i(t)$; $\mathbf{c}_i$ is $p_i \times 1$ complex attenuation vector for the $i$th source ($1 \leq i \leq G$). The received signal vector, written in terms of independent sources, is:

$$\mathbf{r}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{Q}\mathbf{u}(t) + \mathbf{n}(t) = \mathbf{B}\mathbf{u}(t) + \mathbf{n}(t) \qquad (3)$$

where $\mathbf{B} \triangleq \mathbf{A}\mathbf{Q}$. Our objective is to recover the signals $\{u_i(t)\}_{i=1}^{G}$.

Columns of $\mathbf{B}$, called the *generalized steering vectors*, can be estimated, as explained next. Picking up any three sensors, say the $m$th, $p$th and $q$th, two sensor pairs $(m,p)$ and $(m,q)$ can be formed. Using the measurement pair $(r_m(t), r_p(t))$, the following cumulant can be estimated:

$$\mathrm{cum}(r_m^*(t), r_p(t), r_k^*(t), r_l(t)) = \sum_{i=1}^{G} \gamma_{4,u_i} \mathbf{B}^*(m,i)\mathbf{B}(p,i)\mathbf{B}^*(k,i)\mathbf{B}(l,i) \qquad (4)$$

where $\mathbf{B}(m,n)$ denotes the $(m,n)$th element of the matrix $\mathbf{B}$; $\{\gamma_{4,u_i}\}_{i=1}^{G}$ are the fourth-order cumulants of the sources, and $1 \leq k,l \leq M$. Equation (4) is derived using cumulant properties [CP1], [CP3],[CP5], [CP6] in [9], and independence of the source signals. Note that the cumulant of the additive Gaussian measurement noise is zero [1]. Next, using the measurement pair $(r_m(t), r_q(t))$, we compute:

$$\mathrm{cum}(r_m^*(t), r_q(t), r_k^*(t), r_l(t)) = \sum_{i=1}^{G} \gamma_{4,u_i} \mathbf{B}^*(m,i)\mathbf{B}(q,i)\mathbf{B}^*(k,i)\mathbf{B}(l,i) \qquad (5)$$

Defining $\mathrm{cum}(r_m^*(t), r_p(t), \mathbf{r}(t), \mathbf{r}^H(t))$ as the matrix whose $(l,k)$th entry is $\mathrm{cum}(r_m^*(t), r_p(t), r_k^*(t), r_l(t))$, (4) can be expressed as ($1 \leq k,l \leq M$):

$$\mathrm{cum}(r_m^*(t), r_p(t), \mathbf{r}(t), \mathbf{r}^H(t)) = \mathbf{B}\mathbf{\Lambda}\mathbf{B}^H \qquad (6)$$

where we define $\mathbf{\Lambda} \triangleq \mathrm{diag}\{\gamma_{4,u_1}\mathbf{B}(m,1)^*\mathbf{B}(p,1), \cdots, \gamma_{4,u_G}\mathbf{B}(m,G)^*\mathbf{B}(p,G)\}$. Using a similar definition, (5) becomes:

$$\mathrm{cum}(r_m^*(t), r_q(t), \mathbf{r}(t), \mathbf{r}^H(t)) = \mathbf{B}\mathbf{D}\mathbf{\Lambda}\mathbf{B}^H \qquad (7)$$

where $\mathbf{D} \triangleq diag\{\frac{\mathbf{B}(q,1)}{\mathbf{B}(p,1)}, \cdots, \frac{\mathbf{B}(q,G)}{\mathbf{B}(p,G)}\}$. The next step is to estimate the columns of $\mathbf{B}$ using (6) and (7). The solution is based on the idea of rotational invariance of the underlying signal subspace which is the basis of the ESPRIT algorithm [2]. In ESPRIT, the rotational invariance of the signal subspace is induced by the translational invariance of the array, i.e., an identical copy of the array which is displaced in the space is needed. On the other hand, in our cumulant-based algorithm, the same invariance is obtained without any need for an identical copy. In ESPRIT, the signal subspace is extracted from the eigendecomposition of the covariance matrix of the concatenated measurements from the main array and its copy. Here, the signal subspace is extracted from the singular value decomposition of the concatenated matrix of (6) and (7) which, in turn, gives the columns of $\mathbf{B}$, $\{\mathbf{b}_i\}_{i=1}^{G}$, each to within a complex constant. Then, the complex constants which cause rotation of the signal constellation can be compensated for by a procedure explained in [10]. Using these vectors, we can design beamformers to recover the signals $\{u_i(t)\}_{i=1}^{G}$ one at a time.

The received signal at time point $t$ can be expressed as

$$\mathbf{r}(t) = \mathbf{b}_i u_i(t) + \mathbf{B}_{int}\mathbf{u}_{int}(t) + \mathbf{n}(t) \qquad (8)$$

where all the signals except $u_i(t)$ are treated as interferences, $\mathbf{b}_i$ is the generalized steering vector of $u_i(t)$

[1] Our method can be used to suppress non-Gaussian as well as Gaussian noise if the array consists of a sensor whose noise component is independent of those of other sensors. In this case, that sensor measurement is chosen as the first argument of the cumulant matrices in (6) and (7), and it is excluded from the rest of the arguments. For proof see [8].

and $\mathbf{B}_{int}$ is the generalized steering matrix of the interferences.

A number of different criteria lead to the same beamformer which is given by $\mathbf{w}_i = c\mathbf{R}^{-1}\mathbf{b}_i$ where $\mathbf{R}$ is the array covariance matrix, the constant $c$ depends on the criterion used, and, for MVDR criterion, is given by $c_{mvdr} = \frac{1}{\mathbf{b}_i^H \mathbf{R}^{-1} \mathbf{b}_i}$. Consequently, the source signals can be recovered using this beamformer for each signal $u_i(t)$ as follows

$$u_i(t) = \mathbf{w}_i^H \mathbf{r}(t). \tag{9}$$

where $i = 1, \cdots, G$.

## 3. SIMULATIONS

### 3.1. Experiment 1

The scenario consists of three independent binary phase shift keyed (BPSK) sources which are subject to multipath propagation, and arrive at the array in Fig. 1 from four, two and three different directions respectively. The arrival directions and propagation constants were chosen arbitrarily as [50°, 70°, 90°, 100°] and [1, $-0.8 + j0.2$, $-0.3 - j0.7$, $0.6 + j0.6$]; [60°, 80°] and [1, $-0.1 + j0.8$]; [45°, 65°, 85°] and [1, $0.5 - j0.6$, $0.7 + j0.4$] where unity propagation constants belong to direct paths. The direct path SNRs were 10dB. The array elements were assumed to be arbitrarily rotated dipole antennas. 3000 snapshots were taken. The problem of interest is to recover each source message. We tested our cumulant-based beamforming method which assumes no information about the array geometry or response, and the classical MVDR beamformer for which we had to assume that arrival angles of the desired signals (the direct paths from each source) and the array response in those directions are perfectly known. The beamformer outputs from both methods are presented in Fig. 2. Observe that, while cumulant-based beamformer outputs are localized around 1 and $-1$, the MVDR beamformer fails to recover the source messages. Note that spatial smoothing explained in [5] is a remedy to signal cancellation in the MVDR beamformer for coherent signals; however, spatial smoothing is applicable only to uniform linear arrays, whereas the array in this experiment is a nonuniform one. This experiment supports our earlier claim that multiple coherent signals received by an array of arbitrary geometry and unknown response can be recovered by our cumulant-based optimum blind beamformer.

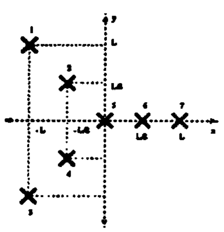

Figure 1: The array geometry used in the first experiment. $L$ is the wavelength.

### 3.2. Experiment 2

In this experiment we compare our method to the spatial smoothing method. Since spatial smoothing is limited to uniform linear arrays, we restrict ourselves here to this case although our method is applicable to any array. We assume that two coherent BPSK signals of equal power and 3000 bits long with zero relative phase impinge on a ten-element uniform linear array from closely spaced directions {0°, 5°} near endfire. Reddy *et al* [3] have shown that, for this case, spatial smoothing loses its decorrelating power for moderate smoothing lengths and therefore results in increased signal cancellation as SNR is increased. In our cumulant-based method we used the pairs $(r_1(t), r_1(t))$ and $(r_1(t), r_2(t))$ where $r_1(t)$ and $r_2(t)$ are the first two sensor measurements. Assuming the desired signal direction is $0^o$ we designed the classical MVDR bemformer. For the smoothed-MVDR beamformer we used a subarray of length 6 for backward and forward smoothing. Figures 3 a-d show outputs of the three beamformers for 0, 10, 20 and 30 dB SNRs. The presence of coherence helps the classical- and smoothed-MVDR beamformer at low SNRs, but, these beamformers deteriorate as SNR is increased [3]. On the other hand, comparison of the first column of Fig. 3 with the other two columns indicates that our method is always better than smoothed-MVDR at equal SNRs, and that our method improves as SNR is increased, because our method combines coherent signal powers effectively instead of trying to decorrelate them. Finally, note that while smoothed-MVDR can utilize only the smoothing subarray, our method uses the entire array–a larger aperture.

## 4. CONCLUSIONS

We have developed a cumulant-based optimum *blind* beamformer for the coherent signals case which is applicable to any *arbitrary* array configuration; it does not require *any knowledge* about array response, and relies *solely* on the measurements. There is no need to estimate the directions of arrival. Our approach is based on the observation that using cumulants of received signals, two matrices can be formed which conform to the ESPRIT architecture. In this approach, multipath powers are effectively utilized instead of decorrelated. This provides increased robustness to interferences and noise. The two matrices permit us to estimate the generalized steering vectors for each source. Then, a number of cumulant-based beamformers can be designed whose optimality have already been shown in the second-order statistics framework. A comparable result using second-order statistics does not exist, because we need at least three arguments to obtain matrices similar to (6) and (7) for the blind beamforming problem.

## 5. REFERENCES


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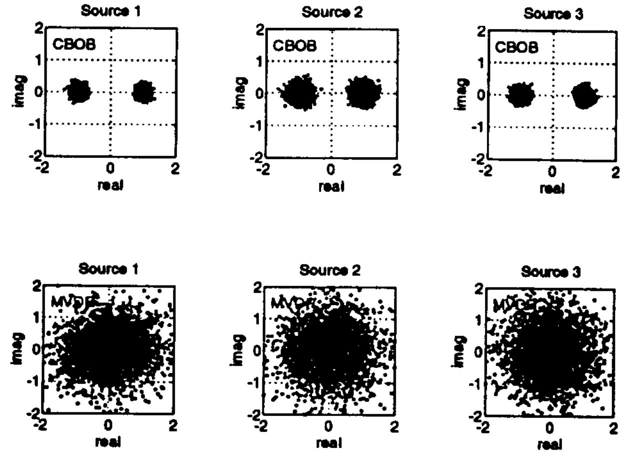


Figure 2: Cumulant-based and MVDR beamformer outputs for Experiment 1. SNR=10dB. "CBOB" refers to cumulant-based optimum beamformer.

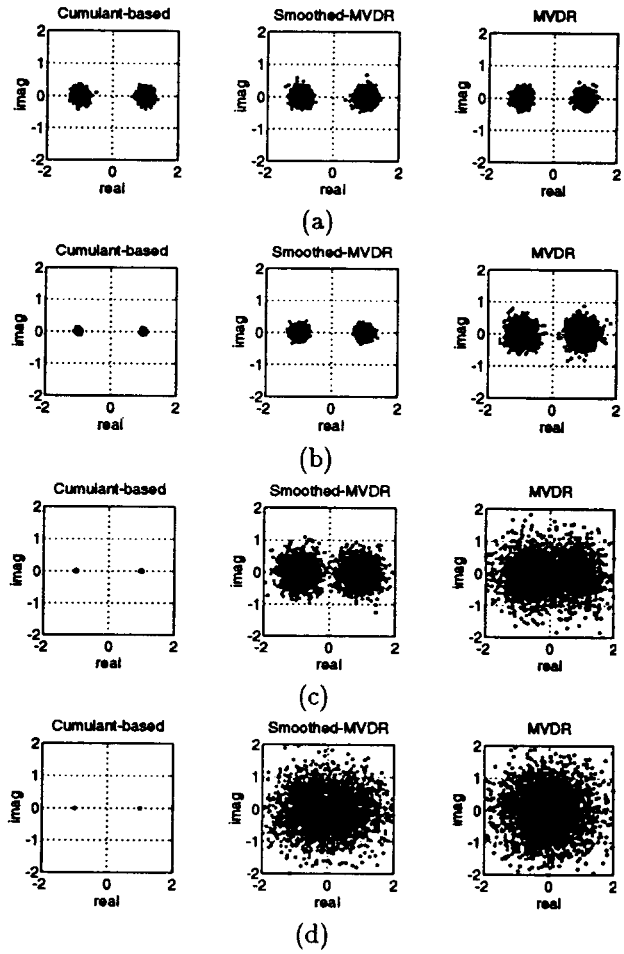


Figure 3: Various beamformer outputs for two coherent signals near endfire from closely spaced directions $\{0^\circ, 5^\circ\}$: (a) SNR=$0dB$, (b) SNR=$10dB$, (c) SNR=$20dB$, and (d) SNR=$30dB$.