CONSIDERATIONS IN THE AUTOCALIBRATION OF QUADRATURE RECEIVERS*

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ABSTRACT

This paper develops a calibration procedure for gain and phase imbalances in quadrature receivers. Quadrature demodulation has many applications in communications and array processing. Frequently, the output of the in-phase (I) and quadrature (Q) channels are considered the real and imaginary parts, respectively, of a complex random process which belongs to the Goodman class. Mismatch in the gain and phase of the I and Q channels results in a departure from this statistical model and a degradation in the performance of subsequent signal processing algorithms. The calibration algorithm presented in this paper estimates the relative imbalance between the I and Q channels, based on data from a sinusoid of known frequency but unknown amplitude and phase.

1. INTRODUCTION

Quadrature receivers have many applications in communications and signal processing. If the responses of the in-phase (I) and quadrature (Q) channels of the receiver are unmatched, the subsequent signal processing algorithms will not achieve their theoretical performance capabilities. This type of non-ideality has received only slight attention in the signal and array processing literature, yet the impact on performance can be significant. Methods are needed for calibrating each quadrature receiver and then, in the array processing case, to calibrate the overall array.

We present new results for the calibration of quadrature receivers, using input signals with unknown parameters. This work represents an extension of earlier work on this problem [1] in that we consider the receiver frequency response over a wide range of frequencies different from the local oscillator (LO) frequency and both gain and phase imbalances are considered, not just gain. This work is also similar to the calibration approach in [2]. Here, it is shown that, with an input sinusoid of known frequency *not* equal to the LO frequency, but unknown amplitude and phase, it is possible to match the in-phase (I) and quadrature (Q) channels of the receiver at that frequency, although it is not possible to determine the absolute frequency response of either channel.

The significance of this calibration problem arises from the use of quadrature receivers in statistical array processing applications, and the ubiquitous but often unstated assumption that receiver outputs, considered as the real and imaginary parts of a complex stochastic process, are subject to a particular subclass of the Gaussian distribution described by Goodman [3] which we refer to as the Goodman class. An uncalibrated receiver with unmatched responses in the I and Q channels will cause a departure from this statistical model and potential breakdown of subsequent signal processing algorithms. Furthermore, when imbalances are present in the I and Q branches, no longer are the direction vectors independent of the relative phase difference between the local oscillator and the received passband signal [1], and therefore direction vectors are not unique for a specific direction-of-arrival.

After this introduction, the next section discusses the model for a non-ideal quadrature receiver. The third section formulates the calibration algorithm. The fourth section illustrates the effectiveness of the calibration procedure in a noisy environment.

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The concluding section discusses the results and suggests future research in this area.

2. QUADRATURE RECEIVER MODEL

Consider the model for a quadrature receiver shown in Fig. 1. The input test signal is a sinusoid of known carrier frequency ω_c (rad/sec) and unknown amplitude and phase C and ψ, respectively. This signal is mixed with the quadrature outputs of the LO; these have known frequency ω_0 and unknown amplitudes G_1 and G_2, and phases ϕ_1 and ϕ_2. In the ideal case $G_1 = G_2 = 1$ and $\phi_1 = \phi_2 = 0$. **A** and **B** are linear time-invariant (LTI) systems which model primarily the lowpass filter that eliminates the sum-frequency components at the mixer outputs, but which can be used also to model other gain and phase discrepancies in the two channels. Our interest is in quantifying the receiver behavior at the difference frequency $\omega_1 = \omega_c - \omega_o$.

Define the responses of **A** and **B**, at frequency ω_1, as

$$A(\omega_1) = ae^{j\alpha} \qquad (1a)$$

$$B(\omega_1) = be^{j\beta} \quad . \qquad (1b)$$

Ideally, $a=b$ and $\alpha=\beta$ so that the channels are identical. In hardware, matching may be difficult to achieve, especially for a large range of frequencies of interest. Assume further that the response of both systems at the sum frequency $\omega_c + \omega_o$ is identically zero. Then straightforward trigonometry shows that the output signals are given by

$$x(t) = \frac{1}{2} CG_1 a\cos(\omega_1 t+\psi-\phi_1+\alpha) \qquad (2a)$$

$$y(t) = \frac{1}{2} CG_2 b\sin(\omega_1 t+\psi-\phi_2+\beta) \,. \qquad (2b)$$

The next section uses this model of the I and Q outputs to a sinusoidal input, to estimate the relative gain and phase imbalances.

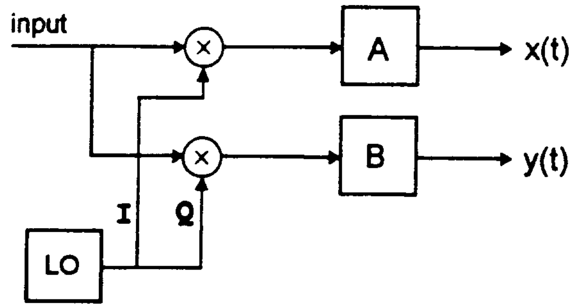

Fig. 1. Quadrature Receiver Model

3. CALIBRATION ALGORITHM

It is clear from inspection of (2) that separate determination of the eight unknown receiver parameters is not possible from observation of $x(t)$ and $y(t)$. However, since the goal is determination of the overall receiver response, we can proceed by absorbing the LO gain and phase imperfections into the model for **A** and **B**; this leads to a simplified model in which $G_1 = G_2 = 1$, $\phi_1 = \phi_2 = 0$, and

$$x(t) = \frac{1}{2} Ca\cos(\omega_1 t+\psi+\alpha) \qquad (3a)$$

$$y(t) = \frac{1}{2} Cb\sin(\omega_1 t+\psi+\beta) \,. \qquad (3b)$$

Suppose that we have observations of $x(t)$ and $y(t)$ over the finite interval $[0,T]$, and that T is an integer multiple of the fundamental period $T_1 = 2\pi/\omega_1$. Then we can obtain the quantities

$$x_r = +\frac{1}{T}\int_0^T x(t)\cos(\omega_1 t)dt \qquad (4a)$$

$$x_i = -\frac{1}{T}\int_0^T x(t)\sin(\omega_1 t)dt \qquad (4b)$$

$$y_r = +\frac{1}{T}\int_0^T y(t)\sin(\omega_1 t)dt \qquad (4c)$$

$$y_i = +\frac{1}{T}\int_0^T y(t)\cos(\omega_1 t)dt \ . \qquad (4d)$$

In actual implementation this might be accomplished by downstream digital processing of samples of $x(t)$ and $y(t)$. Note that in effect we are proposing quadrature detection of $x(t)$ and $y(t)$ at the frequency ω_1. Define the complex quantities

$$X = x_r + jx_i \qquad (5a)$$

$$Y = y_r + jy_i \ . \qquad (5b)$$

Then we have

$$X = \frac{1}{4}Cae^{j(\psi+\alpha)} \qquad (6a)$$

$$Y = \frac{1}{4}Cbe^{j(\psi+\beta)} \ . \qquad (6b)$$

Clearly the absolute response parameters a, b, α, and β cannot be individually determined; however, the relative response is easily obtained:

$$\frac{b}{a} = \frac{|Y|}{|X|} \qquad (7a)$$

$$(\beta-\alpha)_{2\pi} = (arg(Y) - arg(X))_{2\pi} \ . \qquad (7b)$$

From this information, either channel can be corrected so that it matches the other.

Note that this leaves still an overall complex receiver response unknown. However, this procedure ensures that the receiver outputs will be Goodman distributed and thus can be modeled as a complex Goodman input multiplied by a single complex gain. Determination of this complex gain, or rather the relative complex gains across an array of quadrature receivers, is the *co-channel* gain and phase estimation problem, which is the subject of [4], [5], and other papers in array calibration cited therein.

Before going onto a simulation example, a brief discussion is given regarding the calibration when zero-mean, finite power, wide-sense stationary, noise is present in the channels of the receiver. There are a number of locations in the receiver where noise may be present. Ambient and electronic noise may be present at the input in addition to the test input, additive electronic noise may be present in the local oscillator output, and additive electronic noise may be present in the I and Q channels after demodulation. Most likely the most significant noise component is the noise at the input which is amplified by all stages of the receiver system. It can be shown that the estimates of X and Y from (5) are unbiased and consistent for any or all of these noise situations. Therefore, as the observation time T increases the estimates of X and Y approach their true values.

4. SIMULATION EXAMPLE

Two simulation examples are given to illustrate the calibration algorithm performance under noisy calibration conditions. For both simulations wide-sense stationary noise is present at the input.

At a fixed SNR (signal-to-noise ratio) the variance of x_r is examined as a function of T (data observation time). One hundred trials were used for each value of T. Fig. 2 shows the estimated variance as a function of T on a log-log plot. The relationship is linear with a slope of one illustrating that the variance is inversely proportional to T.

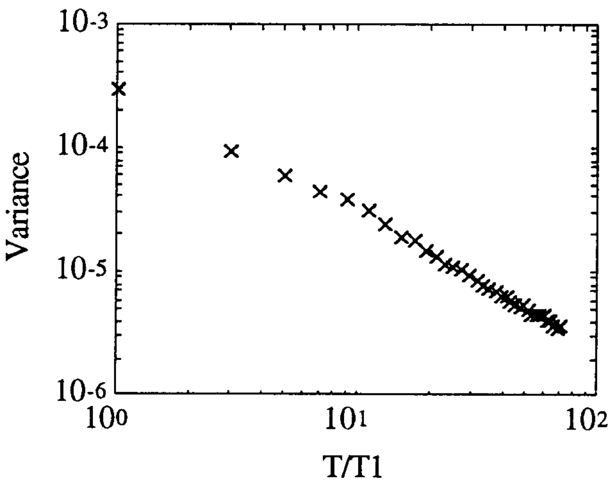

Fig. 2. Variance of the estimate of x_r vs. observation time.

The second simulation demonstrates the calibration algorithm for a number of trials with $T=30T_1$ and an SNR of 10 dB at the output of the quadrature receiver. The actual relative gain error is 0.9502 and the relative phase error is 6.013 degrees. Fig. 3 shows the estimate of the relative gain error, *b/a*, while Fig. 4 shows the estimate of the relative phase error, $(\beta-\alpha)_{2\pi}$, for twenty trials.

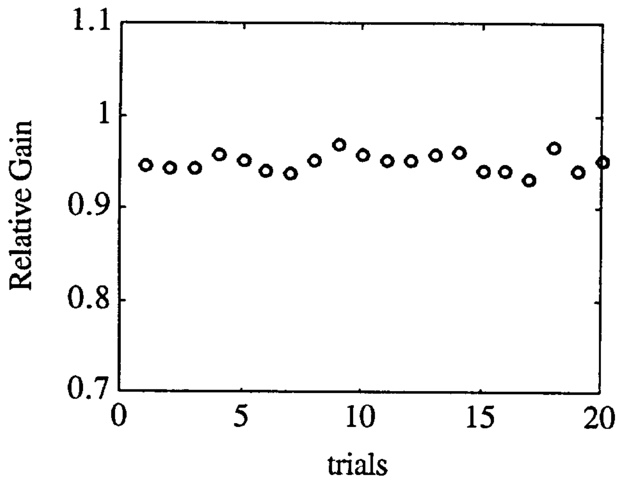

Fig. 3. Estimated relative gain error for 20 trials.

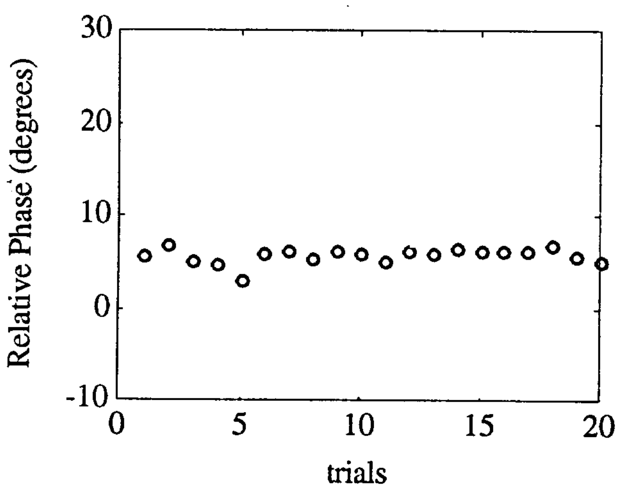

Fig. 4. Estimated relative phase error for 20 trials.

5. CONCLUSION

This paper investigated calibration of quadrature receiver gain and phase errors. An important point made by the paper is that the complex data model frequently used for the quadrature receiver output is invalid if unmatched errors exist in the I and Q channels. The relative gain and phase error for a particular frequency can be computed from the I and Q channel outputs given a sinusoidal input of unknown amplitude and phase but known frequency, different from the local oscillator frequency. Knowing the relative errors, either channel can be compensated to match the other. Calibrating the receiver insures that the complex output data representation belongs to the Goodman class. In the future a more detailed statistical analysis of the calibration approach in a noisy environment may be performed. This paper represents the start of an investigation of the calibration of quadrature receivers.

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