A SELF-CALIBRATION ALGORITHM FOR CYCLOSTATIONARY SIGNALS AND ITS UNIQUENESS ANALYSIS

Patrick C. Yip and Yifeng Zhou

Communications Research Laboratory
McMaster University
Hamilton, Ontario, Canada L8S 4K1
e-mail : yippatr@mcmaster.ca

ABSTRACT

In this paper, a self-calibration DOA estimation algorithm for cyclostationary source signals is presented in which the effects of the sensor gain and phase shift uncertainty have been eliminated. The uniqueness conditions and the asymptotic consistency of the estimates are discussed. An alternating projecting optimization algorithm is provided which lessens the computational load involved in the nonlinear multivariate optimization problem much less. A numerical example is presented to show the effectiveness of the algorithm.

1. INTRODUCTION

Recently, the concept of cyclostationarity has been introduced into array signal processing [1]-[3]. By exploiting the source cyclostationarity, these techniques select the desired source signals at the known cycle frequency and eliminate the interferences and background noise which do not exhibit cyclostationarity or have different cycle frequencies. They have the advantage of not requiring a priori knowledge about the ambient noise environment.

However, these techniques rely on the assumption that either the array sensors have known sensor gian and phase or they have been calibrated when multiple sources are present. In practice, this assumption has almost always been violated due to physical perturbations of the sensors and sensor imperfections, and estimation performances can be adversely affected. Several self-calibration techniques have been proposed [4]-[7] for the conventional methods in which, the sensor gain and phase uncertainties are treated as unknown parameters and are estimated along with the source parameters. These conceptually simple algorithms are computationally intense due to the increased dimension in the optimization process.

The objective of the paper is to provide a self-calibration DOA estimation algorithm for cyclostationary source signals. By assuming that the source signals exhibit cyclostationarity at cycle frequencies α and β, the array cyclic correlation matrices at the cycle frequencies of interest are evaluated. The relating matrix is defined and estimated by using the total least squares (TLS) approach. A criterion function is formed in which the effects of the unknown sensor gain and phase uncertainty have been eliminated. We developed a modified alternating projecting optimization algorithm for optimizing the criterion. We also proved the necessary and sufficient conditions under which the source parameters can be uniquely determined. The estimator is shown to be asymptotically consistent. Finally, a numerical example is included to show the effectiveness of the algorithm.

2. CYCLOSTATIONARITY AND THE ARRAY SIGNAL MODEL

Consider an array consisting of M sensors and $K(K < M)$ narrow-band source signals in the far-field of the array. The medium is assumed to be isotropic and nondispersive. When using analytic representation, the array model is described by

$$\underline{x}(t) = GA(\Theta)\underline{s}(t) + \underline{n}(t), \tag{1}$$

where $\underline{x}(t)$, $\underline{s}(t)$ and $\underline{n}(t)$ are the array data vector, the signal vector and the noise vector, and $G = \text{diag}[\delta_1, \delta_2, \ldots, \delta_M]$ denotes the sensor gain and phase uncertianty matrix. $A(\Theta)$ is the array composite steering matrix and its kth column vector is defined as the steering vector associated with the kth source.

SOURCE CYCLOSTATIONARITY We assume that the source signals $s_k(t)$ are cyclostationary. A random process $x(t)$ is said to exhibit cyclostationarity with cycle frequency α if its cyclic autocorrelation function

$$r_x^{\alpha}(\tau) = < r_x(t, t+\tau)\exp(-j2\pi\alpha t) >, \tag{2}$$

is not identically zero, where notation $< \cdot >$ denotes time average and $r_x(t, t+\tau) = E[x(t)x^*(t+\tau)]$ is the autocorrelation function of $x(t)$. In $r_x(t, t+\tau)$, the conjugate operator can be removed and (2) is called the cyclic conjugate correlation function. Some signals have either nonzero cyclic or cyclic conjugate correlation and some may have both. For example, a quarternary phase-shift keying (QPSK) signal exhibits only nonzero cyclic correlation and a binary phase-shift keying (BPSK) signal has both nonzero cyclic and cyclic conjugate correlation.

ARRAY CYCLIC CORRELATION Assume that the K sources are mutually cyclically uncorrelated at α and β. Also

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assume that the sensor noise processes are cyclically uncorrelated with themselves and with the source signals at α and β. It follows that the array data cyclic correlation matrix can be written as

$$\begin{aligned} R_x^\alpha(\tau) &= GA(\Theta)R_s^\alpha(\tau)A^H(\Theta)G^H \\ R_x^\beta(\tau) &= GA(\Theta)R_s^\beta(\tau)A^H(\Theta)G^H, \end{aligned} \quad (3)$$

where $R_s^\alpha(\tau)$ and $R_s^\beta(\tau)$ are the source signal cyclic correlation matrices at cycle frequencies α and β. The diagonal matrices $R_s^\alpha(\tau)$ and $R_s^\beta(\tau)$ can be related by

$$R_s^\beta(\tau) = \Phi R_s^\alpha(\tau). \quad (4)$$

where $\Phi = \text{diag}[\phi_1, \phi_2, \ldots, \phi_K]$ is defined as the relating matrix. Define the matrix Z as

$$Z = \begin{bmatrix} R_x^\alpha(\tau) \\ R_x^\beta(\tau) \end{bmatrix} = \begin{bmatrix} GA(\Theta) \\ GA(\Theta)\Phi \end{bmatrix} R_s^\alpha(\tau)A^H(\Theta)G^H, \quad (5)$$

and its singular value decomposition is $Z = E[\Sigma^T, \text{O}]D^H$. For the assumed full rank source cyclic correlation matrix, Z has only K nonzero singular values. Assume that diagonal elements of Σ have been arranged in decreasing order and partition matrix E as follows

$$E = \begin{bmatrix} E_{s1} & E_{n1} \\ E_{s2} & E_{n2} \end{bmatrix}, \quad (6)$$

where $E_{s1}, E_{s2} \in R^{M\times K}$. From the SVD analysis, we know that the range space of $[E_{s1}^T, E_{s2}^T]^T$ is identical to that of $[A^T(\Theta\Delta, \Phi A^T(\Theta)\Delta]^T$ and the following relation can be obtained

$$E_{s2} = E_{s1}T^{-1}\Phi T = E_{s1}\Psi, \quad (7)$$

where matrix Ψ can be solved in the least squares sense. Denote $E_s = [E_{s1}, E_{s2}]$. The TLS approach can be applied [8] which yields

$$\Psi_{TLS} = -Q_{12}Q_{22}^{-1}, \quad (8)$$

where Q_{12} and Q_{22} are defined by the following eigendecomposition

$$E_s^H E_s = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \Gamma \begin{bmatrix} Q_{11}^H & Q_{21}^H \\ Q_{12}^H & Q_{22}^H \end{bmatrix}, \quad (9)$$

where the block matrix $Q_{ij} \in R^{K\times K}$ for $i, j = 1, 2$. Since the relationship $\Psi = T\Phi T^{-1}$ holds, the elements of the relating matrix Φ will be estimated from the eigenvalues of $\hat{\Psi}_{TLS}$.

3. SELF-CALIBRATION ALGORITHM

With the estimation of the relating matrix Φ, we can proceed with the self-calibration DOA estimation algorithm. From (3), the mnth element of $R_x^\alpha(\tau)$ and $R_x^\beta(\tau)$ can be written as

$$\begin{aligned} r_{mn}^\alpha &= \delta_m\delta_n^* \sum_{k=1}^{K} \exp\{j\omega_0(\tau_{mk} - \tau_{nk})\}s_k \\ r_{mn}^\beta &= \delta_m\delta_n^* \sum_{k=1}^{K} \exp\{j\omega_0(\tau_{mk} - \tau_{nk})\}\phi_k s_k, \end{aligned} \quad (10)$$

respectively, where s_k is the kth diagonal element of the diagonal matrix $R_s^\alpha(\tau)$ denoting the kth source cyclic correlation. Define l_{mn} as

$$l_{mn} = \frac{r_{mn}^\alpha}{r_{mn}^\beta} = \frac{\sum_{k=1}^{K} \exp\{j\omega_0(\tau_{mk} - \tau_{nk})\}s_k}{\sum_{k=1}^{K} \exp\{j\omega_0(\tau_{mk} - \tau_{nk})\}\phi_k s_k}, \quad (11)$$

for $m, n = 1, 2, \ldots, M$. Equation (11) can also be written as

$$\sum_{k=1}^{K} \exp\{j\omega_0(\tau_{mk} - \tau_{nk})\}(l_{mn}\phi_k - 1)s_k = 0. \quad (12)$$

which in matrix form, becomes

$$[LB(\Theta)\Phi - B(\Theta)]\underline{s} = \underline{0}, \quad (13)$$

where

$$L = \text{diag}[l_{11}, l_{12}, \ldots, l_{MM}]$$
$$\underline{s} = [s_1, s_2, \ldots, s_K]^T.$$

and $B(\Theta)$ is a matrix of $M^2 \times K$ and its kth column $\underline{b}(\theta_k)$ is implicitly defined by

$$\underline{b}(\underline{\theta}_k) = vec[\underline{a}(\underline{\theta}_k)\underline{a}^H(\underline{\theta}_k)], \quad (14)$$

where $vec[\cdot]$ stands for the concatenation of the columns of its matrix argument. In practice, exact L and Φ are usually not available and their estimates $\hat{L}$ and $\hat{\Phi}$ are applied. The source DOA parameters are estimated by minimizing the following criterion function in the least squares sense

$$\begin{aligned} \hat{\Theta} &= \arg\min_{\Theta} J(\Theta) \\ J(\Theta) &= \| [\hat{L}B(\Theta)\hat{\Phi} - B(\Theta)]\underline{s} \|_F, \end{aligned} \quad (15)$$

where F denotes the Frobenius norm. The optimization of $J(\Theta)$ is a multivariate nonlinear optimization process which usually involves high computational load. Since $J(\Theta)$ is a monotonically increasing function of the norm of $\underline{s}$, restrictions on $\underline{s}$ are required to make $J(\Theta)$ a valid criterion function. Let $\underline{\hat{c}}(\underline{\theta}_k) = (\hat{L}\hat{\phi}_k - I)\underline{b}(\underline{\theta}_k)$, where I denotes the identity matrix and define matrix $\hat{C}_k$ and $\underline{s}_k$ as

$$\begin{aligned} \hat{C}_k &= [\underline{\hat{c}}(\underline{\theta}_1), \ldots, \underline{\hat{c}}(\underline{\theta}_{k-1}), \underline{\hat{c}}(\underline{\theta}_{k+1}), \ldots, \underline{\hat{c}}(\underline{\theta}_K)] \\ \underline{s}_k &= [s_1, \ldots, s_{k-1}, s_{k+1}, \ldots, s_K]^T. \end{aligned} \quad (16)$$

Obviously, fixing any one element of $\underline{s}$ would not affect the optimization of $J(\Theta)$ with respect to Θ. If we set $s_k = 1$, then criterion (15) can be rewritten as

$$\hat{\Theta} = \arg\min_{\Theta} \| \underline{\hat{c}}(\underline{\theta}_k) + \hat{C}_k\underline{s}_k \|_F . \quad (17)$$

Fixing Θ and minimizing $J(\Theta)$ with respect to $\underline{s}_1$ yields

$$\underline{\hat{s}}_k = -\hat{C}_k^\dagger \underline{\hat{c}}(\underline{\theta}_k), \quad (18)$$

where $\hat{C}_k^\dagger = (\hat{C}_k^H\hat{C}_k)^{-1}\hat{C}_k^H$ is the pseudo-inverse of $\hat{C}_k^H$. Substituting $\underline{\hat{s}}_k$ back into (17), we obtain

$$\hat{\Theta} = \arg\min_{\Theta} \| P_{\hat{C}_k}^{\perp} \hat{\underline{c}}(\underline{\theta}_k) \|_F, \tag{19}$$

where $P_{\hat{C}_k}^{\perp}$ is the orthogonal projector onto the null space of $\hat{C}_k^H$. A computationally efficient alternating minimization algorithm is then proposed as follows.

- Set threshold ϵ. Obtain an initial source DOA parameter estimate

$$\hat{\Theta}^{(0)} = [\hat{\underline{\theta}}_1^{(0)}, \hat{\underline{\theta}}_1^{(0)}, \ldots, \hat{\underline{\theta}}_K^{(0)}]$$

- Assume that in the ith iteration, the estimate $\hat{\Theta}^{(i)}$ has been obtained.
- For k=1 to K, minimize criterion (19) in the kth source DOA parameter $\underline{\theta}_k$ while fixing all the others. After the Kth iteration, a new estimate $\hat{\Theta}^{(i+1)}$ is formed.
- Stop the algorithm if $\| \hat{\Theta}^{(i+1)} - \hat{\Theta}^{(i)} \| < \epsilon$ and assign the estimate as $\hat{\Theta} = \hat{\Theta}^{(i+1)}$. If the inequality is not satisfied, go back to the previous step and repeat.

UNIQUENESS Assume that the array cyclic correlation matrices are accurately estimated. Define the limit criterion function $J(\Theta)$ as

$$J(\Theta) = \| [LB(\Theta)\Phi - B(\Theta)]\underline{s} \|_F^2 = \| D(\Theta)\underline{s} \|_F^2 . \tag{20}$$

The source parameters are known to be estimated as the minimizing arguments of $J(\underline{\Theta})$ or by resolving the following equation

$$D(\Theta)\underline{s} = 0, \tag{21}$$

It is obvious that the true source parameters are solutions of equation (21). In the following, we establish the necessary and sufficient conditions for (21) without detailed proof [9].

Theorem 1 Let Θ and $\underline{s}$ denote the true source parameters, where $s_k \neq 0$, $k = 1, 2, \ldots, K$. Then, $\underline{s}'$ and Θ' are also solutions of (21) iff

$$\begin{aligned} \tau_m(\underline{\theta}_k') &= \tau_m(\underline{\theta}_k) + \Delta\tau_m \\ \underline{s}' &= g\underline{s} \end{aligned} \tag{22}$$

where τ_m denotes the time delay induced on the mth sensor by the source, $\Delta\tau_m$ is a real and g may be a complex constant.

It is noted that the proof of this theorem is completed using mathematical induction, and the theorem is subject to certain array structure restrictions [9]. The theorem states that the self-calibration algorithm can resolve the sources which induce time delays on the array up to a translation factor and the source cyclic correlation up to a scaling factor. However, the uniqueness of the source DOA parameters also depends on the array structure. In the following, the rotational-invariant and rotational-variant array structures are introduced.

Definition Define τ_{max} and τ_{min} as

$$\tau_{max} = \max_{\underline{\theta}} \tau_m(\underline{\theta}) \text{ and } \tau_{min} = \min_{\underline{\theta}} \tau_m(\underline{\theta}),$$

for $m = 1, 2, \ldots, M$. An array is said to be rotational-invariant if, for an arbitrary constants τ_m and $\underline{\theta}$, satisfying $\{\tau_{min} \leq \tau_m(\underline{\theta}) + \Delta\tau_m \leq \tau_{max} \;\; m = 1, 2, \ldots, M\}$, there exists a $\underline{\theta}'$ such that

$$\tau_m(\underline{\theta}') = \tau_m(\underline{\theta}) + \Delta\tau_m \quad m = 1, 2, \ldots, M. \tag{23}$$

An array structure is called rotatioal-variant if for arbitrary constants $\Delta\tau_m$ and $\underline{\theta}$ satisfying, $\{\tau_{min} \leq \tau_m(\underline{\theta} + \Delta\tau_m \leq \tau_{max} \;\; m = 1, 2, \ldots, M\}$, there does not exist a $\underline{\theta}'$ such that (23) holds.

For rotational-variant and -invariant arrays, we have the following lemmas [9].

Lemma 1 For rotational-variant arrays, the DOA estimates obtained from solving equation (21) can be uniquely determined, but for rotational-invariant arrays, they can be resolved within an arbitrary translation factor in their corresponding time delay induced on the array.

The non-uniqueness of the DOA estimates for rotational-variant arrays can be seen from the array signal models. For rotational-invariant array, we have $A(\Theta') = \Lambda A(\Theta)$ and

$$\underline{x}(t) = GA(\Theta)\underline{s}(t) + \underline{n}(t) = G'A(\Theta')\underline{s}(t) + \underline{n}(t), \tag{24}$$

in which $G' = G\Lambda$. Since $GA(\Theta)$ and $G'A(\Theta')$ are indentical, then Θ and G can not be solved separately.

In the above, the uniqueness conditions on the array structure have been discussed for source parameters solved from the limit criterion function. In practice, the limit criterion function is usually not available and the criterion function is formed from the estimated array cyclic correlation matrix at the cycle frequency of interest. In the following lemma, the asymptotic consistency of the DOA estimates is established in which we assume $\hat{R}_x^{\alpha}$, $\hat{R}_x^{\beta}$ and $\hat{\Phi}$ are all asymptotic consistent estimates.

Lemma 2 The DOA estimate $\hat{\Theta}$ obtained from (15) converges w.p.1. to Θ for rotational-variant arrays, and converge w.p.1. to Θ within a common arbitrary translation factor in their corresponding time delays induced on the array for rotational-invariant arrays.

4. A NUMERICAL EXAMPLE

Assume that the sources and the array sensors are distributed in xy plane. The array contains six sensors located at positions

$$\{(0,0), (\frac{1}{2}, \frac{1}{2}), (1,0), (\frac{3}{2}, \frac{1}{2}), (2,0), (\frac{5}{2}, \frac{1}{2})\}.$$

Two uncorrelated BPSK signals are simulated from the far-field at $\theta_1 = -10°$ and $\theta_2 = 10°$ to the y-axis. The source powers are set to unity and half the unity, respectively. The additive stationary sensor noise is assumed to be a spatially white Gaussian process with zero-mean. The sensor gains and phase are simulated as perturbed around unity and zero 10% and 20%. Fig. 1 shows the mean-squares-error (MSE) of the DOA estimates versus SNR for Cyclic MUSIC [2] and the proposed self-calibration algorithm. In each test, 100 array samples are used and each test is repeated 100 times to obtain the averaged results. In the plot, Cyclic MUSIC is seen to be a biased estimator while the self-calibration algorithm shows a great improvement over Cyclic MUSIC in the presence of unknown sensor gain and phase uncertainty.

5. REFERENCES

[1] W. A. Gardner, "Simplification of MUSIC and ESPRIT by exploitation of cyclostationary", IEEE Proc., Vol. 76, No. 7, pp 845-847, 1988

[2] S. V. Schell, R. A. Calabretta and W. A. Gardner, "Cyclic MUSIC algorithm for signal selective DOA estimation", ICASSP-1989, pp 2278-2281, 1989

[3] G. X. Xu and T. K. Kailath, "Direction-of-arrival estimation via exploitation of cyclostationarity - A combination of temporal and spatial processing", IEEE Tran. SP, Vol. 40, No. 7, pp 1775-1785, 1992

[4] A. Paulraj and T. Kailath, "Direction-of-arrival estimation by eigenstructure methods with unknown sensor gain and phase", Proc. ICASSP-85, pp. 640-643, 1985

[5] A. J. Weiss, A. S. Willsky and B. C. Levy, "Eigenstructure approach for array processing with unknown intensity coefficients", IEEE Trans. SP, Vol. SP-36, pp 1613-1617, 1988

[6] A. J. Weiss and B. Friedlander, "Eigenstructure methods for direction finding with sensor gain and phase uncertainties", Circuits, Systems and Signal processing, Vol. 3, pp 271-300, 1990

[7] C. Wang and J. A. Cadzow, "Direction-finding with sensor gain, phase and location uncertainty", Proc. ICASSP-91, pp. 1429-1432, 1991

[8] G. H. Golub and C. F. Van Loan, "Matrix computation", The Johns Hopkins University Press : London, 1990

[9] Y. Zhou and P. Yip, "A Self-Calibration DOA Estimation Algorithm for Cyclostationary Source Signals", to be submitted for publication.

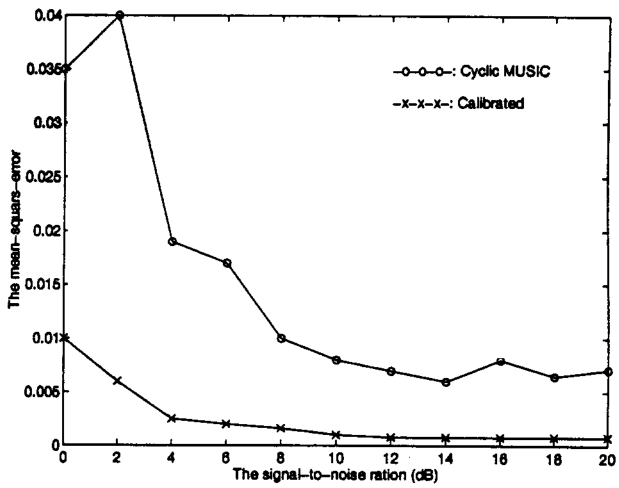

Figure 1: The variation of MSE of the estimates via SNR