ROBUST BEARING ESTIMATION IN THE PRESENCE OF DIRECTION-DEPENDENT MODELLING ERRORS: IDENTIFIABILITY AND TREATMENT

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ABSTRACT

This paper presents identifiability and treatment relative to bearing estimation in presence of modelling errors. It introduces the general case of direction-dependent modelling errors. The classical direction-independent case is only a particular case which takes into account a prior knowledge. This general case introduces the important issue of simultaneous sources and perturbation identifiability which is analysed in this paper. A new self-calibration technique based on MUSIC algorithm and able to treat direction-dependent errors is also proposed. For regularization purpose, a "cost term" is introduced in this algorithm. It confers good robustness to algorithm which usually fails in presence of great gap between model and reality. After reduction of the new multidimensional "increased function", values of azimuths are easily obtained on a monodimensional spectrum. Some simulations support results and verify improvements expected in theory.

1. INTRODUCTION

The direction finding algorithms based on eigenstructure methods [1] or maximum likelihood methods have good performances if the underlying model is correct. Recent studies have shown that parametric estimation techniques are quite sensitive to the array response knowledge accuracy [2]. Their performances decrease drastically with a bad knowledge on the array manifold and can even fail. It is therefore necessary to calibrate the array or to use a self-calibration algorithm. In the literature several authors have proposed self-calibration methods.

We see a particular interest in the self-calibration method of MUSIC proposed by B.Friedlander [3] because of its simplicity at implementation and its relative robustness. In certain application fields, for example in radar, arrays uncertainties are direction-dependent which make Friedlander's self-calibration method fail. In this paper we propose a self-calibration algorithm based on MUSIC, able to deal with direction-dependent errors. This paper introduces the important issue of simultaneous sources and perturbations identifiability.
In section 3, we introduce the parametric modelisation of the perturbation. Section 4 deals with the important problem of the identifiability of the system, which unknowns are directions of arrival and nuisance parameter vectors. In section 5, we propose a robust self-calibration algorithm able to treat direction-dependent errors. Section 6 presents simulations where position errors are treated like direction-dependent phase errors. Section 7 includes some conclusions.

2. NOTATIONS

N	Number of sources.
M	Number of sensors.
$x(t)=[x_1(t),\dots,x_M(t)]^T$	Received signal.
θ	Vector of azimuths.
$A=[a(\theta_1) \quad a(\theta_2) \quad \dots \quad a(\theta_N)]$	True steering vectors matrix.
ρ_n	The steering vectors are known to within this coefficient.
$\{a_0(\theta)\}$	Theoretical array manifold. $a(\theta)=\Gamma a_0(\theta)$
$C(\theta)$	Factorisation of $a(\theta)$, $a(\theta)=C(\theta)\delta$
$\delta=\operatorname{diag}\{\Gamma\}$	Vector of the nuisance parameter.
$p=\dim(\delta)$	
δ_0	Nominal value of the nuisance parameter.
$s_n(t)$	Vector of radiated signals.
$n(t)$	Vector of noise.
$\hat{U}$	Estimated eigenvectors matrix of the signals covariance matrix.
$\hat{\Pi}_n=\hat{U}_n\hat{U}_n^H$	Estimated projection matrix on the noise subspace.
$w=[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$	
(x_i,y_i)	Position of the ith sensor
k	Iteration number.
Ψ	Covariance matrix of the complex gains.
$\mathcal{S}\{A\}$	Space generated by column vectors from A.

Subscript RM refers to Robust MUSIC.

3. PARAMETRIC MODELISATION OF THE PERTURBATIONS

Consider N radiating narrowband sources observed by an arbitrary array of M sensors. Let $x(t)$ denotes the true vector of the signals at the array element outputs, given by:

$$x(t)=\sum_{n=1}^{N} a(\theta_n,\delta)s(t)+n(t)=A(\theta,\delta)s(t)+n(t) \qquad (1)$$

where $s(t)$ is the vector of radiated signals, $n(t)$ is the noise vector. The element (i,j) of matrix A is the true transfer function between the j^{th} source and the i^{th} sensor. Its parameters are the vector of azimuths θ and the vector of nuisance parameter which parametrises deviation to nominal model δ_0. δ can be direction-independent or direction-

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dependent δ_θ [4]. We assume that during the data collection, interval δ is fixed. Our model is restricted to the case of a linear relation between the true steering vector $a(\theta,\delta)$ and the theoretical one $a_0(\theta)$. Vector $a(\theta,\delta)$ may be expressed:

$$a(\theta,\delta)=\Gamma(\delta)a_0(\theta)=C(\theta)\delta \quad (2)$$

with known $\Gamma(\delta)$ so that $C(\theta)$ is a matrix constituted by known linear combinations of elements of $a_0(\theta)$.

In the case of **direction-dependent** nuisance parameter vector δ_{θ_n} the received signal is:

$$x(t)=\sum_{n=1}^{N}C(\theta_n)s_n(t)\delta_{\theta_n}+n(t) \quad (3)$$

δ_{θ_n} may include wavefront distortion specific of source n or unknown direction-dependent variation δ_θ of the nuisance parameter δ. The unknowns of this problem are $\delta_{\theta_1},\ldots,\delta_{\theta_N}$ and the azimuths $\theta_1,\ldots,\theta_N$.

4. SIMULTANEOUS BEARING AND ERROR PARAMETER IDENTIFIABILITY

Let's note R_x the covariance matrix $R_x=U_s\Lambda_sU_s^H+U_n\Lambda_nU_n^H$.

Identifiability is recovered if the knowledge of the signal subspace $\mathcal{U}_s=\mathcal{U}(u_1,\ldots,u_N)$ and the model of the steering vector $a(\theta,\delta)$ or $a(\theta,\delta_\theta)$ determine the D.O.A. and the errors on sensors without ambiguity.

4.1. A first necessary condition obtained by comparing the number of equations and the number of unknowns

According to the model (1) we obtain N equations, for vectors with M complex components. Therefore the system is constituted of $2NM$ real scalar equations. The steering vectors are known but for an arbitrary multiplicative complex constant ρ_n, the same is then true for δ. An additional constraint has to be introduced. We obtain the system:

$$\begin{cases}\rho_n a(\theta_n,\delta_{\theta_n})=C(\theta_n)\delta_{\theta_n} & n=1,\ldots,N\\ w^T\delta_{\theta_n}=1 & \text{additional constraint on } \delta\end{cases} \quad (4)$$

4.1.1. Case of direction-dependent errors

It is necessary to determine $2pN$ (elements of vector δ_θ) + $2N$ (coefficients ρ_n) + N (azimuths θ_n) real parameters to determine. Thus there are $2pN+3N$ unknowns and $2MN+2$ equations, so the identifiability condition is $2MN+2\geq 2pN+3N$. In conclusion, to localise sources with direction-dependent nuisance parameters it is necessary that:

$$\boxed{p<M} \quad (5)$$

4.1.2 Case of direction-independent errors

It is necessary to determine $2N$ (due to the coefficients $\{\rho_n\}, n=1,..,N$) + $2p$ (due to the elements of vector δ) + N (due to the azimuths θ_n) real parameters with $2MN+2$ equations. The identifiability condition is:

$$\boxed{p\leq MN-\tfrac{3}{2}N+1} \quad (6)$$

If $p=M$, the condition is $N>1$. Thus the problem has a unique solution if there are two sources or more. (Case of Friedlander)

If $p<M$ the problem can be solved whatever the number of sources may be.

4.2 Identification of the steering vectors with the signal subspace in the presence of perturbations

For methods based on second order statistics, MUSIC in particular, the span $\mathcal{S}\{a(\theta_1,\delta)\ldots a(\theta_N,\delta)\}=\mathcal{S}\{\theta,\delta\}$ is identified as a whole. The general identifiabity condition for θ and δ is the non ambiguity of $\mathcal{S}\{\theta,\delta\}$, i.e.:

$$\mathcal{S}\{\theta,\delta\}=\mathcal{S}\{\theta',\delta'\}\Leftrightarrow \theta=\theta' \text{ and } \delta=\delta' \quad (7)$$

Another possible formulation is as follows: consider Π_n projector matrix on the noise subspace for $\mathcal{S}\{\theta_0,\delta_0\}$:

$$\Pi_n a(\theta,\delta)=0 \text{ has the unique solution } \begin{cases}\theta=(\theta_1,\ldots,\theta_N)\\ \delta=\delta_0\end{cases} \quad (8)$$

We can now propose a local condition for identifiability. Let's note a variation da_k of the steering vector compatible with the model (2). If the system is identifiable, the perturbed vector a_k+da_k must not be confined in the signal subspace. The perturbation must verify the following condition:

$$\Pi_n da_k=0 \Leftrightarrow da_k=0 \quad (k=1,\ldots,N) \quad (9)$$

Let's analyse this relation for the two types of errors.

4.2.2 Case of direction-dependent errors

In the case of direction-dependent perturbations:

$$da_k=\frac{\partial a}{\partial\theta_k}d\theta_k+\sum_{i=1}^{p}\frac{\partial a}{\partial(\delta_{\theta_k})_i}d(\delta_{\theta_k})_i=Q_k d\gamma \quad (10)$$

$$\text{where } Q_{k(Mxp+1)}=\left[\frac{\partial a}{\partial\theta_k},\frac{\partial a}{\partial(\delta_{\theta_k})_1},\ldots,\frac{\partial a}{\partial(\delta_{\theta_k})_p}\right]$$

Assuming the observation reported on the eigenvalue basis, $\Pi_n Q_k d\gamma$ has only M-N non null components. By keeping only this M-N non null components Π_n is a matrix of size $(M\text{-}N)\text{x}M$. Therefore:

$$\Pi_n da_k=[\Pi_n Q_k]_{(M-Nxp+1)}d\gamma \quad (11)$$

where $\text{rank}(\Pi_n Q_k)\leq \min(p+1,M-N)$.

To obtain $d\gamma=0$ as the unique solution of equation (11), $\Pi_n Q_k$ must be ranked $p+1$. Thus we deduce the necessary condition:

$$p\leq M-N-1 \quad (12)$$

Remark that as we choose the first component of δ and a equal to 1, rank of $\Pi_n Q_k$ becomes p.

Therefore the condition becomes:

$$\boxed{p \le M - N} \tag{13}$$

Only a fine physical analysis of the perturbations can allow "reduced" modelisation (2) with a small dimension of vector δ. For example, for symmetry reasons, certain operational constraints can provide similar perturbations on several sensors. Another situation well adapted to modelisation (2) is the case of direction dependent coupling between sensors. Let's consider a linear array in which sensors are only coupled with neighbours with a coupling coefficient $\alpha(\theta)$.

$$\boldsymbol{a}(\theta) = \boldsymbol{Z}\boldsymbol{a}_0(\theta) \quad \text{with } \boldsymbol{Z} = \begin{bmatrix} 1 & \alpha(\theta) & 0 & \ldots & 0 \\ \alpha(\theta) & 1 & & & \\ 0 & & & & 0 \\ & & & 1 & \alpha(\theta) \\ 0 & \ldots & 0 & \alpha(\theta) & 1 \end{bmatrix} \tag{14}$$

In this way $\boldsymbol{a}(\theta)$ can be written according to eq. (2) with:

$$C(\theta) = \begin{bmatrix} 1 & a_{02} \\ a_{02} & a_{01} + a_{03} \\ \cdot & \cdot \\ a_{0M-1} & a_{0M-2} + a_{0M} \\ a_{0M} & a_{0M-1} \end{bmatrix} \quad \text{and} \quad \delta = \begin{bmatrix} 1 \\ \alpha(\theta) \end{bmatrix} \tag{15}$$

In the above example the dimension of the nuisance parameter vector δ is $p=2$.

4.2.2 Case of direction-independent errors

The system:

$$\begin{cases} \Pi_n Q_1 \begin{bmatrix} d\theta_1 \\ d\delta \end{bmatrix} = 0 \\ \Pi_n Q_2 \begin{bmatrix} d\theta_2 \\ d\delta \end{bmatrix} = 0 \\ \ldots\ldots\ldots\ldots \\ \Pi_n Q_N \begin{bmatrix} d\theta_N \\ d\delta \end{bmatrix} = 0 \end{cases} \Leftrightarrow \underbrace{\begin{bmatrix} \Pi_n \frac{\partial \boldsymbol{a}}{\partial \theta_1} & 0 & \ldots & 0 \\ 0 & \Pi_n \frac{\partial \boldsymbol{a}}{\partial \theta_2} & 0 & 0 \\ \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & 0 & \Pi_n \frac{\partial \boldsymbol{a}}{\partial \theta_N} \end{bmatrix}}_{(N(M-N)\mathrm{x}N)} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ \ldots \\ d\theta_N \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} \Pi_n \frac{\partial \boldsymbol{a}_1}{\partial \delta_1} & \ldots & \Pi_n \frac{\partial \boldsymbol{a}_1}{\partial \delta_N} \\ \ldots & \ldots & \ldots \\ \Pi_n \frac{\partial \boldsymbol{a}_N}{\partial \delta_1} & \ldots & \Pi_n \frac{\partial \boldsymbol{a}_N}{\partial \delta_N} \end{bmatrix}}_{(N(M-N)\mathrm{x}p)} d\delta = 0 \tag{16}$$

must have an unique solution. It is of the form:

$$\Pi_n Q_{(N(M-N)\mathrm{x}N+p)} d\gamma = 0 \tag{17}$$

where $\mathrm{rank}(\Pi_n Q) \le \min(p+N, N(M-N))$.

To obtain $d\gamma = 0$ as the unique solution of equation (17) $\Pi_n Q$ must be ranked $p+N$. Thus we deduce the necessary condition:

$$p \le N(M-N-1) \tag{18}$$

The same remark as in the previous section provides the condition:

$$\boxed{p \le N(M-N)} \tag{19}$$

We conclude that these last two necessary identifiability conditions (13), (19) are far more restrictive that those obtained in the previous section (5), (6).

Note that identifiability can be recovered by introduction of a prior knowledge of the array model.

We know that resolution is closely connected to the Cramer Rao Bound (CRB). Obviously the considered sources must verify this condition for correct identifiability. On an other side, the nominal array is supposed non ambiguous in this work for simplicity 's sake

Remark: Usually, the research of "optimal array configuration" is made by a CRB approach. The robustness to ambiguity and non identifiability problems in the presence of array manifold perturbations can open the way of a more practical approach of array design.

Let's note $\Delta\theta$ the span resulting from variations of array manifold versus θ and $\Delta\delta$ the span resulting from variations of array manifold versus δ. The condition of non ambiguity is

$$\Delta\theta \cap \Delta\delta = \varnothing \tag{20}$$

5. TREATMENT OF DIRECTION-DEPENDENT ERRORS

The aim of this section is to propose an efficient method for localizing several emitters with direction-dependent perturbations δ_θ on sensors. A first multidimensional approach consists in applying MUSIC with a steering vector dependent of all the parameters.

$$F(\theta, \delta_\theta) = \boldsymbol{a}(\theta, \delta_\theta)^H \hat{\Pi}_n \boldsymbol{a}(\theta, \delta_\theta) \tag{21}$$

where $\hat{\Pi}_n$ is the estimate of the noise projector. Minimisation is made independently on δ and θ. Let $\delta_\theta(\theta)$ be :

$$\delta_\theta(\theta) = \mathrm{argmin}_\delta \{F(\delta_\theta, \theta)\} \tag{22}$$

minimisation being made with suitable constraint to avoid trivial solution $\delta = 0$. Then a modified MUSIC can be proposed by searching the local minimum of:

$$F_M = F(\theta, \delta_\theta(\theta)) = \boldsymbol{a}(\theta, \delta_\theta(\theta))^H \hat{\Pi}_n \boldsymbol{a}(\theta, \delta_\theta(\theta)) \tag{23}$$

We further propose to introduce a prior knowledge (δ_θ close to δ_0 in Ψ norm) by a penalty function (see eq. 24). Thus we assume the nuisance parameter vector be extracted from a Gaussian law, $\delta_\theta \in \boldsymbol{N}(\delta_0, \Psi)$. The expression of robust MUSIC with the notations introduced previously becomes:

$$F_R(\theta, \delta_\theta) = F(\theta, \delta_\theta) + (\delta_\theta - \delta_0)^H \Psi^{-1} (\delta_\theta - \delta_0) \tag{24}$$

where Ψ is a definite positive matrix. Minimisation with respect to δ_θ is easy and gives $F_{RM}(\theta) = F_R(\delta_\theta(\theta), \theta)$

with:

$$F_R(\delta_\theta(\theta),\theta) = \delta_0{}^H \Psi^{-1}\left(\Psi - \left(a(\theta,\delta_\theta(\theta))^H \hat{\Pi}_n\, a(\theta,\delta_\theta(\theta)) + \Psi^{-1}\right)^{-1}\right)\Psi^{-1}\delta_0 \quad (25)$$

Arguments of the minimum provides estimates $\hat{\theta}_n$ and corresponding values $\hat{\delta}_{\theta_n}$. In order to increase the performances of this algorithm it is implemented recursively. The procedure is then constituted by two steps, first the directions of arrival are determinated by minimisation of F_{RM}. The values of $\delta_\theta(\theta)$ are then calculated and in (24) nominal value δ_0 is updated by the new values.

If δ is assumed independent of the azimuths, the different values of δ can be averaged. This algorithm also gives, of course, good results for direction-independent uncertainties.

Our algorithm is able to treat location, phase and gain errors, even when all errors are simultaneously present. In the literature it would be called a global self-calibration method. All these nuisance parameters are of course treated like direction-dependent errors.

6. SIMULATION RESULTS

To examine the performances of the above estimation algorithm, a scenario involving a six element uniform circular array and three sources was simulated. The three narrowband sources are located in the far field of the array at the directions: -30°, 0° and 35°. An additive uncorrelated noise is injected with sensor noise SNR 30dB for the three sources. 100 snapshots are used to estimate the correlation matrix.

The positions of sensors 5 and 6 are known with a bad accuracy. For example these last two sensors are far from the boat fastener and submitted to the wind. The position error on sensor 5 is $(\Delta x_5, \Delta y_5)$, on sensor 6 $(\Delta x_6, \Delta y_6)$. This position errors can be seen as direction-dependent phase errors.

The nuisance parameter vector is constituted of p=3 elements. Factorisation (2) can be applied, so the obtained matrix $C(\theta)$ and vector δ_θ are:

$$C(\theta) = \begin{bmatrix} a_1(\theta) & 0 & 0 \\ a_2(\theta) & 0 & 0 \\ a_3(\theta) & 0 & 0 \\ a_4(\theta) & 0 & 0 \\ 0 & a_5(\theta) & 0 \\ 0 & 0 & a_6(\theta) \end{bmatrix} \text{ and } \delta_\theta = \begin{bmatrix} 1 \\ \delta_1 \\ \delta_2 \end{bmatrix} \quad (26)$$

with:

$$\delta_i = e^{j2\pi(\Delta x_i \cos(\theta) + \Delta y_i \sin(\theta))} \quad (27)$$

In order to clarify the figures we draw the inverse of the functions, so that the minima are now peaks. Fig. 1 depicts the result of Friedlander's self-calibration algorithm. This algorithm does not take into account the direction-dependence of the phase error. It diverges. Fig. 2 depicts the result of the new algorithm, who takes into account the direction-dependence. In a few iterations it converges with accuracy to the azimuths. This result is not particular to the choices made in this simulation. We obtain the same result for a great number of simulations.

The perturbations in this simulation are severe. For small perturbations, this algorithm doesn't need iterations.

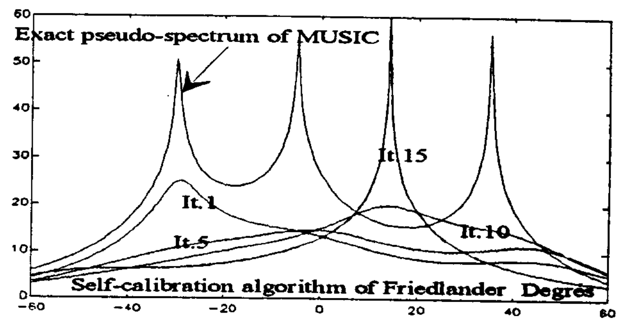

Fig. 1: Result of Friedlander's self-calibration algorithm in case of direction dependent errors. Obviously it diverges.

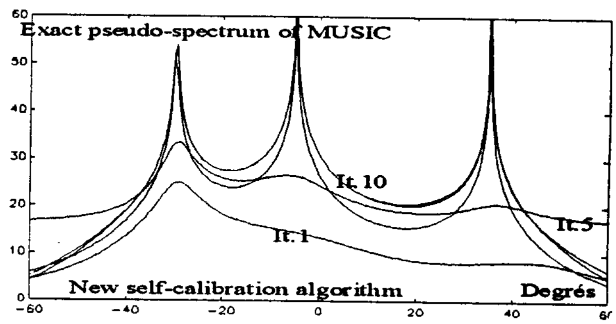

Fig. 2: Result of the new algorithm. It converges in a few iterations.

7. CONCLUSION AND PERSPECTIVES

Practical experimentations have shown that errors on sensors are often direction-dependent. The usual self-calibration algorithms presented in the literature fail in such context. In this paper the identifiability conditions for such a system, when nuisance parameter vectors are direction-dependent, have been studied. We propose a self-calibration algorithm who takes into account the direction-dependence of the sensor errors. Simulations illustrate the good results provided. The method is robust and is easy to implement.

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