MMSE ANTENNA DIVERSITY EQUALIZATION OF A JAMMED FREQUENCY-SELECTIVE FADING CHANNEL

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Abstract

For digital radiocommunications systems operating on a jammed frequency-selective fading channel, the receiver performance can be improved by using the joint antenna diversity and equalization techniques to combat both time- and frequency-selective fades and jammers effects. The optimum, in the sense of minimum mean-squared error (MMSE), structure of the linear equalizer (LE) and the decision feedback equalizer (DFE) for coherent receiver antenna diversity has been derived for an unjammed environment [1]. In the present paper, the structure of the MMSE antenna diversity equalizer is determined for a jammed multipath channel. We show that is composed of the multidimensional matched filter (MMF) followed by a symbol-rate sampler and the MMSE LE or DFE equalizer for single antenna receivers.

1. Introduction

In digital radiocommunications over a frequency-selective fading channel, adaptive equalization is required at the demodulator to mitigate the effects of intersymbol interference resulting from the time-variant multipath propagation of the signal through the channel [2].

The degradation of the receiver performance, due to time-variant multipaths (fading) and jammed environment, can be reduced by using antenna diversity both to combat time- and frequency-selective fades and to reject undesired signals [3,4]. So, in the present paper, adaptive array is considered as a technique of antenna diversity [5].

The maximum likelihood sequence estimation (MLSE) equalizer for coherent receiver antenna diversity (CRAD) is an optimal technique of equalization in the sense that it minimizes the probability of a sequence error. We have shown that it can be realized by the multidimensional matched filter (MMF), followed by a symbol-rate sampler and the MLSE-based modified Viterbi algorithm for single antenna receivers [6,7]. This equivalent structure, called MLSE antenna diversity equalizer, is valid not only for an additive white gaussian noise (AWGN) multipath channel but also for a jammed one. Although the MLSE antenna diversity equalizer is applied to GSM systems in [6], it can be also used in other digital cellular systems such as American Digital Cellular systems.

If the multipath spread is large with respect to the symbol interval, for example on high-rate (2400 bauds) HF links, simpler equalization techniques as linear equalizer (LE) and decision feedback equalizer (DFE) have been proposed to limit the receiver computational complexity [8,9]. The optimum, in the sense of minimum mean-squared error (MMSE), structure of the LE and the DFE equalizer for CRAD has been derived for an unjammed environment [1]. In the present paper, the structure of the MMSE antenna diversity equalizer is determined for a jammed multipath channel. We show that is composed of the MMF followed by a symbol-rate sampler and the MMSE LE or DFE equalizer for single antenna receivers.

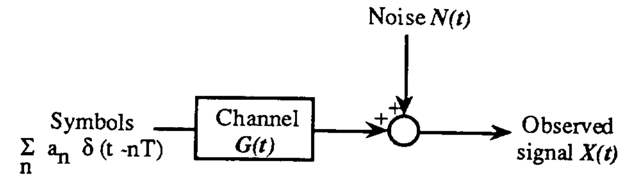

Figure 1. Multidimensional Baseband System Model

In the following section, the system model is described. The mean-squared error (MSE) for CRAD is expressed for a stationary multidimensional noise in Section 3. It is called multidimensional MSE. Then, the structure of the announced MMSE antenna diversity equalizer is derived in Section 4. Finally, a MMF estimator is proposed in Section 5.

2. System Model

The communication system is described as follows. With the equivalent baseband notation, it is well-known that a quadrature amplitude modulation (QAM) signal within a finite time interval I can be defined as:

$$u(t) = \sum_{n=1}^{N} a_n \, v(t-nT), \ t \in I, \tag{1}$$

where a_n is a transmitted symbol, T is the symbol interval, and v(t) denotes the transmitted signal pulse. The symbol sequence (a_n) is assumed to be a white sequence: $E[a_n a_{n-p}^*] = p_a \delta_p$, where E denotes expectation, and δ_p is the Kronecker delta.

The multidimensional signal $X(t)$ received on the K antennas can be displayed with eq.(1) as follows:

$$X(t) = \sum_{n=1}^{N} a_n \, G(t-nT) + N(t), \ t \in I, \tag{2}$$

where the vector $G(t)$ is the impulse response of the equivalent baseband multidimensional channel, and $N(t)$ denotes the vector of the additive noises $N_k(t)$ observed on the K antennas (Figure 1). The k^{th} element of the vector $G(t)$ is the impulse response of the channel that is referring to the k^{th} antenna. Channel includes the transmitted signal pulse v(t), the modulator, the transmission medium, the receiver filter and the demodulator. Noise includes an additive white (or colored) gaussian noise and all undesired signals. Noise is assumed to be a stationary multidimensional signal with the power spectral density equal to the matrix $R(f)$.

3. Multidimensional Mean-Squared Error

The mean-squared error (MSE) between the observed equalizer output signal and the desired one is expressed for a stationary multidimensional noise in this section. The desired output signal is composed of the transmitted symbols.

The direct-form structure of the DFE is illustrated in Figure 2. It consists of a feedforward filter (FFF) and a feedback filter (FBF). The former is adjusted to reduce the intersymbol interference of future symbols while the latter substracts the interference of past symbols. The past symbols $\hat{a}_n$ may be known (training sequence) or provided by the decision device. FFF and FBF are infinite-length filters. Their impulse responses are respectively $W^{\dagger}(-t)$ and b_n, where † denotes the conjugate transposition. The linear equalizer is deduced from this structure by removing the FBF.

The equalizer output error is defined by

$$e_n = z_n - a_n \tag{3}$$

where z_n is the DFE output signal given by

$$z_n = \int_I W^{\dagger}(t)\, X(t+nT)\, dt - \sum_p b_p \hat{a}_{n-p} \,. \tag{4}$$

For the sake of mathematical tractability, we make the customary assumption that the past decisions are correct, that means:

$$\hat{a}_{n-p} = a_{n-p}\,, \qquad p > 0\,. \tag{5}$$

Substituting the received signal $X(t)$ by its expression, given in eq.(2), into eq.(4) and applying the assumption expressed by eq.(5) in the result, the error e_n defined by eq.(3) becomes

$$e_n = \sum_p \left(h_p - \delta_p - b_p\right) a_{n-p} + r_n \tag{6}$$

where the complex coefficient h_p and the output noise sample r_n are respectively defined by

$$h_p = \int_I W^{\dagger}(t)\, G(t+pT)\, dt\,. \tag{7}$$

and

$$r_n = \int_I W^{\dagger}(t)\, N(t+nT)\, dt\,. \tag{8}$$

For a white sequence (a_n), the power spectral density of the equalizer output error e_n, which is established in eq.(6), can be

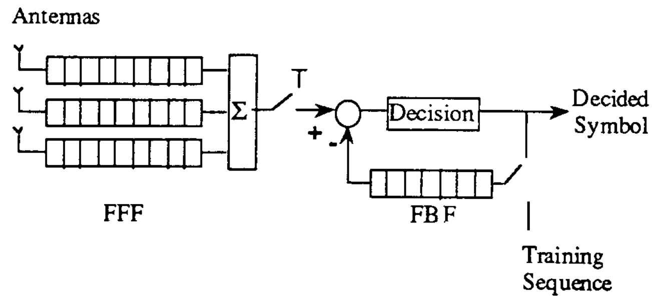

Figure 2. Direct-Form Antenna Diversity DFE Equalizer

displayed as follows:

$$p_e(f) = p_a \left| h(f) - 1 - b(f) \right|^2 + p_r(f) \tag{9}$$

where h(f) is the Fourier transform of h_p defined by eq.(7):

$$h(f) = \frac{1}{T} \sum_p W^{\dagger}(f-\tfrac{p}{T})\, G(f-\tfrac{p}{T})\,, \tag{10}$$

b(f) is the frequency response of the FBF, and $p_r(f)$ is the power spectral density of the output noise r_n expressed by eq.(8):

$$p_r(f) = \frac{1}{T} \sum_p W^{\dagger}(f-\tfrac{p}{T})\, R(f-\tfrac{p}{T}) W(f-\tfrac{p}{T}) \tag{11}$$

The multidimensional MSE is then given with eq.(9) by

$$MSE = E\left[|e_n|^2\right] = T \int_{-1/2T}^{+1/2T} p_e(f)\, df\,. \tag{12}$$

4. MMSE Antenna Diversity Equalizer

The structure of the MMSE antenna diversity equalizer of a jammed frequency-selective fading channel is presented in this section.

4.1. Minimization of the MSE for a Given FBF

Using eq.(9) in eq.(12) and minimizing first with respect to the FFF for a given FBF, it can be shown that :

$$W(f) = \frac{p_a\left[1 + b^*(f)\right]}{p_a\, \gamma(f) + 1} R^{-1}(f) G(f) \tag{13}$$

where $\gamma(f)$ is defined by

$$\gamma(f) = \frac{1}{T} \sum_p G^{\dagger}(f-\tfrac{p}{T})\, R^{-1}(f-\tfrac{p}{T}) G(f-\tfrac{p}{T})\,. \tag{14}$$

Substituting the expression of $W(f)$, given by eq.(13), into eq.(10) and eq.(11) and the results into eq.(9), the multidimensional MSE, established in eq.(12), becomes

$$MSE = T \int_{-1/2T}^{+1/2T} \frac{p_a\left|1 + b(f)\right|^2}{p_a\, \gamma(f) + 1}\, df\,. \tag{15}$$

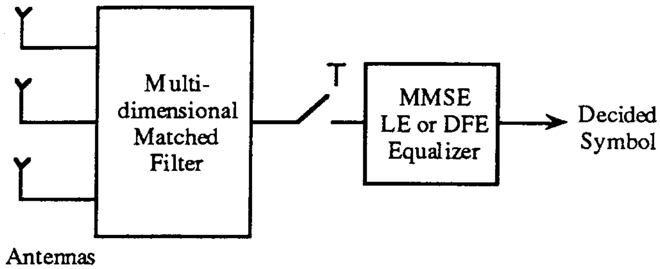

Figure 3. MMSE Antenna Diversity Equalizer

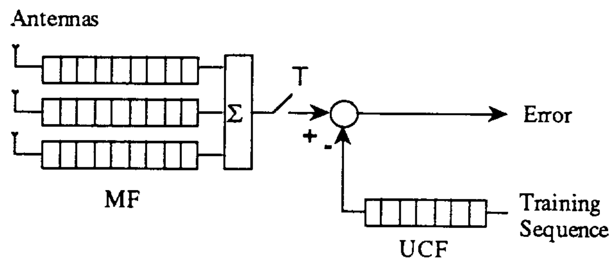

Figure 4. MMF Estimator

4.2. MMSE Antenna Diversity Linear Equalizer

The MMSE antenna diversity linear equalizer (LE) is deduced from eq.(13) when b(f)=0:

$$W_{LE}(f) = \frac{p_a}{p_a\,\gamma(f) + 1} R^{-1}(f)G(f) . \qquad (16)$$

For a LE, the multidimensional MSE, given by eq.(15), becomes

$$MSE_{LE} = T \int_{-1/2T}^{+1/2T} \frac{p_a}{p_a\,\gamma(f) + 1}\, df . \qquad (17)$$

Observing eq.(16), it can be noted that the FFF is composed of the Multidimensional Matched Filter (MMF) [6]:

$$W_{MMF}(f) = R^{-1}(f)\, G(f) , \qquad (18)$$

followed by a symbol-rate sampler and a T-spaced tapped delay line filter. The symbol-rate sampled MMF output signal y_n is deduced from eq.(2) and eq.(18):

$$y_n = \sum_p \gamma_p\, a_{n-p} + \varepsilon_n \qquad (19)$$

where ε_n is the noise component with autocorrelation function: $E[\varepsilon_n \varepsilon_{n-p}^*] = \gamma_p$, and γ_p is the inverse Fourier transform of $\gamma(f)$ defined by eq.(14). Thus, the T-spaced delay line filter is nothing more than the MMSE linear equalizer for single antenna receivers [10] (Figure 3).

4.3. MMSE Antenna Diversity DFE Equalizer

The FBF that minimizes the Toeplitz form of eq.(15) is computed from

$$1 + b(f) = \frac{s(f)}{s_0} , \qquad (20)$$

where s_n is the impulse response of the minimum-phase filter defined by the frequency response s(f) which verifies [10]

$$|s(f)|^2 = \frac{p_a\,\gamma(f) + 1}{p_a} . \qquad (21)$$

Hence, the FFF given by eq.(13) becomes with eq.(20) and eq.(21)

$$W_{DFE}(f) = \frac{MSE_{DFE}}{1 + b(f)} R^{-1}(f)G(f) \qquad (22)$$

where the MSE expressed in eq.(15) is

$$MSE_{DFE} = \frac{1}{|s_0|^2} . \qquad (23)$$

Observing eq.(22), it can be noted that the FFF is composed of the Multidimensional Matched Filter (MMF), defined by eq.(18), followed by a symbol-rate sampler and an anticausal T-spaced tapped delay line filter. Furthermore, it can be observed that the anticausal T-spaced delay line filter and the FBF that are obtained are respectively the FFF and the FBF of the MMSE DFE for single antenna receivers [10] (Figure 3).

5. MMF Estimator

As defined by eq.(18), the MMF depends on the channel frequency response **G**(f) and the noise power spectral density **R**(f). In practice, these channel and noise characteristics are unknown, and consequently MMF must be estimated. In this section, we show how the multidimensional filter (MF) and the uncausal T-spaced tapped filter (UCF) which appear in Figure 4 can be used to estimate the MMF.

A matched filter estimator is proposed by G. Ungerboeck to realize the MLSE equalizer for single antenna receivers [7]. The MMF estimator presented below is an extended version of the matched filter estimator for single antenna receivers to CRAD.

The error between the symbol-rate sampled MF output signal and the UCF output signal is given by

$$e'_n = \int_I W^\dagger(t)\, X(t+nT)\, dt - \sum_p i_p a_{n-p} \qquad (24)$$

where $W^\dagger(-t)$ and i_n are respectively the MF impulse response and the UCF impulse response. The UCF is submitted to the following constraint:

$$i_0 = 1 . \qquad (25)$$

Substituting the received signal $X(t)$ by its expression, given in eq.(2), into eq.(24) and applying the constraint expressed by eq.(25) in the result, the error e'_n becomes

$$e'_n = e_n \qquad (26)$$

where e_n is the error given by eq.(6) and the T-spaced tapped filter b_n is an uncausal filter which is defined by

$$i_n = \delta_n + b_n . \qquad (27)$$

We can deduce from eq.(26) and eq.(27) that the MF which minimizes the mean-squared error (MSE'):

$$MSE' = E\left[\left|e'_n\right|^2\right] \tag{28}$$

under the constraint expressed in eq.(25) is the multidimensional filter given by eq.(13):

$$W(f) = \frac{p_a\, i^*(f)}{p_a\, \gamma(f) + 1}\, R^{-1}(f) G(f) \tag{29}$$

where $\gamma(f)$ is defined by eq.(14). Furthermore, using eq.(27) in eq.(15), the MSE' defined by eq.(28) becomes

$$MSE' = T \int_{-1/2T}^{+1/2T} \frac{p_a\,|i(f)|^2}{p_a\, \gamma(f) + 1}\, df\,. \tag{30}$$

It can be straightforward shown by applying the Cauchy-Schwarz inequality in eq.(30) that the UCF which minimizes the MSE' under the constraint expressed in eq.(25) is

$$i(f) = MMSE' \frac{p_a\, \gamma(f) + 1}{p_a} \tag{31}$$

where MMSE' is the minimum value of the MSE':

$$MMSE' = \frac{p_a}{p_a\, \gamma_0 + 1}\,. \tag{32}$$

Thus, substituting the UCF by its expression given by eq.(31) into eq.(29) yields

$$W(f) = MMSE'\, W_{MMF}(f)\,. \tag{33}$$

where $W_{MMF}(f)$ is given by eq.(18) and defines the MMF. Hence, observing eq.(31) and eq.(33), it can be noted that the MMF and the coefficients γ_p which appear in the expression of the MMF output signal, given by eq.(19), can be computed from the optimum MF and the optimum UCF.

As defined in Section 2, the impulse response $G(t)$ of the multidimensional channel includes the transmitted signal pulse v(t) which is band-limited over [-1/T,+1/T]. Hence, without loss of optimality, MMF can be realized by a T/2-spaced filter an antenna, once the received signals are low-pass filtered. Thus, the MMF estimator can be a finite-duration impulse response (FIR) filter with adjustable coefficients. It is deduced from eq.(33) that adaptive algorithms, such as the stochastic gradient (LMS) algorithm or the recursive least squares (RLS) algorithm, can accomplish the adjustment of the coefficients of the MMF estimator.

6. Conclusion

As for the MLSE antenna diversity equalizer presented in paper [6], we have shown in the present paper that, once the MMF is performed, only an usual technique of equalization for single antenna receivers is required to complete the MMSE antenna diversity equalizer.

Furthermore, an adaptive adjustment of the coefficients of the MMF estimator has been proposed in the paper. It can be used to realize the MLSE antenna diversity equalizer.

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