CLASSIFICATION OF PSK SIGNALS USING THE DFT OF PHASE HISTOGRAM

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ABSTRACT

A method is presented for classifying multi-level PSK signals in the presence of additive white Gaussian noise (AGWN). The technique is based on the Discrete Fourier Transform (DFT) of a phase histogram. The probability of correct classification is given and it is found that the technique performs well at low SNR. The benefits of this technique are that it is simple to implement and requires no prior knowledge of the SNR of the signal for the classification.

1. INTRODUCTION

The automatic classification of modulation type of a communications signal finds applications in the fields of Electronic Surveillance, spectrum management and signal interception where it is an important sorting parameter in a complicated problem. There are also applications in modulation diverse communication systems in which the system may receive a variety of modulation types. The initial trend was to treat modulation recognition as a non-deterministic pattern recognition problem [1] which works well at a high SNR, but is poor at low SNR. Consequently some research effort has been applied to deterministic forms of pattern recognition [2][3][4]. Some of these methods rely upon knowledge of the SNR of the signal for decision parameters, but this is difficult to obtain in a true signal environment. The method presented shows a technique for classifying PSK signals without using SNR information, and the performance is found to work well at low SNR.

Phase samples of the incoming signal are collected and placed in a histogram. After a sufficient number of these have been collected, the histogram is passed through a DFT in order to exploit the periodicity of the histogram. The DFT bin numbers correspond to the number of levels of each of the PSK types considered, and are converted into magnitude squared where the largest of these identifies the PSK type (figure 1).

The theoretical development given below shows that the number of bins should be a power of two to avoid the effects of spectral leakage, and a closed form expression for the probability of misclassification is derived. It is found that the number of histogram bins used in the process does not affect the classification procedure when aliasing effects are made insignificant. Finally the error probability for this method is compared to that of the method of statistical moments, and it is found to compare favourably.

2. THEORETICAL DEVELOPMENT

A time frame of the signal of interest is captured and digitised. This signal is then converted into an analytic form, the carrier is removed by complex mixing and the phase samples are extracted. It is assumed that the carrier frequency component is accurately known. The received signal is the sum of an ideal PSK signal and AGWN. The phase p.d.f. of multi-level PSK in AGWN may be developed from a carrier wave (CW) p.d.f. $f(\phi)$, which is described in Fourier series form as [5]:

$$f(\phi)=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}b_n\cos(n\phi) \qquad -\pi<\phi\leq\pi \tag{1}$$

When M level PSK is considered, the p.d.f. $f_M(\phi)$ becomes :

$$f_M(\phi)=\frac{1}{M}\sum_{k=0}^{M-1}f\left(\phi+\frac{2\pi(k+0.5)}{M}-\pi\right) \tag{2}$$

The Fourier series form of this p.d.f. is given by [6]

$$f_M(\phi)=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^n b_{(nM)}\cos(nM\phi) \tag{3}$$

The Fourier series coefficients b_m are a function of the SNR ρ, which is given by [5] as :

$$b_m=\frac{\sqrt{\rho\pi}e^{-\frac{\rho}{2}}}{2}\left[I_{\frac{m+1}{2}}\left(\frac{\rho}{2}\right)+I_{\frac{m-1}{2}}\left(\frac{\rho}{2}\right)\right] \tag{4}$$

$I_v(x)$ is the modified Bessel function of order v, which is of an integer plus a half order. It is found that as m is increased b_m decreases, and as ρ is decreased the separation between harmonic magnitudes increases [6]. At low SNR the Fourier series is dominated by the mean and first harmonic. This final property will be used when evaluating the probability of false classification.

2.1 Histogram Representation

The phase samples $\phi(n)$ are used to build up a phase histogram with N bins and L samples to approximate the p.d.f.. The following theory characterises the error between the true p.d.f. and the histogram approximation.

It is known [7] that the variance σ_i^2 between the true p.d.f. and the histogram estimate for a particular histogram bin i is :

$$\sigma_i^2 \approx \frac{1}{L\Delta} f(\phi_i) \quad (5)$$

where L is the number of samples, Δ is the bin width, which is assumed to be small, and i is the histogram bin number. By virtue of the central limit theorem, the errors of all the bins will be normally distributed for large L. The mean variance of the error terms is expressed by :

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}\sigma_i^2 \quad (6)$$

where $\hat{\sigma}^2$ is the noise variance of the N bin histogram. As the histogram has equally spaced points in the interval $[-\pi,\pi]$,

$\Delta = \frac{2\pi}{N}$. It can be shown that the noise variance is :

$$\hat{\sigma}^2 \approx \frac{N}{4\pi^2 L} \quad (7)$$

2.2 Discrete Fourier Transform

The N points of the histogram are operated on by the discrete time Fourier transform to exploit spectral peaks corresponding to the harmonic terms. N is made to be a power of 2 in order that the harmonic terms will coincide with the frequency bins, thus avoiding the effects of spectral leakage. The bin number corresponds to the harmonic number (where the D.C. component is on bin 0), and from (1) it is seen that M level PSK will be characterised by a series of spectral lines on the bins which are a multiple of M.

It can be shown by the sampling theorem that harmonics of order $\frac{N}{2}$ and higher will be subjected to aliasing. In order to avoid the aliasing of the fundamental harmonics for any of the PSK schemes presented to the system for classification, N must be at least four times the highest symbol number.

The magnitude squared of the DFT is used as it is simple to calculate, and this is scaled by $\frac{4}{N^2}$ to produce D(k) which will provide magnitude squared values for the harmonic terms. When a harmonic component is not present in a bin that bin has only AGWN present and D(k) will be Rayleigh distributed [8] with variable y and p.d.f. :

$$p(y) = \pi^2 L \exp\left(-\pi^2 L y\right) \quad y>0 \quad (8)$$

This result shows that the noise floor is independent of the number of histogram bins N and implies that N may be made large enough to remove any significant effects of aliasing without affecting the noise floor.

When a frequency bin is occupied by a harmonic signal with the histogram noise, the bin is distributed with a non-central Chi-squared distribution, with two degrees of freedom [9]. It can be shown that the distribution g(x) of a bin D(k) containing a harmonic of amplitude $\frac{b_m}{\pi}$ and the histogram noise is given by :

$$g(x) = \pi^2 L \exp\left(-L\left[b_m^2 + \pi^2 x\right]\right) I_0\left(2\pi b_m L\sqrt{x}\right) \quad x>0 \quad (9)$$

Where $I_0(z)$ is the modified Bessel function of zero order. It should be noted that this expression is also independent of N.

2.3 Classification

The classification is achieved by finding the maximum DFT magnitude for the bins which are of interest, $D(\alpha_n)$ where α_n is the number of states in the n^{th} PSK signal. The classified signal is M-PSK where :

$$\alpha_M \in MAX\left[D(\alpha_n)\right] \quad (10)$$

e.g. when 1,2,4 & 8 PSK are to be classified, bins 1,2,4 & 8 of D(k) are examined, and if bin 4 is the maximum then the signal is classified as 4 PSK.

2.4 Probability Of False Classification

Consider the bin containing the signal x with distribution g(x), and n noise bins which are i.i.d. with distribution p(y). The probability that the signal lies in the interval $x, x+\delta x$ is given by

$$g(x)\delta x \quad (11)$$

The condition for correct classification is that the noise signals are less than x. The probability of correct classification in the interval is therefore :

$$g(x)[1-\Phi(x)]\delta x \quad (12)$$

where

$$\Phi(x) = \int_x^\infty p(y)dy \quad (13)$$

When all of these contributions are summed and in the limit of $\delta x \rightarrow 0$, the probability of correct classification is given by :

$$p_{corr} = \int_0^\infty g(x)[1-\Phi(x)]^n dx \tag{14}$$

Which can be re-written as :

$$p_{corr} = \sum_{i=0}^{n} \frac{n!(-1)^i}{(n-i)!i!} \int_0^\infty g(x)\Phi^i(x)dx \tag{15}$$

The probability of error is given by :

$$p_{err} = 1 - p_{corr} = \sum_{i=1}^{n} \frac{n!(-1)^{i+1}}{(n-i)!i!} \int_0^\infty g(x)\Phi^i(x)dx \tag{16}$$

From (8), $\Phi(x)$ is given by :

$$\Phi(x) = \exp(-\pi^2 Lx) \tag{17}$$

Using (9) and (17) and [10] it can be shown that

$$\int_0^\infty g(x)\Phi^i(x)dx = \frac{1}{i+1}\exp\left[-\frac{ib_m^2 L}{i+1}\right] \tag{18}$$

Therefore the error probability is given by :

$$p_{err} = \sum_{i=1}^{n} \frac{n!(-1)^{i+1}}{(n-i)!(i+1)!}\exp\left[-\frac{ib_m^2 L}{i+1}\right] \tag{19}$$

3. RESULTS

Plots of error probability against SNR are given in figure 2 for the case of CW, BPSK, QPSK and 8PSK being examined with a sample length of 1024 points. This is compared with simulated results, and it is found that the model is accurate for CW and BPSK, but QPSK deviates slightly, and 8PSK deviates further from the theory which indicates that the Gaussian assumption of the noise becomes less accurate. However the simulated results show that the error probability is better than that which the theoretical model suggests, and the two tend to converge at error probabilities less than 1% which are the main areas of interest.

This is compared with classification using the 8^{th} statistical moment [2] (figure 3) and at a 1% error probability. The first column of Table 1 shows the SNR gain of the new technique and it is found that the new technique is better in every case except 8 PSK. This is because the statistical moments technique assumes that a signal with a moment greater than that of 8PSK will not be present. The second column is the same comparison when 16 PSK is also included, and it is found that the new technique performs better in all cases.

Finally a comparison is drawn with the method of maximum likelihood classification [4] (figure 4) and it is found that the proposed method is close in error performance in all cases except that of 8PSK where the method is outperformed by 6.5 dB and lies close to the error performance of QPSK. It should be noted that the 'optimum' method requires a heavy computational overhead for each sample along with knowledge of the SNR of the signal.

4. CONCLUSIONS

A new method has been presented for the classification of multi-level PSK signals which requires no prior knowledge of the SNR of the signal unlike other deterministic methods. The algorithm is extremely simple and fast to implement requiring no complicated thresholding calculations. The error performance is found to be good and in certain cases it out-performs more complicated techniques. The technique works well at high and low SNR, and is proposed as an attractive method for the classification of PSK signals.

5. REFERENCES

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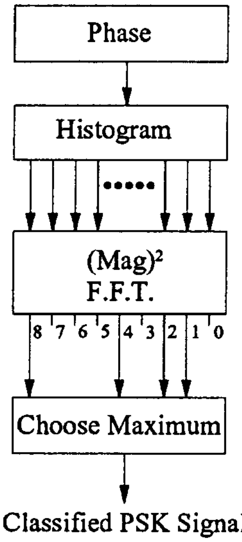

Figure 1 : Algorithmic description

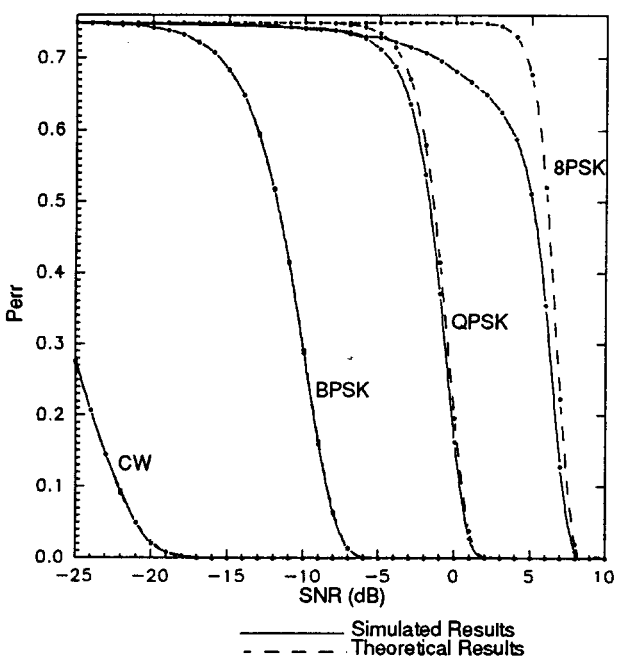

Figure 2 : Plots of classification error probability against SNR For CW, BPSK, QPSK, 8PSK for the DFT of phase histogram method

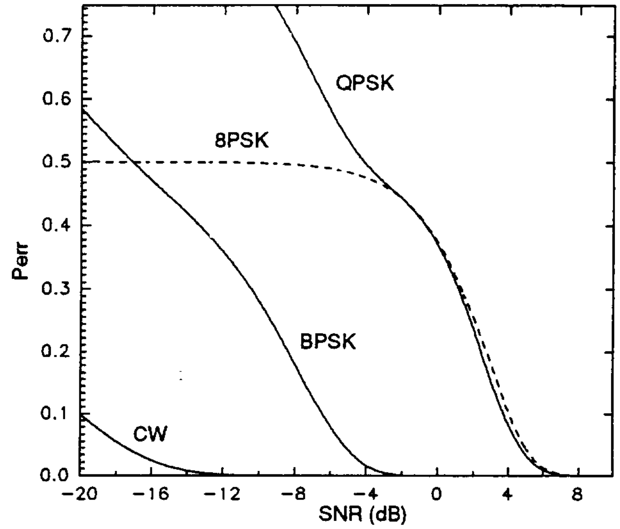

Figure 3 : Plot of error probability v SNR for the statistical moment classifier

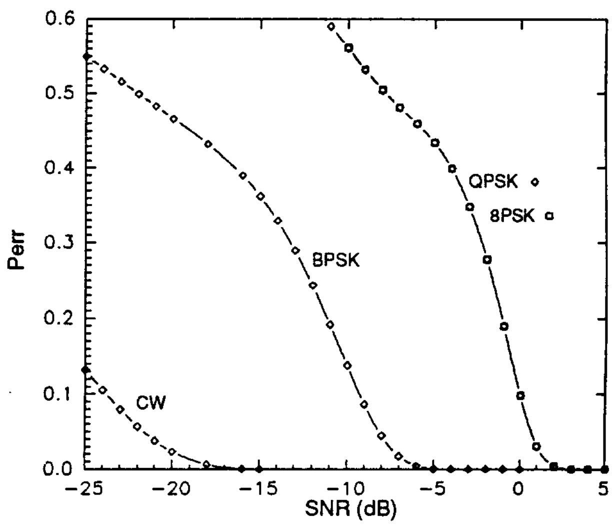

Figure 4 : Plot of error probability v SNR for the optimum classifier

	CW-8PSK	CW-16PSK
CW	4.3	3.9
BPSK	3	2.7
QPSK	4.3	4.2
8PSK	-2.5	Large
16PSK		Large

Table 1 : SNR gain (dB) of presented method, compared to 8th statistical moment when the error probability is 1%