PARALLEL FEEDFORWARD EQUALIZATION - A NEW NONLINEAR ADAPTIVE ALGORITHM

Jian Zhan, Jin S. Zhang, Fu Li
Portland State University
Portland, OR 97207-0751

Zhigang Fan
Xerox Corporation
Webster, NY 14580

ABSTRACT

It was proved in [1] that feedforward schemes are universally capable of approximating any measurable function to any desired degree of accuracy. In this paper, we propose a new realization of such feedforward scheme to the channel equalization problem - Parallel Feedforward Equalization (PFE). An important feature of the new approach is the decomposition of any equalization into linear and nonlinear components. the new approach chooses $F_j(\cdot)$ $(j = 1, \cdots)$ from a family of nonlinear functions to approximate the nonlinear component decomposed from the desired mapping $f(\cdot)$. The other new idea proposed in this paper is a measure called Nonlinearity Distribution which characterizes the nonlineality in multipath fading channels. The architecture of the new equalization consists of parallel feedforward nonlinear filters, each of them has a specifically tailored nonlinear function $F_j(\cdot)$.

1. INTRODUCTION

Adaptive equalization can be used to improve digital data transmission on wireless links with time-varying multipath distortion, and is broadly defined as a signal processing method for the mitigation of intersymbol interference (ISI) in high-rate data transmission over unknown channels. There are two general types of equalizations: linear and nonlinear equalizations. Attempting to compensate for the channel distortion, the linear equalizer places a large gain in the vicinity of the spectral null and, as a consequence, significantly enhances the additive noise present in the received signal. Nonlinear equalizers are often used in mobile communication where the channel distortion is too severe for a linear equalizer to handle. Actually, a linear equalizer dose not perform well for the channels with spectral nulls in their frequency response. During the past decades, there were three major nonlinear equalization methods developed: a decision feedback equalization (DFE) [2], a symbol-by-symbol detection algorithm based on the maximum a posteriori probability (MAP) criterion [3], and a sequence detection algorithm (MLSE) criterion [4]. The MAP and MLSE require the knowledge of channel characteristics and statistical distribution of the noise which corrupts the signal. For time-varying channels in which the intersymbol interference (ISI) spans many symbols, these probabilistic algorithms become impractical due to an exponentially growing computational complexity with ISI span. In DFE, the nonlineality in the filter is unknown. The basic idea is that once an information symbol has been detected, the ISI which causes on future symbols is estimated and subtracted out prior to the implementation of the direct-form structure of the DFE which consists of a feedforward filter (FFF) and a feedback filter (FBF). The latter is driven by decisions of the output of the detector and its coefficients are adjusted to cancel the ISI on the current symbol that results from past detected symbols. The coefficient adjustment is performed in the linear equalizer by the relatively simple gradient LMS algorithm or by the fast converging RLS algorithm. The question is, in the DFE, how many taps in FFF and FBF have to be implemented in terms of different circumstance with different ISI? Is it possible to know how severe the channel distortion occurs in current situation? It is desirable to have some kind of measures to quantify the nonlineality of the mapping between the input and desired output data. In this paper we propose a new nonlinear adaptive algorithm of parallel feedforward equalization (PFE) which decomposes the given input and output data mapping into linear and nonlinear components. This algorithm is self-organized approach in which the linear filter with fixed taps is first constructed and then coefficients are adjusted by LMS or RLS. If the desired reduction of ISI is not reached, the nonlinear approximation is added to further mitigate ISI. If the desired mitigation of ISI is still not satisfied, a parallel filter with same architecture is added. In principle, this algorithm can reach arbitrary accuracy of mitigating ISI.

2. THE NEW NONLINEAR ADAPTIVE ALGORITHM OF PARALLEL FEEDFORWARD EQUALIZATION

Equalization by adjusting the coefficients of filter adaptively can be considered as mapping $f(\cdot)$: $R_i^n \rightarrow R_i$, with n delay elements. The given channel, no matter how severe the distortion of signal occurs, is a mathematical model of arbitrary data mapping pairs. The capability of feedforward schemes to approximate any desired precision is proven in [1]. The problem is what kind of algorithm is used to implement the arbitrary approximation. Our new algorithm is aiming at addressing this issue.

2.1 Linear Component of PFE in Equalizer

It is known that an optimum receiver for a digital communication signal consists of matched filters which sample the received signals periodically at the symbol rate. If the received signal samples are corrupted by intersymbol interference, the symbol-spaced samples are further processed by either a linear or nonlinear equalizer. The frequency-selective channel characteristics in mobile radio generally result in channel spectral nulls. As a consequence, linear equalizers are ruled out since it is generally recognized that their performance is poor on such channel characteristics. The question is, for the time-varying channel model, when or where the channel characteristics become severe and need nonlinear equalizer to handle? If the fading multipath channel is implemented well with linear equalizer, it is not necessary to use nonlinear equalizer and pay the price of the increasement of computational complexity. Along with this idea, It is obvious that the fixed schemes of nonlinear equalizer such as DFE, MLSE, and MAP do not fit to the real world due to the fact how many taps needed in equalizer design are unknown. The idea of adapting both the scheme and coefficients of equalizer comes up naturally, that is an underlying thought of PFE algorithm.

As mentioned above, an equalizer can be considered a arbitrary mapping $f(\cdot)$: $R_i^n \rightarrow R_i$. For any given mapping data pairs, a linear approximation is implemented first. The LMS algorithm is used to derive weights W shown in Fig.1.

$$\begin{aligned} w_j(n) &= w_j(n-1) + \mu(-\partial E(w)/\partial w_j) \\ &= w_j(n-1) + \mu \sum_{i=1}^{r} (y_i - y_i^s) x_{ij} \\ (j &= 1, ..., n) \end{aligned} \quad (1)$$

This step of estimation is optimal in the sense of LMS. The mean squared error $E = \sum_{i=1}^{r} e_i^2$ is derived to measure if the linear equalizer satisfies the required estimation. If, in some circumstances, the linear equalizer is enough to mitigate ISI, the equalization stops in this step. The computational complexity is only proportional to $2N$, where N is the total equalizer length. If the accuracy of approximation using linear scheme does not reach the desired precision, a nonlinear scheme is added to mitigate ISI. Actually this step reveals the linear component of PFE. The coefficients in linear equalizer, as linear component parameters, are determined by conventional algorithm LMS

2.2 Nonlinear Component of PFE in Equalizer

After the linear equalization is implemented in first step, we come to an important issue - nonlinear equalization in the equalizer.

Definition 2.1 Let $M \rightarrow N(X_i \in M, y_i \in N)$ be arbitrary mapping $\Phi : R_i^N \rightarrow R_i$ $i = 1, ..., r$, $N < r$, after linear optimal estimation (LMS), there are an actual output y_i^l and an set $V, y_i^l \in V$. There exists another mapping $f : V \rightarrow N$, then f is called nonlinear function for the given mapping Φ, and the curve of relationship between y_i^l and y_i is denoted Nonlinearity Distribution (ND).

This definition describes that an arbitrary mapping $\Phi : R_i^n \rightarrow R_i$ can be decomposed into two steps: linear mapping and nonlinear mapping. The relationship between y_i^l and y_i unveils information of what kind nonlinear functions should be used. If the channel distortion of trained data set is linear, then the ND should be a straight line. Since the time-varying channel may appear with arbitrary nonlinearity, the ND could be arbitrary, the approach to be developed in this algorithm should fit to any distorted channel. There have been several methods developed to choose nonlinear function. In this paper, the Legendre functions are used as a set of orthogonal functions to approximate arbitrary nonlinear function. The architecture of added nonlinear function is shown in Fig.2.

The Legendre function are :

$$\psi_{ij}(t) = \sqrt{j + 1/2} P_{ij}(t), \quad -1 \leq t \geq 1, \quad i = 1, 2 \quad (2)$$

$$P_{ij}(t) = \frac{1}{2^j j!} \frac{d^j}{dt^j} (t^2 - 1)^j, \quad j = 0, 1, 2, \cdots \quad (3)$$

The adapting scheme is illustrated in Fig.3. When the multipath channel fading become worse, linear equalizer performs poorly, the errors are larger than one can tolerated, a nonlinear function $F_i(\cdot)$ is added automatically to further reduce estimation error. This behavior of adaptation acts like the mechanism of self-organized. When the new nonlinear function $F_i(\cdot)$ is added, the previous linear scheme and coefficients are not changed. The first four Legendre functions are chosen to approximate $F_i(\cdot)$

$$f_i^{nl}(t) = C_{i0}\psi_{i0}(t) + C_{i1}\psi_{i1} + C_{i2}\psi_{i2}(t) + C_{i3}\psi_{i3}(t) \quad (4)$$

and parameters $C_{i0}, C_{i1}, C_{i2}, C_{i3}$ are determined by

$$C_{ij} = \int_{-1}^{1} F_i(t)\psi_{ij}(t)dt, \quad i = 1,2 \quad j = 0,1,2,3 \quad (5)$$

After nonlinear approximation, the error $e_i = (F_i(\cdot) - f_i^{nl})$ may still exists. If $\sum_{i=1}^{r} e_i^2$ is less than the desired error, the processing of adaptive equalization stops here. When the remained errors do not satisfy the required criteria, the another parallel feedforward filter should be added to further reduce the remained errors.

2.3 Parallel Feedforward Filter

As mentioned above, when the channel distortion is too severe to handle, and one nonlinear function f_i^{nl} alone is not enough to mitigate the distortion, a parallel feedforward filter is connected as shown in Fig.4. This self-organized behavior makes the PFE an universal approximator. The algorithm used in the parallel equalizer is same as the first one. The difference between the first and second feedforward filter is that the remained error of the first feedforward equalizer is used as the desired outputs to be approximated in the second (parallel) feedforward equalizer. When the added linear parallel equalizer can reach required precision of estimation, it is necessary to cascade nonlinear function $F_2(\cdot)$. In this sense, this new algorithm organizes the architecture of equalizer and automatically adding the linear and nonlinear blocks to reach the desired accuracy. The computational complexity of nonlinear approximation is only $4N$. This is also analytical approach instead of using recursive calculation. The simulation demonstrates the self-organization equalization is effective and useful to decrease distortion in the channel, in fact, it is an arbitrarily accurate estimator.

3. SIMULATION

The given random mapping pairs $R_i^6 \rightarrow R_i$, $i = 0, \cdots, 14$

$$\begin{bmatrix} -.98 & .15 & .55 & 1.0 & .68 & .53 \\ -.25 & -.98 & .15 & .55 & 1.0 & .68 \\ -.65 & -.25 & -.98 & .15 & .55 & 1.0 \\ -.70 & -.65 & -.25 & -.98 & .15 & .55 \\ .86 & -.70 & -.65 & -.25 & -.98 & .15 \\ .83 & -.86 & -.70 & -.65 & -.25 & -.98 \\ .28 & -.83 & -.86 & -.70 & -.65 & -.25 \\ -.46 & .28 & -.83 & -.86 & -.70 & -.65 \\ .35 & -.46 & .28 & -.83 & -.86 & -.70 \\ .015 & .35 & -.46 & .28 & -.83 & -.86 \\ -.55 & .015 & .35 & -.46 & .28 & -.83 \\ -.27 & -.55 & .015 & .35 & -.46 & .28 \\ .84 & -.27 & -.55 & .015 & .35 & -.46 \\ -.93 & .84 & -.27 & -.55 & .015 & .35 \end{bmatrix} \Longrightarrow \begin{bmatrix} .71 \\ .713 \\ -.499 \\ -.0043 \\ -.1726 \\ -1.00 \\ .0223 \\ -.7603 \\ -.7016 \\ .1621 \\ -.2819 \\ -.7581 \\ .4404 \\ .5874 \end{bmatrix} \quad (6)$$

The steps of the proposed algorithm are :

1. Using RLM linear approximation, the parameters of weights are [.132, .2184, .0188, .1595, .2953, .1808]. The mean square error (MSE) of linear is $E = \sum_{i=0}^{14}(e_i^l)^2$ =2.5284. The nonlinearity distribution (ND) is shown in Fig.5.

2. Compressing data scale from $[-1, 1]$ to $[-.75, .75]$ along x pivot, and rotating the nonlinear curve in ND -45 degree to get nonlinear piecewise curve $F_i(\cdot)$ as shown in Fig.6, this is because the Lengendre basis functions almost equal to one, when $x \rightarrow 1$ and $x \rightarrow -1$.

3. Using Legendre orthogonal functions to approximate $F_i(\cdot)$, the coefficients of basis functions are C_1= [.0282, .0412, -.0342, -.0296], the output y_1^{nl} is

$$\begin{aligned} y_1^{nl} = (1/2)^2 c_{10} + (3/2)^2 c_{11} y_1^l + \\ (5/2)^2 c_{12}(3/2(y_1^l)^2 - 1/2) \\ +(7/2)^2 c_{13}(5/2(y_1^l)^3 - 3/2 y_1^l) \end{aligned} \quad (7)$$

where the y_1^l is the output linear filter.

4. Rotating the nonlinear curve 45 degree and expanding the data scale from $[-.75, .75]$ to $[-1.1]$. After nonlinear approximation, the MSE is $E = \sum_{i=0}^{14}(e_i^{nl})^2$ = .4936, the accuracy is improved a lot as shown in Fig.7. If the accuracy of the first feedforward filter is not satisfied, the second feedforward filter is established with same algorithm as described previously. After the second feedforward filter is added, the MSE $E = \sum_{i=0}^{14}(e_i^{nl})^2$ = .0737, the ND is shown in Fig.8.

REFERENCES

[1] K. Zhou, J. G. Proakis, and F. Ling, "Decision-feedback equalization of time dispersive channels with coded modulation," IEEE Transactions on Communications, vol. 38, pp.18-24, Jan. 1990.

[2] K. Hornik, M. Stinchcombe and H. White, "Multilayer Feedforward Networks are Universal Approximators" Neural Networks, Vol. 2, pp. 359-366, 1989.

[3] K. Abend and B. D. Fritchman, "Statistical Detection for communication channels with intersymbol interference," Proc. IEEE, pp.779-785, May, 1970.

[4] G. D. Forney, Jr.,"Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interferences," IEEE Transactions on Information Theory, vol IT-18, pp.363-378, May, 1972.

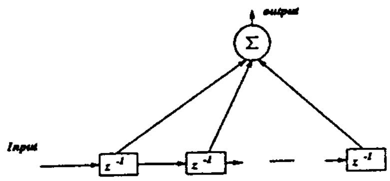

Fig.1 Traditional FIR Linear Filter

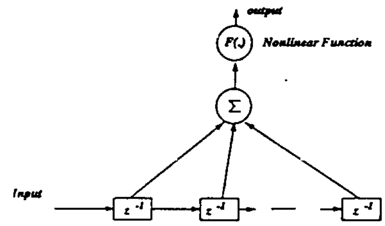

Fig.2 Achitecture of Nonlinear Equalizer Proposed Here

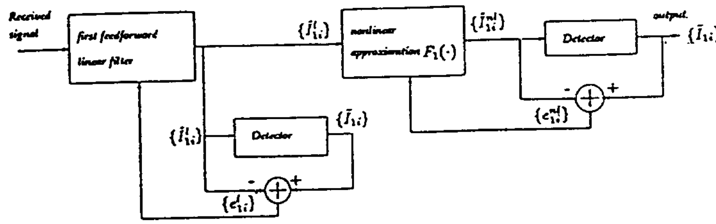

Fig.3 The first adaptive nonlinear feedforward equalizer with the new algorithm

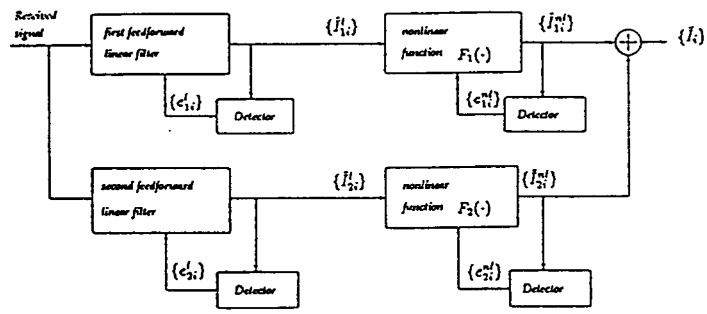

Fig.4 The new adaptive nonlinear equalizer with parallel NLFF architecture, where $\{\tilde{I}^{l}_{2i}\} = \{e^{nl}_{1i}\}$

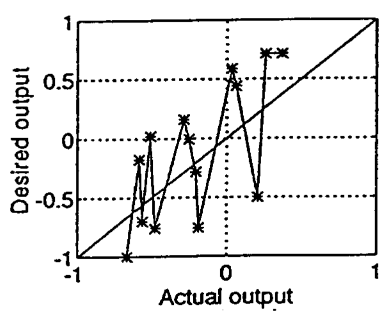

Fig.5 ND after the first linear feedforward filter

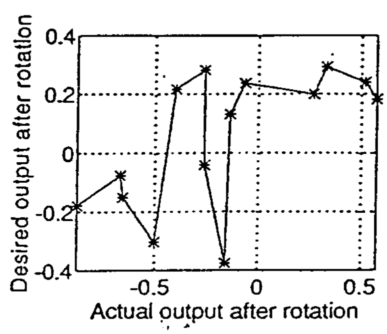

Fig.6 Nonlinear piecewise curve $F_1(\cdot)$

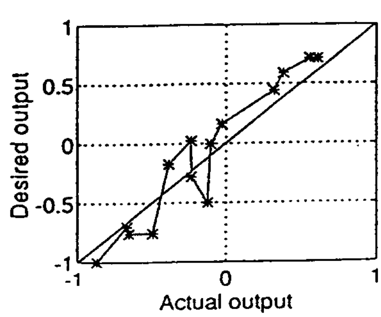

Fig.7 NDC after the first nonlinear approximation

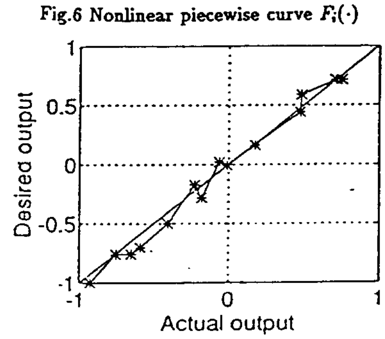

Fig.8 ND after adding the parallel nonlinear filter